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Super Twisting Control of Linear Induction Motor Considering End Effects with Unknown Load Torque

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Abstract— **This paper proposes a super twisting sliding mode control technique for linear induction motors (LIMs) with unknown load torque, taking into consideration the dynamic end effects. First, LIM's dynamic end effects are presented by Ducan's T-model, then following this model is controlled by a designed super twisting controller (STC) for flux tracking and speed tracking purpose. Simultaneously, an open loop flux observer and a reduced order load torque observer are designed based on Lyapunov's analysis. Finally, simulation results show that the designed observer-based super twisting controller has great tracking performance and the system is robust with disturbances and uncertainties, and flux observer and reduced torque observer show good estimate performance with nominal system and input-to-state stability (ISS) property with uncertainty system.**

I. INTRODUCTION

Linear induction motors (LIMs) are widely applied in the industrial applications, such as elevators, maglev trains, electric vehicles and medical equipments, etc [1]. Comparing with rotary induction motor (RIM), the most obvious advantage of LIM is a simple structure which has no gears and does not need mechanical rotary-to-linear converters. Different with RIM, the whole structure of LIM is asymmetric and its secondary part consists of a sheet of aluminum with a back core of iron. When the primary part moves, new continuous eddy currents appear at the entry of the primary part and disappear at the exit part [2]. This case causes the varies of equivalent inductances and resistances, which is the so-called dynamic end effects [3-4].

To describe this case above, Ducan [5] introduced an end effect factor Q to describe the phenomena when airgap flux grows and decays gradually varies, showing the per phase equivalent circuit by adjusting the magnetising branch of the equivalent circuit of the RIM. The equivalent state space equations were given to describe the equivalent mathematical model of LIM.

Because of the complexity of LIM's model, the control methods strongly challenge the classical control strategies from RIMs to LIMs. It is well known that the control objective of LIM is to achieve speed tracking and flux tracking. Considering end effects, several researchers made efforts to apply apposite control methods into LIM in the past several years. Kang [2] applied the classical field oriented control (FOC) method into it. However, he simplified the model by neglecting the eddy-current loss, this control scheme can not accurate reflect the LIM's dynamic response. After that Pucci introduced state space equation of the model [6] and controlled this model with FOC [7]. Other control methods are proposed to apply into LIM, such as input-output feedback linearization control [8], neutral network control [9], combined vector and direct thrust control [10]. However in practical, those control methods are not robust to parameter uncertainties and disturbances, such as resistance variations, load torque disturbance, etc. These controllers' properties will become worse when disturbances occur.

To guarantee the LIM's performance even in worse case, different nonlinear robust control methods are proposed to settle this problem. Traditional sliding mode control (SMC) [11] is a type of nonlinear control and robust to system uncertainties and parameter variations, it is widely used in aircrafts and robotics etc [17]. However, this method may appear chattering phenomenon, due to the discontinuous control action in digital equipment generating a finite sampling frequency. A solution to deal with this problem is super twisting algorithm [12]. The STA with the advantage of reducing the chattering problem robustness, is generally applied into induction motors (IMs) [13-16].

Another considering factor of LIM is flux estimation because it is well known that the rotor flux can not be measured in the practical system. When without considering end effects, a lot of flux observer methods are given to estimate flux [16], when considering the influence of end effects, this situation will become more complex. Similarly, load torque estimation is also a research problem. As a result of the mechanical dynamic of the LIM is slower than electric one in practical system, so we can assume the derivative of unknown load torque is zero.

In this paper, we proposes a super twisting sliding mode control (STC) technique for linear induction motors (LIMs) with unknown load torque, taking into consideration the dynamic end effects. To estimate the rotor flux, an open rotor flux observer is designed in this paper based on Lyapunov's theory. For the estimation of load torque, based on assumption of the derivative of load torque is zero, a reduced order load torque observer is designed based on Lyapunov's method.

The remaining of this paper is organized as follows: in Section II, the mathematical model of LIM with end effects is presented. In Section III, a super twisting controller is

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designed for the linear induction motor considering end effects for the tracking of desired speed and flux modulus, then following an open rotor flux observer and a reduced torque observer are presented. Simulation results are presented in Section IV. Finally, some comments conclude the work in Section V.

II. Linear Induction Motor Model with Dynamic End Effects

The space-vector dynamic model of the LIM, taking into consideration its dynamic end effects, in the inductor part flux reference frame (α,β) is given as follows [6]:

$$\dot{i}_{s\alpha} = -\gamma i_{s\alpha} + \beta\alpha\psi_{r\alpha} + \beta\frac{n_p\pi}{h}v\psi_{r\beta} + \frac{u_{s\alpha}}{\delta} \quad (1)$$

$$\dot{i}_{s\beta} = -\gamma i_{s\beta} + \beta\alpha\psi_{r\beta} - \beta\frac{n_p\pi}{h}v\psi_{r\alpha} + \frac{u_{s\beta}}{\delta} \quad (2)$$

$$\dot{\psi}_{r\alpha} = -\eta\psi_{r\alpha} + \varsigma i_{s\alpha} - \frac{n_p\pi}{h}v\psi_{r\beta} \quad (3)$$

$$\dot{\psi}_{r\beta} = -\eta\psi_{r\beta} + \varsigma i_{s\beta} + \frac{n_p\pi}{h}v\psi_{r\alpha} \quad (4)$$

and the mechanical equation of the motion of LIM is:

$$\dot{v} = F_e - (D/M)v - T_L/M \quad (5)$$

where F_e is the electromagnetic thrust, is defined as [10]:

$$F_e = \mu(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}) \quad (6)$$

where v is the motor speed, $u_{s\alpha}$ and $u_{s\beta}$ are the stator voltages, $i_{s\alpha}$ and $i_{s\beta}$ are the stator currents, $\psi_{r\alpha}$ and $\psi_{r\beta}$ are the rotor fluxes, T_L is the load torque, n_p is the number of pole pairs, and M is the motor mass. Moreover, the variables $\gamma, \alpha, \beta, \varsigma, \eta, \delta, \mu$, $\hat{T}_r$ are defined as follows:

$$\gamma = \frac{1}{\hat{\sigma}\hat{L}_s}\left[R_s + \hat{R}_r(1 - \frac{\hat{L}_m}{\hat{L}_r}) + \frac{\hat{L}_m}{\hat{L}_r}(\frac{\hat{L}_m}{\hat{T}_r} - \hat{R}_r)\right]$$

$$\alpha = (\frac{1}{\hat{T}_r} - \frac{\hat{R}_r}{\hat{L}_m}), \beta = \frac{\hat{L}_m}{\hat{\sigma}\hat{L}_s\hat{L}_r}, \varsigma = (\frac{\hat{L}_m}{\hat{T}_r} - \hat{R}_r)$$

$$\eta = \frac{R_r + \hat{R}_r}{\hat{L}_r}, \delta = \hat{L}_s\left(1 - \frac{\hat{L}_m^2}{\hat{L}_r\hat{L}_s}\right), \mu = \frac{3n_p\pi\hat{L}_m}{2Mh\hat{L}_r}$$

and $Q, f(Q), \hat{R}_r, \hat{L}_m, \hat{L}_s, \hat{L}_r, \hat{T}_r$ are given:

$$Q = \frac{\tau_m R_r}{(L_m + L_{\sigma r})v}, f(Q) = \frac{1 - e^{-Q}}{Q}$$

$$\hat{R}_r = R_r f(Q), \hat{L}_m = L_m[1 - f(Q)]$$

$$\hat{L}_s = L_{\sigma s} + \hat{L}_m, \hat{L}_r = L_{\sigma r} + \hat{L}_m$$

$$\hat{T}_r = \frac{\hat{L}_r}{R_r + \hat{R}_r} = \frac{L_{\sigma r} + L_m(1 - f(Q))}{R_r + R_r f(Q)}$$

III. Controller and Observer

The control objective is to force the linear induction motor speed v and the squares of the rotor flux modulus $\psi_m = \psi_{r\alpha}^2 + \psi_{r\beta}^2$ to track the desired references v_{ref} and $\psi_{m,ref}$, even in the case of load torque disturbance and parameter uncertainties. We first solve the control problem, then solve the problem of flux observer and torque observer.

A. *Super Twisting Controller*

In order to solve the control problem, we derive the expressions of the tracking error dynamics $z_1 = v - v_{ref}$ and $z_2 = \psi_m - \psi_{m,ref}$, the error dynamics can be written as follows:

$$\dot{z}_1 = \mu(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}) - (D/M)v - T_L/M - \dot{v}_{ref} \quad (7)$$

$$\dot{z}_2 = -2\eta\psi_m + 2\varsigma(\psi_{r\alpha}i_{s\alpha} + \psi_{r\beta}i_{s\beta}) - \dot{\psi}_{m,ref} \quad (8)$$

Now, define the vector $\chi_1 = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T$ and its dynamics results

$$\dot{\chi}_1 = f_1 + GI + d \quad (9)$$

where $I = \begin{bmatrix} i_{s\alpha} & i_{s\beta} \end{bmatrix}$, $f_1 = \begin{bmatrix} f_{11} & f_{21} \end{bmatrix}^T$, $d = \begin{bmatrix} -T_L/M & 0 \end{bmatrix}^T$, and

$$G = \begin{bmatrix} -\mu\psi_{r\beta} & \mu\psi_{r\alpha} \\ 2\varsigma\psi_{r\alpha} & 2\varsigma\psi_{r\beta} \end{bmatrix} \quad (10)$$

with

$$f_{11} = -(D/M)v - \dot{v}_{ref} \quad (11)$$

$$f_{21} = -2\eta\psi_m - \dot{\psi}_{m,ref} \quad (12)$$

To avoid chattering phenomena, we introduced sigmoidal functions approximate the classical sign function, defined as:

$$S_{\varepsilon_i}(z_i) = tanh(z_i/\varepsilon_i), i = 1, 2 \quad (13)$$

By choosing sufficiently small ε_i, we can guarantee this function approximate classical sign function as follows:

$$\lim_{\varepsilon\to 0} S_\varepsilon(z) = sign(z) \quad (14)$$

The sigmoidal functions are robust to unknown unmatched and bounded perturbations, these functions decompose the control law into several sub-problems of lower order.

For the stabilization of equation one proposes a desired dynamics as

$$\dot{\chi}_1 = f_1 + GI + d = -K_1 S_\varepsilon(\chi_1) \quad (15)$$

where $K_1 = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$, k_1 and k_2 are postive values to be designed. From equation (15) can calculate the current vector as reference signal. Define $I_{ref} = \begin{bmatrix} i_{s\alpha,ref} & i_{s\beta,ref} \end{bmatrix}^T$, while

$$I_{ref} = G^{-1}(-K_1 S_\varepsilon(\chi_1) - f_1 - \hat{d}) \quad (16)$$

The value of matrix G equals to

$$\det(G) = -2\mu\varsigma(\psi_{r\alpha}^2 + \psi_{r\beta}^2) = -\frac{3n_p\pi\hat{L}_m}{Mh\hat{L}_r}(\frac{\hat{L}_m}{\hat{T}_r} - \hat{R}_r)\psi_m \quad (17)$$

It is obvious that G^{-1} exits under the assumption that $\psi_m \neq 0$. The reference currents guarantee that z_1, z_2 decay asymptotically to zero. In order to force the stator currents to be equal to the stator reference signals. Define the variable $\chi_2 = \begin{bmatrix} z_3 & z_4 \end{bmatrix}^T = \begin{bmatrix} i_{s\alpha} - i_{s\alpha,ref} & i_{s\beta} - i_{s\beta,ref} \end{bmatrix}^T$, and their dynamics are:

$$\dot{\chi}_1 = -K_1 S_\varepsilon(\chi_1) + G\chi_2 \quad (18)$$

$$\dot{\chi}_2 = f_2 + U/\delta - \dot{I}_{ref} \quad (19)$$

where $f_2 = \begin{bmatrix} f_{12} & f_{22} \end{bmatrix}^T$, $U = \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}^T$ and

$$f_{12} = -\gamma i_{s\alpha} + \beta\alpha\psi_{r\alpha} + \beta\frac{n_p\pi}{h}v\psi_{r\beta} \tag{20}$$

$$f_{22} = -\gamma i_{s\beta} - \beta\frac{n_p\pi}{h}v\psi_{r\alpha} + \beta\alpha\psi_{r\beta} \tag{21}$$

A super twisting sliding mode controller is designed for perturbation and chattering elimination by choosing z_3, z_4 as sliding functions. One proposes the following super twisting controller:

$$u_{s\alpha} = -k_a|i_{s\alpha} - i_{s\alpha,ref}|^{\frac{1}{2}}sign(i_{s\alpha} - i_{s\alpha,ref}) + u_{s\alpha,1}$$

$$\dot{u}_{s\alpha,1} = -k_{a,1}sign(i_{s\alpha} - i_{s\alpha,ref})$$

$$u_{s\beta} = -k_b|i_{s\beta} - i_{s\beta,ref}|^{\frac{1}{2}}sign(i_{s\beta} - i_{s\beta,ref}) + u_{s\alpha,1}$$

$$\dot{u}_{s\beta,1} = -k_{b,1}sign(i_{s\beta} - i_{s\beta,ref}) \tag{22}$$

Differentiate the sliding functions twice:

$$\dot{z}_3 = f_{12} + u_{s\alpha}/\delta - \frac{di_{s\alpha,ref}}{dt}$$

$$\dot{z}_4 = f_{22} + u_{s\beta}/\delta - \frac{di_{s\beta,ref}}{dt}$$

$$\ddot{z}_3 = \dot{f}_{12} + \frac{d}{dt}\left(\frac{u_{s\alpha}}{\delta}\right) - \frac{d^2 i_{s\alpha,ref}}{dt}$$

$$\ddot{z}_4 = \dot{f}_{22} + \frac{d}{dt}\left(\frac{u_{s\beta}}{\delta}\right) - \frac{d^2 i_{s\beta,ref}}{dt} \tag{23}$$

We assume that there are positive constants, such that within the region the following inequalities hold $\forall t, x \in \mathrm{X}, u_{s\alpha}, u_{s\beta} \in \mathrm{Z}$:

$$\left|\dot{f}_{12} - \frac{d^2 i_{s\alpha,ref}}{dt}\right| \le \Phi_{s\alpha}, \left|\dot{f}_{22} - \frac{d^2 i_{s\beta,ref}}{dt}\right| \le \Phi_{s\beta}$$

$$0 < \Gamma_m \le \frac{1}{\delta} \le \Gamma_M \tag{24}$$

$\forall t, x \in \mathrm{X}, u_{s\alpha}, u_{s\beta} \in \mathrm{Z}$.

The corresponding sufficient conditions for the finite time convergence to the sliding manifolds are

$$k_{a,1} > \frac{\Phi_{s\alpha}}{\Gamma_m}, k_a^2 \ge \frac{4\Phi_{s\alpha}}{\Gamma_m^2}\frac{\Gamma_M(k_{a,1} + \Phi_{s\alpha})}{\Gamma_m(k_{a,1} - \Phi_{s\alpha})}$$

$$k_{b,1} > \frac{\Phi_{s\beta}}{\Gamma_m}, k_b^2 \ge \frac{4\Phi_{s\beta}}{\Gamma_m^2}\frac{\Gamma_M(k_{a,1} + \Phi_{s\beta})}{\Gamma_m(k_{a,1} - \Phi_{s\beta})} \tag{25}$$

then the sliding functions can keep globally uniformly stable by choosing suitable parameters.

B. Open Loop Rotor Flux Observer

In this part, $\hat{\psi}_{r\alpha}$ and $\hat{\psi}_{r\beta}$ are the rotor flux to be estimated, we can design the following open-loop rotor flux observer

$$\dot{\hat{\psi}}_{r\alpha} = -\eta\hat{\psi}_{r\alpha} + \varsigma i_{s\alpha} - \frac{n_p\pi}{h}v\hat{\psi}_{r\beta} \tag{26}$$

$$\dot{\hat{\psi}}_{r\beta} = -\eta\hat{\psi}_{r\beta} + \varsigma i_{s\beta} + \frac{n_p\pi}{h}v\hat{\psi}_{r\alpha} \tag{27}$$

which is a copy of the rotor flux dynamic equations in which the flux estimate replaces the true flux. It requires the measurements of (v, $i_{s\alpha}$,$i_{s\beta}$). Define the flux estimation errors as $\tilde{\psi}_{r\alpha} = \psi_{r\alpha} - \hat{\psi}_{r\alpha}$ and $\tilde{\psi}_{r\beta} = \psi_{r\beta} - \hat{\psi}_{r\beta}$.

The resulting estimation error dynamics are given by

$$\dot{\tilde{\psi}}_{r\alpha} = -\eta\tilde{\psi}_{r\alpha} - \frac{n_p\pi}{h}v\tilde{\psi}_{r\beta} \tag{28}$$

$$\dot{\tilde{\psi}}_{r\beta} = -\eta\tilde{\psi}_{r\beta} + \frac{n_p\pi}{h}v\tilde{\psi}_{r\alpha} \tag{29}$$

Now, we consider the Lyapunov function

$$V = \frac{1}{2}\tilde{\psi}_{r\alpha}^2 + \frac{1}{2}\tilde{\psi}_{r\beta}^2 \tag{30}$$

whose time derivative along the trajectories of equation (30) is given by

$$\dot{V} = -\eta(\tilde{\psi}_{r\alpha}^2 + \tilde{\psi}_{r\beta}^2) = -2\eta V = -\frac{2(R_r + \hat{R}_r)}{\hat{L}_r}V \tag{31}$$

integrating with respect to time on the time interval [0,t], we obtain

$$V(t) = V(0)e^{-2\eta t} \tag{32}$$

Assume the maximum speed is $V_{\max}$, then the scope of parameter η is

$$0 < \eta_{\min} < \eta = \frac{R_r + \hat{R}_r}{\hat{L}_r} < \eta_{\max} \tag{33}$$

which implies that the norm of the rotor flux error vector $\tilde{\psi}_r(t) = \begin{bmatrix} \tilde{\psi}_{r\alpha}(t) & \tilde{\psi}_{r\beta}(t) \end{bmatrix}^T$ decays exponentially with rate of convergence η, for any initial estimation error $V(0) = \frac{1}{2}\tilde{\psi}_{r\alpha}^2(0) + \frac{1}{2}\tilde{\psi}_{r\beta}^2(0)$.

C. Reduced First Order Torque Observer

In this part, a reduced order observer is designed to estimate load torque [17]. Due to the fact that mechanical dynamic of the motor is much slower than electric one, we assume the derivative of load torque is zero, then the motor mechanical equations are given as follows:

$$\dot{v} = \mu(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}) - (D/M)v - T_L/M \tag{34}$$

$$\dot{T}_L = 0 \tag{35}$$

Let the change of variable as follows:

$$\kappa = T_L + \lambda v(\lambda > 0) \tag{36}$$

Based on the above equation, we have

$$\dot{\kappa} = -(\lambda/M)\kappa + (\lambda^2/M)v - (D/M)v + \lambda\mu(i_{s\beta}\hat{\psi}_{r\alpha} - i_{s\alpha}\hat{\psi}_{r\beta}) \tag{37}$$

Let $\hat{\kappa}$ be an estimate of κ generated by

$$\dot{\hat{\kappa}} = -(\lambda/M)\hat{\kappa} + (\lambda^2/M)v - (D/M)v + \lambda\mu(i_{s\beta}\hat{\psi}_{r\alpha} - i_{s\alpha}\hat{\psi}_{r\beta}) \tag{38}$$

The estimate torque is given by

$$\hat{T}_L = \hat{\kappa} - \lambda v, \hat{\kappa}(0) = \hat{T}_L(0) + \lambda v(0) \tag{39}$$

Define $\tilde{\kappa} = \kappa - \hat{\kappa}$, $\tilde{T}_L = T_L - \hat{T}_L$, $\tilde{\psi}_{ra} = \psi_{ra} - \hat{\psi}_{ra}$, $\tilde{\psi}_{rb} = \psi_{rb} - \hat{\psi}_{rb}$, then the estimation error dynamics

$$\dot{\tilde{\kappa}} = -(\lambda/M)\tilde{\kappa} + \lambda\mu(i_{s\beta}\tilde{\psi}_{r\alpha} - i_{s\alpha}\tilde{\psi}_{r\beta}) \tag{40}$$

$$\tilde{T}_L = \tilde{\kappa} \tag{41}$$

Consider the Lyapunov function

$$V = \frac{1}{2}\tilde{\kappa}^2 \tag{42}$$

From previous part we assume $V_{\max}$ is maximum speed, then the scope of parameter μ is

$$0 < \mu_{\min} < \mu = \frac{3n_p\pi\hat{L}_m}{2Mh\hat{L}_r} < \mu_{\max} \tag{43}$$

The time derivative of V is

$$\begin{aligned}
&\dot{V} = -(\lambda/M)\tilde{\kappa}^2 + \lambda\mu(i_{s\beta}\tilde{\psi}_{r\alpha} - i_{s\alpha}\tilde{\psi}_{r\beta})\tilde{\kappa} \\
&\leq -(\lambda/2M)\tilde{\kappa}^2 + \left(\lambda\mu^2 M/2\right) \\
&(|i_{s\alpha}| + |i_{s\beta}|)(1 + |i_{s\alpha}| + |i_{s\beta}|)(\tilde{\psi}_{r\alpha}^2 + \tilde{\psi}_{r\beta}^2) \\
&\leq -(\lambda/2M)\tilde{\kappa}^2 + \left(\lambda\mu_{max}^2 M/2\right) \\
&(|i_{s\alpha}| + |i_{s\beta}|)(1 + |i_{s\alpha}| + |i_{s\beta}|)(\tilde{\psi}_{r\alpha}^2 + \tilde{\psi}_{r\beta}^2)
\end{aligned} \tag{44}$$

In practice, the currents $i_{s\alpha}(t), i_{s\beta}(t)$ are bounded on $t \in [0, \infty)$, from the previous section we know $\tilde{\psi}_{ra}(t), \tilde{\psi}_{rb}(t)$ exponentially converge to zero, which demonstrating that $\tilde{T}_L(t)$ exponentially converge to zero for any initial condition. When the perturbation terms $(\tilde{\psi}_{r\alpha}, \tilde{\psi}_{r\beta})$ are nonzero and bounded, its property ensures the input-to-state stability (ISS) property, namely $\tilde{\kappa}$ remains bounded, that means $\tilde{T}_L$ is bounded [18]. In fact the flux observer is based on the open flux loop flux observer (27-28), which is ensured to remain bounded.

IV. SIMULATION

In this section the performance of the proposed control scheme is simulated with Matlab/Simulink software. The nominal parameters of linear induction motor are given in Tabel I.

The variables $\gamma, \alpha, \beta, \varsigma, \eta, \delta$ and μ have related physical meanings, as explained in [7]. In Fig 1, the waveforms of those parameters are showed for the motor under simulation in a speed range varying between 0 and 5 m/s.

The gain parameters have been chosen for the controller as $k_a = 2500$, $k_{a,1} = 50000$, $k_b = 2500$, $k_{b,1} = 50000$, $\varepsilon_1 = 0.01$, $\varepsilon_2 = 0.01$, $k_1 = -100$, $k_2 = -50$, $\lambda = 500$.

TABLE I

PARAMETERS OF LINEAR INDUCTION MOTOR

Inductor resistance Rs [Ω]	11
Induced-part resistance Rr [Ω]	32.57
Inductor inductances Ls [H]	0.6376
3-phase magnetizing inductance Lm [H]	0.5175
Induced-part inductances Lr [H]	0.7578
Primary mass M [Kg]	20
Viscous friction D [m/s]	20
Pole-pairs n_p	3
Inductor length τ_m [m]	1.5
Pole pitch h [m]	0.1

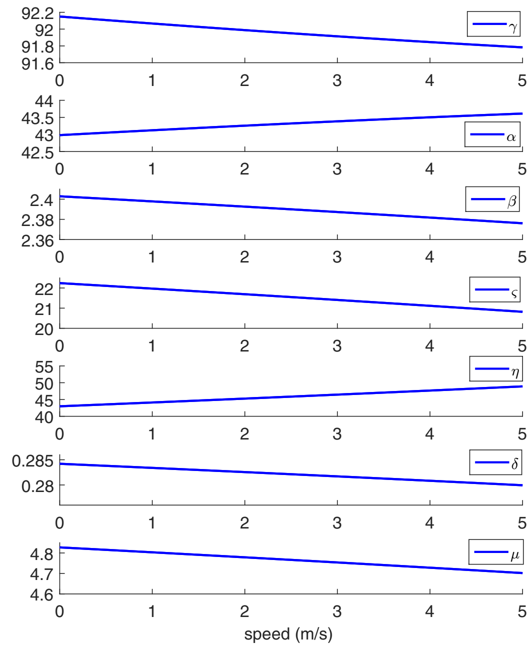
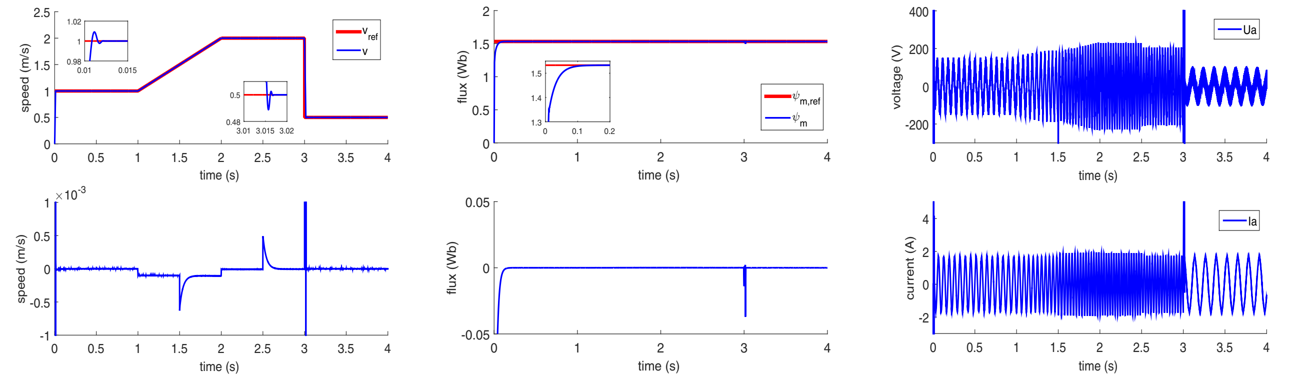


Fig. 1. Waveforms of $\gamma, \alpha, \beta, \varsigma, \eta, \delta$ and μ when speed v varies

A. Control Performance Analysis

In this part, a typical type of reference signal is given to test the performance of the observer-based super twisting controller, as shown in Fig 2.

To test the influence of external disturbance, an 100 Nm load force is added at $t = 1.5$ s and removed at $t = 2.5$ s. The simulation results for the reference and actual speed and speed tracking error are shown in Fig 2 (a). The simulation results for the reference and actual flux modulus and flux modulus tracking error are shown in Fig 2 (b). The primary voltage of u-phase U_a and current voltage of i-phase I_a are shown in Fig 2 (c). case As shown in Fig 2 (a), the speed tracking errors have a fast convergence rate, even in case of external load torque disturbance and parametric uncertainties. Satisfactory flux modulus tracking performances are shown in Fig 2 (b). In Fig 2 (c), it's easy to see that the phase and amplitude vary with different speed signal and no burr, and the primary voltage response and primary current evolve in a reasonable region.

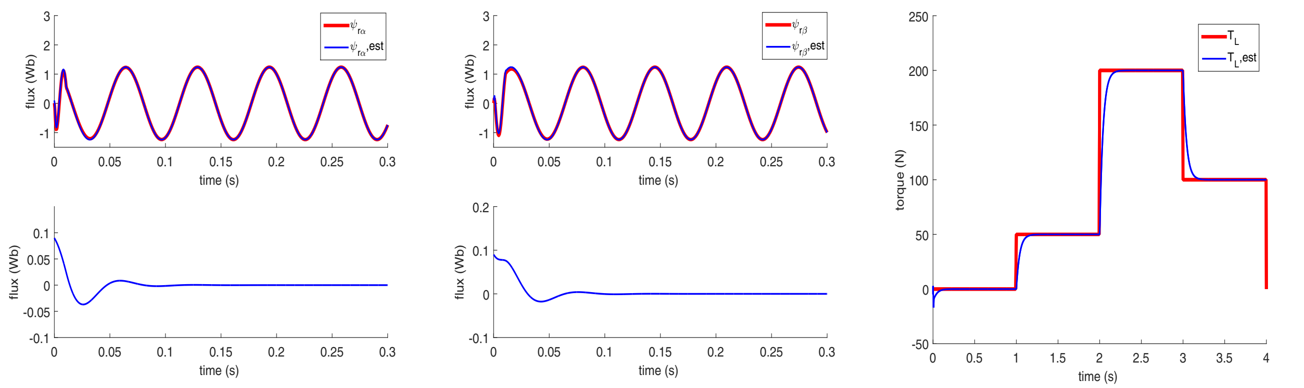


(a) Speed tracking performance with load force disturbance: reference and real speed; speed tracking error

(b) Flux tracking performance with load force disturbance: reference and real flux $|\psi_m|$; flux tracking error

(c) Primary voltage of one phase and primary current of one phase

Fig. 2. Tracking performance and control input with load disturbance

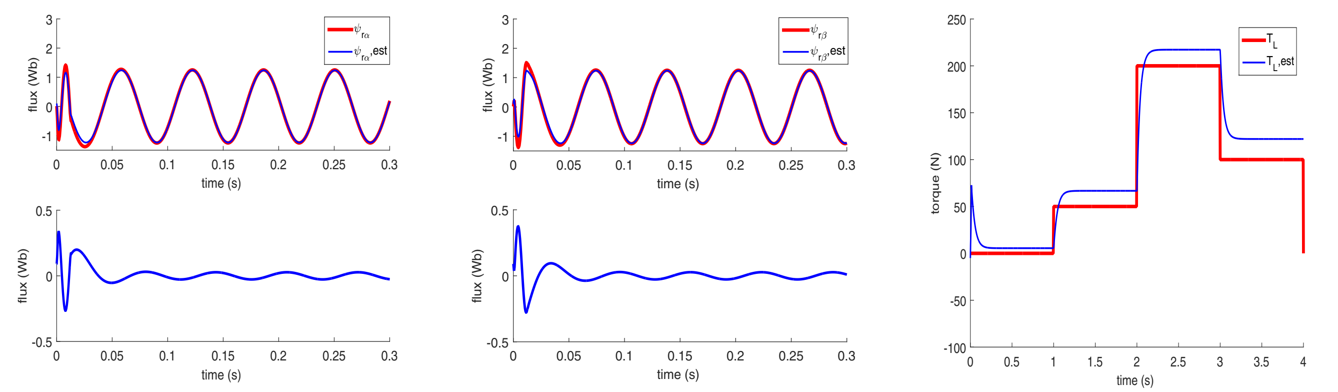


(a) Open-loop rotor flux observer: rotor flux $\psi_{r\alpha}$ and its estimate; rotor flux $\psi_{r\alpha}$ estimation error

(b) Open-loop rotor flux observer: rotor flux $\psi_{r\beta}$ and its estimate; rotor flux $\psi_{r\beta}$ estimation error

(c) Load torque estimator: load torque T_L and its estimate

Fig. 3. Observer performance without parameter uncertainities



(a) Open-loop rotor flux observer with $\bar{R}_s$ and $\bar{R}_r$: rotor flux $\psi_{r\alpha}$ and its estimate; rotor flux $\psi_{r\alpha}$ estimation error

(b) Open-loop rotor flux observer with $\bar{R}_s$ and $\bar{R}_r$: rotor flux $\psi_{r\beta}$ and its estimate; rotor flux $\psi_{r\beta}$ estimation error

(c) Load torque estimator with $\bar{R}_s$ and $\bar{R}_r$: load torque T_L and its estimate

Fig. 4. Observer performance with parameter uncertainities

Satisfactory performance has been illustrated even in situations of external load torque disturbance and parametric uncertainties.

B. Observer Performance Analysis

In this part, the open flux observer and load torque observer are simulated to verify the observer performance, and the related parameters are given in Table I. The motor (with initial conditions $\tilde{\psi}_{r\alpha}(t) = \tilde{\psi}_{r\beta}(t) = 0.1\text{Wb}$) is controlled by the super twisting control (with control parameters, rotor speed and flux modulus references, and applied load torque). The observer initial conditions are set to zero. For the load torque observer, the design parameter is chosen as (all values are in SI units) $\lambda = 500$. All initial conditions for the load torque estimator are set to zero while the rotor flux estimates are generated by the reduced order flux observer. The rotor flux vector (α,β) components along with the corresponding estimation errors are reported in Fig 3 (a) and Fig 3 (b), they show that exponentially converging estimation is achieved. The estimate of the load torque T_L is reported in Fig 3 (c): exponentially converging load torque estimation is obtained.

Considering parametric uncertainties, we assume the actual $\bar{R}_s$ and $\bar{R}_r$ to be $R_s + \Delta R_s$ and $R_r + \Delta R_r$, where $\Delta R_s = 0.2R_s$, $\Delta R_r = 0.3R_r$. The rotor flux vector (α,β) components along with the corresponding estimation errors are shown in Fig 4 (a) and Fig 4 (b), they show that estimated error is bounded. Similarly, the estimated torque error is bounded, as shown in Fig 4 (c).

In case of the nominal system we can find that the flux estimate errors have a fast convergence rate and go to zero, as shown in Fig 3 (a, b). Similarly, torque estimate has the same case from Fig 3 (c). When considering parametric uncertainties, both the flux estimate error and torque estimate error have the bounded properties, as shown in Fig 4 (a, b) and Fig 4 (c), which ensure the input-to-state stability (ISS) property.

V. CONCLUSION

In this paper, an observer-based super twisting sliding mode control technique has been designed for linear induction motors, taking into consideration the dynamic end effects. The proposed controller has a good tracking performance without chattering phenomena. Moreover, it is robust to external load torque disturbance and parametric variations. The proposed open loop flux observer and reduced torque observer show good estimate performance with nominal system and input-to-state stability (ISS) [18] property with perturbed system.

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