Active Disturbance Rejection Control of Linear Induction Motor

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Abstract—This paper proposes the theoretical framework and the experimental application of the active disturbance rejection control to linear induction motors. Such a nonlinear control (ADRC) technique can be viewed as a particular kind of input-output linearization control, where the nonlinear transformation of the state is not computed by means of the model, but it is estimated online. Such an approach permits to cope with un-modeling dynamics, as well as uncertainty in the knowledge of the model parameters and exogenous disturbances. The effectiveness of the proposed ADRC control law has been verified both by numerical simulations and experimentally on a suitably developed test setup. Moreover, the results have been compared with those achievable with the model-based feedback linearization control.

Index Terms—Extended state observer (ESO), linear induction motor (LIM), rejection of disturbances, state feedback control.

I. INTRODUCTION

Linear induction motors (LIMs) have been largely studied for several years [1], [2]. Among the main reasons of their interest there is the capability to develop a direct linear motion without the need of any gear-box for the motion transformation from rotating to linear. This advantage presents, as counterpart, the disadvantage of an increase of the complexity of the machine, which presents the so-called dynamic end-effects, caused by the relative motion between the short inductor and the induced part track. As a result, starting from the assumption that the rotating induction machine (RIM) presents a nonlinear space-vector dynamic model [3], the dynamic model of the LIM presents further significant nonlinearities, caused by the dynamic end-effects. Moreover, these end-effects introduce high uncertainties in the model of the whole system. Consequently, this causes additional difficulties in the control of the electromechanical variables. Although the LIM presents a far more complicated nonlinear model than the RIM, the approach adopted in the literature for its control has been usually to simply extend the classic control techniques from RIMs to LIMs. In particular, as for high-performance control techniques, the corresponding versions of the field oriented control (FOC) [4]–[6] or the direct thrust control [7] have been developed. These control techniques have been partially adapted from the RIM counterpart [3], so as to take into consideration the end-effects of the LIMs. Recently, also sensorless control methods have been applied to the LIM [8], which have been adapted from the RIM counterpart [9].

The control system theory, however, offers an important corpus of nonlinear control methodologies for dealing with a highly nonlinear systems. Among them, one is the so-called input-output feedback linearization (FL). Actually, few applications of the input–output FL to RIM control are provided by the scientific literature [10]–[12]. The current state of the art is described in [13]. A very limited number of papers in literature deal with the FL of LIMs. Papers like [14] and [15] apply the FL control technique to LIMs, but by adopting the RIM dynamic model for the definition of the control law, with consequent limitations due to an inadequate model. Only recently, the input–output FL control has been addressed in a systematic way, by suitably adopting a dynamic model of the LIM taking into consideration its dynamic end-effects [16]. In particular, [17], [18] define the theoretical framework of the FL control for LIMs and present the experimental results. Works [19] and [20] present some improvements of [17], [18], where the proposed FL control has been made adaptive with respect to the variations of the inductor resistance ([19]), or to the induced part time constant ([20]). However, the development of such control techniques requires the knowledge of an accurate dynamic model of the LIM. The adopted dynamic model, on the basis of which these techniques have been developed, had been previously presented in [16]. Modeling of LIMs has been faced up in the scientific literature almost 20 years ago, and only recently it has been approached again, thanks to the very recent interest which has been growing about LIMs, due to their potential applications in many fields. Among the most significant applications there are the following:

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1) high-speed transport systems like MAGLEVs (Magnetically Levitated Systems), see Japanese Linimo magnetic levitation train line near Nagoya;
2) modern subways, see Tokyo’s Toei Oedo Line;
3) lifting systems in deep mines;
4) sliding doors in low floor trams, see the Citadis and the Eurotram;
5) Electromagnetic Aircraft Launch System, see United States Navy.

In general, it can be stated that nonlinear controllers, based on the FL, are derived following two steps. In the first step, the model of the system is led to a canonical form consisting of a chain of integrators expressed in new state-space coordinates, where an auxiliary control input is adopted. As a result, the true control input is the superposition of a nonlinear function of the state and the auxiliary input. In absence of uncertainties and endogenous disturbances, the above nonlinear function can be assumed known and, consequently, it can be suitably compensated. In the second step, a linear control technique imposes the desired dynamics to the closed-loop system. In the FL, however, the state-feedback computed analytically suffers from an evident problem arising from the uncertainties on both the dynamic model and its parameters, characterized by a significant complexity, see [17]. A way to overcome such a limitation of the FL is the adoption of the so called active disturbance rejection control (ADRC) [21]–[23]. On this basis, this paper proposes the application of the ADRC to LIMs. To the best of the authors’ knowledge, such a technique has never been applied to the LIM (except for [24] which represents the preliminary version of this work): actually only few examples of the ADRC are shown in literature applied to the RIM [25], [26].

In ADRC, the order of the dynamic model of the system is augmented, considering the abovementioned nonlinear function of the state, which allows the model to be linearized, as a further state variable. While in the classic FL method this function is assumed known, in the ADRC method this function is estimated by means of an extended state observer (ESO). It follows that ADRC could be considered as an “adaptive robust version” of the FL control technique, since the state feedback term is estimated online. In this way, not only the problems due to the uncertainties on the parameters are addressed, but it is also possible to cope with unmodeled dynamics and exogenous disturbances (such as the load thrust).

The application of this control technique to the LIM is more significant than to the RIM, since the dynamic model of the LIM presents additional nonlinearities and uncertainties caused by the dynamic end-effects, leading to more complex feedback terms. To this aim, the proposed technique will make the control algorithm more robust, because it is not necessary to take explicitly into consideration all the nonlinearity effects, and variations of the parameters during the different working conditions.

The paper is organized as follows. In Section II, the space-vector model of the LIM, including the end-effects, is given. Section III deals with the ADRC law and it constitutes the main part of this paper. It is divided into three parts, in the first the extended models are defined, in the second the ESOs are designed for the previously defined extended models, and in the third two state feedback linear controllers are designed so that the induced part flux and the speed loops satisfy the assigned requirements. In Section IV, The proposed control techniques has been tested by means of numerical simulations, and it has been compared with the classic FL. Finally, in Sections V and VI, the proposed ADRC has been tested and validated experimentally on a suitably developed test setup.

II. SPACE-VECTOR STATE MODEL OF THE LIM INCLUDING END-EFFECTS

The ADRC of LIMs is derived from the inductor and induced part voltage equations of the LIM including the end-effects, as represented in [16]. A reference frame rotating at the speed of the induced part flux is chosen, with the direct axis $x$ aligned with the induced part flux space-vector. The following equations are taken from [16], just giving a slight different form, for a better explanation of the proposed ADRC:

$$\frac{di_{sx}}{dt} = -\gamma i_{sx} + \frac{\rho}{\tau_p} v i_{sy} + \frac{\alpha L_m i_{sy}}{\psi_r} + \beta \alpha \psi_r + \frac{u_{sx}}{\delta L_s}$$  \hspace{1cm} (1)

$$\frac{di_{sy}}{dt} = -\gamma i_{sy} - \frac{\rho}{\tau_p} v (i_{sx} + \beta \psi_r) - \frac{\alpha L_m i_{sy}}{\psi_r} + \frac{u_{sy}}{\delta L_s}$$  \hspace{1cm} (2)

$$\frac{d\psi_r}{dt} = - (\alpha - \eta) \psi_r + \alpha L_m i_{sx}$$  \hspace{1cm} (3)

$$\frac{d\psi_r}{dt} = \frac{\rho}{\tau_p} v + \frac{\alpha L_m i_{sy}}{\psi_r}$$  \hspace{1cm} (4)

$$\frac{dv}{dt} = \mu (\psi_r i_{sy}) - \frac{F_r}{M} - \frac{F_{eb}}{M}$$  \hspace{1cm} (5)

$$F_{eb} = \theta [\psi_r^2 + L_m^2 (i_{sx}^2 + i_{sy}^2) + L_s (\psi_r i_{sx})]$$  \hspace{1cm} (6)

where $\psi_r = \psi_{rx}$ and $\psi_r = 0$. The variables $\alpha$, $\beta$, $\eta$, $\mu$, and $\theta$ are defined as follows:

$$\alpha = \frac{1}{T_r} - \frac{\dot{R}_r}{L_m}, \hspace{1cm} \beta \equiv \frac{\dot{L}_m}{\delta L_s L_r}, \hspace{1cm} \eta = - \frac{\dot{R}_r}{L_m}$$

$$\mu = \frac{3}{2} \pi \frac{\dot{L}_m}{\delta L_s L_r M^2}, \hspace{1cm} \theta = \text{sign}(v) \frac{3 L_r}{2 L_r^2} \left( 1 - e^{-Q} \right)$$

$$\gamma = \frac{1}{\delta L_s} \left( R_s + \frac{\dot{R}_r}{L_r} \left( 1 - \frac{\dot{L}_m}{L_m} \right) + \frac{\dot{L}_m}{L_r} \left( \frac{\dot{L}_m}{L_r} - \dot{R}_r \right) \right)$$  \hspace{1cm} (7)

For the definition of the symbols, see Table I.

It should be noted that, differently from RIM case, where all coefficients are constant, in the LIM case they are all speed dependent, and thus time-varying, parameters. As for the definition of the speed-varying electrical parameters, which is the braking force due to the dynamic end effects, (symbols with \(\cdot\)), the reader can refer to [16]. Moreover, $F_{eb}$ is a term which does not exist in the RIM counter part, and it influences significantly the speed dynamics. This justifies the use of the ADRC technique in the LIM more than in the RIM when a linearization procedure is used in order to increase the dynamic performance. Indeed, in the FL control technique of LIM [17] very complex state feedback terms are considered, moreover the parameters of
TABLE I

LIST OF SYMBOLS

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<td>$u_{sx}, u_{sy}$</td>
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<td>$i_{sx}, i_{sy}$</td>
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<td>$\psi_{sx}, \psi_{sy}$</td>
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where $f$ is called total flux disturbance and it is defined as follows:

$$ f = -q_1 \psi_r + (\alpha - \eta)^2 \psi_r - \alpha \hat{L}_m (\alpha - \eta) i_{sx} + q_2 i_{sx} $$

$$ - \alpha \hat{L}_m \gamma i_{sx} + \alpha \hat{L}_m \frac{p \pi}{\tau_p} i_{sy} + \frac{\alpha^2 \hat{L}_m^2 \gamma^2}{|\psi_r|} + \hat{L}_m \beta \alpha^2 \psi_r $$

with

$$ q_1 = \frac{R_r \hat{L}_r + R_r \hat{L}_m (1 + f(Q))}{L_r^2} \frac{T_a}{T_m} \left( 1 - \frac{1}{1 + \frac{\tau_m}{T_r} \tau} e^{-\frac{\tau_m}{T_r} \tau} \right) $$

$$ q_2 = R_r \left( \frac{L_m^2}{L_r^2} (1 + f(Q)) + 1 - \frac{2 L_m f(Q)}{L_r} \right) $$

$$ \cdot \frac{\tau_m}{T_m} \left( 1 - \frac{1}{1 + \frac{\tau_m}{T_r} \tau} e^{-\frac{\tau_m}{T_r} \tau} \right) $$

with $Q = \frac{\tau_m}{T_r} \tau$ and $f(Q) = \frac{1 - e^{-Q}}{Q}$, while $b_\psi = \frac{\alpha \hat{L}_m}{\sigma \tau_p}$. Now if an extra state variable $x_{\psi_3} = f$ is defined, the flux extended model becomes

$$ \dot{x}_1 = x_2, \quad \dot{x}_2 = x_3 + b_\psi u_{sx}, \quad \dot{x}_3 = f $$

2) Speed Extended Model: The procedure used for obtaining the speed extended model is analogous to that used for defining the flux extended model, while, in this case, the speed is assumed as measured output. Even in this case, from the model expressed by (1)–(6), using the linearization procedure used in [17], and defining $x_{\psi_1} = \dot{v}$ and $x_{\psi_2} = \ddot{v} = \alpha$, the following equations can be written:

$$ \dot{x}_1 = x_2, \quad \dot{x}_2 = \xi + b_\psi u_{sy} $$

where $\xi$ is called total speed disturbance defined as follows:

$$ \xi = + \left( q_3 - \mu (\alpha - \eta) \right) \psi_r i_{sy} + \mu \alpha \hat{L}_m i_{sx} i_{sy} - \gamma \mu \psi_r i_{sy} $$

$$ + \left( q_4 - \frac{\mu \tau_p}{M} (\alpha - \eta) \right) \psi_r^2 + \frac{\mu \psi_r}{M} \hat{L}_m i_{sx} $$

$$ + L^2_{dr} \left( q_2 - 2 \frac{\mu \tau_p}{M} \psi_r^2 - \left( \mu \psi_r + 2 \frac{\mu \psi_r}{M} \right) \hat{L}_m \right) $$

$$ \cdot \left( \frac{p \pi}{\tau_p} \psi_{sx} + \frac{\alpha \hat{L}_m i_{sy} i_{sx}}{|\psi_r|} + \beta \frac{p \pi}{\tau_p} \psi_r - \psi_r \right) $$

with

$$ q_3 = \frac{3 \mu \pi \tau_p}{2} \frac{\hat{L}_r \hat{L}_m}{L_r^2} \frac{T_a}{T_m} \left( 1 + \frac{1 - e^{-Q}}{Q} \right) $$

$$ q_4 = \frac{3 L_r}{2 L_r^2 p \tau_p M} \frac{a}{v} \left( \frac{L_m}{L_r^2} \left( 1 - e^{-Q} \right) \left( f(Q) - e^{-Q} \right) \right) $$

$$ \left( \hat{L}_r - L_m f(Q) \right) - Q e^{-Q} $$

with $Q = \frac{\tau_m}{T_r} \tau$ and $f(Q) = \frac{1 - e^{-Q}}{Q}$, while $b_v = \frac{\mu \psi_r + 2 \frac{\mu \psi_r}{M} L_r^2}{\sigma \tau_p}$.

III. ACTIVE DISTURBANCE REJECTION CONTROL LAW

The ADRC approach is based on the construction of an extended model of order $n + 1$, where $n$ is the order of the system to be controlled. An additional state variable, consisting of all the nonlinear terms depending on the state and on the parameters of the system, including also external disturbances, must be properly defined. This additional state variable is called total disturbance and will be better defined later. The model is expressed in a canonical form consisting of a chain of integrators, where the last equation represents the dynamics of the total disturbance. Then, an ESO must be designed, so to be able to estimate the total disturbance. Finally, a control law is determined consisting of two components, the first compensates the total disturbance (by means of the estimate provided by the ESO) and the second assigns the desired behavior to the system.

A. Extended Models

Two distinct extended models are proposed, i.e., the flux and the speed extended model. The first expresses the dynamics of the direct component of the inductor current and the induced part flux, and the other expresses the dynamics of the quadrature component of the inductor current and the speed. Both models are derived from (1)–(6).

1) Flux Extended Model: From model (1)–(6), using the linearization procedure used in [17], and defining $x_{\psi_1} = \psi_r$, and $x_{\psi_2} = \psi_r$, the following equations can be written:

$$ \dot{x}_1 = x_2, \quad \dot{x}_2 = f + b_\psi u_{sx} $$

These terms are time-varying due to the end-effects. Conversely, in the ADRC approach, the state feedback terms are estimated online by means of two ESOs, so all the nonlinearities, uncertainties, and variations of the parameters, due to the end-effects, have not been taken into consideration directly, while they have been compensated adaptively.
Now if an extra state variable \( x_{\nu 3} = \xi \) is defined, the speed extended model becomes

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3 + b_s u_s y, \\
\dot{x}_3 &= \dot{\xi}.
\end{align*}
\]  \hfill (16)

Models (11) and (16) show that both extended models have the same structure, so a similar control methods can be applied.

Now, in order to apply a control law with active rejection of the disturbances, the control variables are chosen as follows:

\[
\begin{align*}
\upsilon_{sx} &= \frac{1}{b_s} (-\hat{\dot{x}}_{\psi 3} + \dot{\nu}_x), \\
\upsilon_{sy} &= \frac{1}{b_s} (-\hat{\dot{x}}_{\psi 3} + \dot{\nu}_y)
\end{align*}
\]  \hfill (17)

where \( \hat{\dot{x}}_{\psi 3} \) and \( \hat{\dot{x}}_{\psi 3} \) are the estimates of \( x_{\psi 3} \) and \( x_{\psi 3} \), respectively, while \( \dot{\nu}_x \) and \( \dot{\nu}_y \) are designed so that the models (11) and (16) satisfy the design requirements. The design procedure for the two inputs \( \dot{\nu}_x \) and \( \dot{\nu}_y \) will be better analyzed in the following. Therefore, initially the attention is focused on the estimation of the total disturbances, which is carried out by means of two ESOs. These two ESOs are designed in order to estimate the state of the extended models and, as it can be viewed from the model, the estimate of the total disturbance corresponds to the estimate of the third state variable.

At this point, two remarks have to be given in order to highlight the principal differences with the FL technique.

**Remark 1:** On the basis of the above considerations, the differences between the classic FL and the proposed ADRC technique are clear. Indeed, in the classic FL the control inputs are also designed as in (17) and (18), but the total disturbances \( f = x_{\psi 3} \) and \( \xi = x_{\psi 3} \) are analytically computed as in (8) and (13), with a clear problem arising from the uncertainties on both the model formulation and the model parameters knowledge due to the complexity of the formulation of (8) and (13). With the proposed ADRC, these terms are estimated and no-knowledge on the structure of these terms is needed. In this way, not only the problems due to the uncertainties on the parameters are addressed, but also problems of unmodeled dynamics are sorted out.

**Remark 2:** Looking at [17], two approximations have been assumed in order to linearize the model. In particular in [17], it is assumed that \( F_r \approx 0 \), meaning that the rate of change of the load force is assumed null. Moreover, also the braking thrust \( F_{\nu b} \) is approximated (see [17, Eq. (35)]). All these approximations are not necessary here, because the exact knowledge of these quantities is not needed, since they are adaptively estimated. For these reasons, the proposed approach presents robustness features also against the exogenous disturbances such as the load thrust.

Since the flux and speed extended models have the same structure, it is convenient to focus our attention on a general third-order extended model given by

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3 + b_s u, \\
\dot{x}_3 &= h
\end{align*}
\]  \hfill (19)

where \( h \) and \( x_1 \) are, respectively, the total disturbance and the output. Then, the obtained results for the model (19) will be particularized for the two models under study (11) and (16).

### B. ESO for a Third-Order Extended Model

The ESO chosen for the state estimation of model (19), proposed in [23], whose set of equations is given by

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 - \epsilon g_1 \left( \frac{\dot{x}_1 - x_1}{\epsilon^2} \right) \\
\dot{x}_2 &= \dot{x}_3 - g_2 \left( \frac{\dot{x}_1 - x_1}{\epsilon^2} \right) + bu \\
\dot{x}_3 &= -\epsilon^{-1} g_3 \left( \frac{\dot{x}_1 - x_1}{\epsilon^2} \right)
\end{align*}
\]  \hfill (20)

where \( \epsilon \) is a suitable positive parameter, and the functions \( g_i(\cdot), i = 1, 2, 3 \), can be either linear or nonlinear functions.

Now, let us define the following estimation errors:

\[
\eta_i = \frac{e_i}{\epsilon^3}, i = 1, 2, 3
\]  \hfill (21)

where \( e_i = x_i - \hat{x}_i \). By means of (19) and (20), the dynamics of the variables \( \eta_i \) can be written as follows:

\[
\begin{align*}
\epsilon \dot{\eta}_1 &= \eta_2 - g_1(\eta_1) \\
\epsilon \dot{\eta}_2 &= \eta_3 - g_2(\eta_1) \\
\epsilon \dot{\eta}_3 &= -\epsilon h - g_3(\eta_1)
\end{align*}
\]  \hfill (22)

The structure of \( g_i(\eta_1), i = 1, 2, 3 \), characterizes the behavior of the ESO. In this paper, a linear ESO (LESO) is considered. With this choice, fixing \( g_i(\eta_1) = \beta_i \eta_1 \), with \( \beta_i \) positive constants, equations (22) can be written as follows:

\[
\epsilon \dot{\eta} = A \eta + c b h
\]  \hfill (23)

where

\[
\eta = [\eta_1 \eta_2 \eta_3]^T, \quad A = \begin{bmatrix} -\beta_1 & 1 & 0 \\ -\beta_2 & 0 & 1 \\ -\beta_3 & 0 & 0 \end{bmatrix},
\]

and \( b = [0 \ 0 \ -1]^T \).

A theorem showing the stability of model (23) is shown in [22]. This aspect is particularly important since the stability of model (23) implies the convergence to zero of the estimation error (21). In particular, it is possible to prove that for a given \( \epsilon > 0 \), choosing \( \beta_i \) such that \( A \) is Hurwitz, and assuming that the equivalent disturbance is bounded, then \( \eta \) converges within a ball around the origin. Moreover, for \( \epsilon \to 0 \), the ball shrinks to the origin. This implies that the final error can be made as small as possible by acting on \( \epsilon \).

**Remark 3:** With regard to the question of bounded equivalent disturbance, looking at (8) and (13), it is possible to note that the only possibility for the equivalent disturbance to blow up to infinity is when \( |\psi_r| \to 0 \). However, this situation does not occur in the normal operation, since the flux must be strictly greater than zero in any working condition for physical reasons. Moreover, this is a common feature with the FL control and with the FOC, where one existence condition for the decoupling of the inductor currents is that the flux amplitude is different from zero.

It is worthwhile to note that the ESO (20) is able to estimate the equivalent disturbance \( h \) since \( \dot{x}_3 = h \), and it requires only
the knowledge of the input $u$ and of the coefficient $b$, no information about the equivalent disturbances, constituted by the complex terms (8) and (13), is necessary. Moreover, besides the equivalent disturbances, it is able to estimate the state variables $x_1$ and $x_2$.

In the following, these results will be particularized in order to be used for the flux and the speed extended models.

1) LESO for the Flux Extended Model: The flux extended model can be obtained putting $x_i = x_{\psi,i}$, for $i = 1, 2, 3$, $h = f$, and $b = b_0$. The variable $x_1 = x_{\psi,1}$ is assumed known and, as it will be seen in the follows, it is given by a suitable observer. The parameters $\beta_i = \beta_{\psi,i}$, for $i = 1, 2, 3$, are obtained so that the eigenvalues of the matrix $A$ of model (23) belong to the left half part of the complex plane. In particular, the three eigenvalues are chosen real and coincident. More precisely, the characteristic polynomial of $A$, is given by

$$\Delta_\psi(\lambda) = \lambda^3 + \beta_{\psi,1}\lambda^2 + \beta_{\psi,2}\lambda + \beta_{\psi,3} = (\lambda + \omega_\psi)^3$$  \hspace{1cm} (24)

where $\omega_\psi$ is roughly the bandwidth of the LESO for the flux.

2) LESO for the Speed Extended Model: The speed extended model can be obtained putting $x_i = x_{v,i}$, for $i = 1, 2, 3$, $h = \xi$ and $b = b_v$. The variable $x_1 = x_{v,1}$ is the measured speed, and the parameters $\beta_i = \beta_{v,i}$, for $i = 1, 2, 3$, are chosen as said for the flux extended model. More precisely, the characteristic polynomial of $A$ is given by

$$\Delta_v(\lambda) = \lambda^3 + \beta_{v,1}\lambda^2 + \beta_{v,2}\lambda + \beta_{v,3} = (\lambda + \omega_v)^3$$  \hspace{1cm} (25)

where $\omega_v$ is roughly the bandwidth of the LESO for the speed.

C. Design of Flux and Speed Controllers

As explained above, the flux and speed extended models have the same structure (19). It follows that also the flux and speed controllers will have the same structure and can be designed using the same approach. With reference to (19), and according to (17) and (18), the control law is defined as

$$u = \frac{1}{b} (\dot{x}_3 + u_0).$$  \hspace{1cm} (26)

If the ESO is designed such that the estimation error converges to zero, then $\dot{x}_3 \rightarrow x_3$, and the model (19) becomes

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u_0.$$  \hspace{1cm} (27)

Such a structure corresponds to a double integrator. This model is reachable and, consequently, a state feedback control law based on the assignment of the eigenvalues can be derived, but, as well known, it does not allow steady-state null errors to be obtained. Consequently, in order to have perfect tracking of constant reference signals, the state of model (27) is augmented by adding a third variable $z$, whose dynamics is

$$\dot{z} = x_{1,\text{ref}} - x_1$$  \hspace{1cm} (28)

where $x_{1,\text{ref}}$ is the desired value of $x_1$. As it is easy to verify, model (27), (28) is reachable and by means of the control law

$$u_0 = -k^T \dot{x}$$  \hspace{1cm} (29)

with $k = [k_1 \ k_2 \ k_3 \ 0]$, and $\dot{x} = [x_1 \ x_2 \ z]^T$, the eigenvalues of the dynamic matrix of the model can be assigned. Moreover, a zero steady-state error can be achieved for the variable $x_1$. The characteristic polynomial of this matrix is

$$\Delta(\lambda) = \lambda^3 + k_1\lambda^2 + k_2\lambda + k_3$$  \hspace{1cm} (30)

where the parameters $k_1$, $k_2$, and $k_3$ are determined assuming that the desired eigenvalues are $\lambda_1 = -\zeta\omega_n + j\omega_n \sqrt{1 - \zeta^2}$, $\lambda_2 = -\zeta\omega_n - j\omega_n \sqrt{1 - \zeta^2}$ and $\lambda_3 = \sigma$, where $\omega_n$ and $\zeta$ are the natural frequency and the damping factor, respectively, while $\sigma$ is a negative real number. The implementation of the above control law can be carried out using the state estimated by the ESO, since it requires the knowledge of $x_1$ and $x_2$, whereas the knowledge of $x_3$ allows us to compensate the total disturbance $h$.

D. Design Data of the ESO and Controllers

The parameters of the ESOs and controllers are given below.

1) Flux Model: The ESO for flux is designed with $\omega_\psi = 5$ rad/s and $\epsilon = 0.05$. The parameters $\beta_i = \beta_{\psi,i}$, for $i = 1, 2, 3$, are obtained from (24). The control law is given by (17) with $\nu'_\psi = -k^T_{\psi} x_{\psi}$, where $k_{\psi}$ is determined so that the zeros $\lambda_1$, $\lambda_2$, and $\lambda_3$ of the polynomial (30) are chosen with $\omega_n = 10$, $\zeta = 0.9$ and $\sigma = -150$. The state is $x_{\psi} = [x_{\psi,1} \ x_{\psi,2} \ z_{\psi}]^T$ in which $z_{\psi}$ is the output of an integrator supplied by the flux tracking error $(\psi_{\text{ref}} - \psi_{\psi,1})$.

2) Speed Model: The ESO for speed is designed with $\omega_v = 5$ rad/s and $\epsilon = 0.05$. The parameters $\beta_i = \beta_{v,i}$, for $i = 1, 2, 3$, are obtained from (25). The control law is given by (18) with $\nu'_v = -k^T_{v} x_{v}$, where $k_{v}$ is determined so that the zeros $\lambda_1$, $\lambda_2$, and $\lambda_3$ of the polynomial (30) are chosen with $\omega_n = 12$, $\zeta = 1$, and $\sigma = -150$. The state is $x_v = [x_{v,1} \ x_{v,2} \ z_v]^T$ in which $z_v$ is the output of an integrator supplied by the speed tracking error $(v_{\text{ref}} - v_{v,1})$.

For the implementation of the presented control law, the induced part flux estimation is necessary. Therefore, in order to cope with this problem, a suitable observer have to be designed. In this paper, the observer developed in [8] has been used, and it has been tuned by means of the procedure shown in [27].

In Fig. 1, the block diagram of the overall control scheme of the LIM drive, based on the ADRC, is shown, while in Fig. 2 the block diagram of the ADRC scheme is given.

IV. SIMULATION RESULTS

The proposed control techniques has been tested by means of numerical simulations in MATLAB–Simulink environment. The Simulink model includes the ADRC control law, the two ESOs, and the LIM model. The LIM model used for the simulation tests is the same as the one used in the experimental tests, whose parameters and rated data are shown in Table II.

In particular, three tests have been made. The first is a direct comparison between the classic FL and the proposed ADRC, performed adopting a FL technique neglecting the end-effects. Such comparative test has been carried out to show the robustness of the proposed method against the unmodeled dynamics (the dynamic end-effects typical of LIMs in this case). The second test has been carried out with specific regard to the sensitivity analysis of the controller, evaluating the induced part flux
and the speed tracking errors with respect to the variations of the inductor and induced part resistances, which are the two main parameters that can change during the normal operation (heating, load, etc.). Both the first and the second tests are needed to confirm what claimed in Remark 1. Indeed, one of the main difference between the classic FL and the proposed ADRC technique is that in the FL the nonlinear feedback terms are analytically computed as in (8) and (13), with a clear problem arising from the uncertainties on both the model formulation and the model parameters knowledge. With the proposed ADRC, these terms are estimated online, so no-knowledge on the structure of these terms is needed.

Finally, the third test corresponds to a rapid sequence of contemporaneous variations of load and induced part flux, and this test is carried out to prove what stated in Remark 2.

With regard to the first test, the machine has been initially operated at zero speed and with an induced part flux of 0.4 Wb, at 1 s the speed is increased from zero up to 4 m/s, and finally at 2.5 s a contemporary load force (0–80 N) and induced part flux (0.4-0.8Wb) step commands have been given to the LIM drive. During this test, the end-effects have been neglected in the development of the FL in order to show the robustness of the proposed method against the unmodeled dynamics. The results of this test are shown in Figs. 3–6.

The speed figure clearly shows that, the adoption of a very performing control technique like FL, without considering all the effects related to the dynamics of the plant (in this case the LIM dynamic end-effects), leads to an improper behavior of the drive, especially during fast speed transient. As matter of fact, some speed oscillations are visible during the first instants of the start-up, close to zero speed, when the drive is getting close to the reference speed and finally when the load force is applied. In these instants, the effects of a not proper field orientation condition, caused by the unmodeled dynamics, play a significant role in the drive performance. This is confirmed by the presence of a nonnull speed steady-state tracking error, caused itself by the unmodeled dynamics of the FL. All these effects are, on the contrary, absolutely not present in case of adoption of the...
ADRC. In this specific case, speed transients are smooth in all instants, connoting a proper field orientation working condition. As a result, the speed tracking error is null not only at steady state, but also during transients. Coherently with what commented on the speed curve, the induced part flux curve presents a visible nonnull steady-state tracking error in case of adoption of the FL, which becomes zero in case of adoption of the ADRC. The biggest beneficial effects arising from the adoption of the ADRC are observable in the electromagnetic force curve. As a matter of fact, in case of adoption of the FL, the thrust presents a big initial overshoot at each speed transient, followed by long and big oscillations before settling down in a steady state. This is not the case when the ADRC is adopted, presenting only a first overshoot at each speed transient, followed by a very fast settling down toward steady state. Finally, even the three-phase inductor currents present an envelope that is much smoother in case of the ADRC than in the case of the FL.

Moreover, with specific reference to the test in Figs. 3–6, the integral absolute error (IAE) indexes have been computed in order to evaluate the tracking errors obtained both with the classic FL and with the proposed ADRC. The results are shown in Table III, where the IAE indexes for the speed and the induced part flux have been computed on the basis of the following equations:

\[
IAE_{|\psi_r|} = \int_0^T |\psi_r(t)| - |\psi_{r,ref}(t)| \, dt \tag{31}
\]

\[
IAE_v = \int_0^T |v(t) - v_{ref}(t)| \, dt. \tag{32}
\]

The analysis of the IAE indexes, as computed for both the speed and the induced part flux tracking errors, confirm what stated above. The adoption of the ADRC permits a reduction of the IAE indexes of 97% in case of the speed and 57% in case of the induced part flux, showing, clearly, the higher performance that can be achieved.

The second test has been carried out on the same kind of simulation test shown in Figs. 3–6, and evaluating the IAE indexes, computed as in (31) and (32), when the resistances Rs and Rr vary in the range between 20% and 200% of their rated values (\(R_s = R_{s,\text{nom}} = 11 \Omega\) and \(R_r = R_{r,\text{nom}} = 31.57 \Omega\)). The corresponding results are shown in Figs. 7–10. These error surfaces clearly show that the biggest tracking error, on both the induced part flux and speed loops, occur for contemporary big variations of the inductor and induced part resistances. In particular, the contemporary reduction of the inductor and induced part resistances causes the biggest tracking errors. This effect is more visible on the speed loop than on the induced part flux loop. The direct comparison between the corresponding surfaces obtained respectively with the ARDC and the FL clearly highlights that the obtained tracking error with the ARDC is at least one order lower than that with the FL. This is identically true for the
Fig. 7. IAE index, in the FL case, computed as in (31), when the resistances $R_s$ and $R_r$ vary in the range between 20% and 200%.

Fig. 8. IAE index, in the FL case, computed as in (32), when the resistances $R_s$ and $R_r$ vary in the range between 20% and 200%.

Fig. 9. IAE index, in the ADRC case, computed as in (31), when the resistances $R_s$ and $R_r$ vary in the range between 20% and 200%.

Fig. 10. IAE index, in the ADRC case, computed as in (32), when the resistances $R_s$ and $R_r$ vary in the range between 20% and 200%.

Fig. 11. Speed and speed tracking error during the third simulation test.

induced part flux and the speed loops and confirms the higher robustness of the control action versus parameters’ variation achievable with the ADRC.

Finally, during the third test, a rapid sequence of contemporary variations of the load and of the induced part flux has been given to the drive. In particular the machine has been operated at the constant speed of 2 m/s, while the load thrust and the induced part flux have been varied simultaneously between 0–80 N and 0.4–0.8 Wb, respectively. The results are shown in Figs. 11–14. Coherently with the above shown results, the speed figure clearly shows that, adopting the FL, some consistent speed oscillations are visible during every load application, due to the not proper field orientation condition caused by the unmodeled dynamics. This effect is, on the contrary, absolutely not present in case of adoption of the ADRC. In this specific
Fig. 12. Induced part flux and induced part flux tracking error during the third simulation test.

Fig. 13. Electromagnetic thrust during the third simulation test.

Fig. 14. Inductor currents during the third simulation test.

TABLE IV
IAE INDEXES FOR THE TEST IN FIGS. 11–14

<table>
<thead>
<tr>
<th></th>
<th>FL</th>
<th>ADRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IAE_{\psi}$</td>
<td>0.2153</td>
<td>0.1163</td>
</tr>
<tr>
<td>$IAE_v$</td>
<td>0.7535</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

Fig. 15. Photograph of the experimental test setup.

case, speed transients after load application are smooth, connoting a proper field orientation working condition. Coherently, the induced part flux curve presents a visible nonnull steady-state tracking error in case of adoption of the FL, which becomes zero in case of adoption of the ADRC. Exactly as shown in the first test, the biggest beneficial effects arising from the adoption of the ADRC are observable in the electromagnetic force curve. As a matter of fact, in case of adoption of the FL, the thrust presents long and big oscillations before settling down at steady state which are not present when the ADRC is adopted.

Moreover, with reference to the test in Figs. 11–14, the IAE indexes have been computed, as in the previous test, in order to evaluate the tracking errors obtained both with the classic FL and with the proposed ADRC. The results are shown in Table IV. The analysis of the IAE indexes, as computed for both the speed and the induced part flux tracking errors, confirm what stated above. The adoption of the ADRC permits a reduction of the IAE indexes of 99% in case of the speed and 46% in case of the induced part flux.

V. EXPERIMENTAL SETUP

A test setup has been suitably built for the experimental verification of the proposed ADRC. The machine under test is a LIM model Baldor LMAC1607C23D99. The LIM has been equipped with a linear encoder Numerik Jena LIA series. The employed test setup consists of the following:

1) three-phase LIM with parameters shown in Table II;
2) frequency converter which consists of a three-phase diode rectifier and a 7.5 kVA, three-phase VSI;
Fig. 16. (a) Linear speed of the drive, (b) induced part flux amplitude, and (c) currents $i_{sx}$ and $i_{sy}$ during a speed reversal from 0.3 to $0.3 \text{ms}^{-1}$.

Fig. 17. (a) Linear speed of the drive, (b) induced part flux amplitude, and (c) currents $i_{sx}$ and $i_{sy}$ during a start-up from 0 to $0.6 \text{ms}^{-1}$.

3) a dSPACE card (DS1103) with a PowerPC 604e processor for fast floating-point calculation at 400 MHz and a fixed-point digital signal processor (DSP) TMS320F240.

The test setup is equipped also with a torque controlled PMSM model Emerson Unimotor HD 067UDB305BACRA mechanically coupled to the LIM by a pulley-strap system, to determine an active load for the LIM. As far as the control algorithm a sampling frequency equal to 10 kHz has been set, while pulse-width modulation (PWM) frequency of 5 kHz have been chosen.

Fig. 15 shows a photograph of the test setup.

VI. EXPERIMENTAL RESULTS

The proposed ADRC technique has been experimentally tested on the abovedescribed test setup. As a first test, a speed reference with a square waveform of magnitude equal to 0.3 m/s at no load has been given to the LIM drive. The frequency of such a square waveform is determined by the length of the induced part track. Whenever the inductor reaches the end of the track, the sign of the speed reference is inverted. Fig. 16(a) shows the reference and measured linear speed of the LIM during such a test. Fig. 16(b) shows the corresponding estimated induced part flux. Fig. 16(c) shows the corresponding direct and quadrature components of the inductor currents $i_{sx}$, $i_{sy}$. The measured speed suitably tracking its reference with high dynamic performance, showing the correct behavior of the LIM drive. As for the induced part flux waveform, the ADRC correctly controls it at a constant value, with very small transient deviations with respect to the reference value of 0.8 Wb. As for the current waveforms, $i_{sx}$ exhibits a waveform equal in shape and proportional in amplitude to that of the induced part flux, as expected, confirming the correct field orientation working condition. As for $i_{sy}$, it exhibits a waveform with step variations occurring in correspondence to each step variation of the reference speed, as expected. Fig. 17(a)–(c) show the same kind of waveforms obtained during a start-up test of the drive from 0 to 0.6 m/s at no load. The analysis of all these waveforms shows the correct behavior of the drive under such working conditions and its high dynamic performance. Finally, Fig. 18(a)–(c) shows the same kind of waveforms obtained during a start-up test of the drive from 0 to 0.3 m/s after which a step load force of 100 N (close to the rated load of the LIM) is applied (after $t = 2 \text{s}$). It can be clearly observed that the speed controller immediately reacts to the application of the load force, with a null speed tracking error at a steady state (thanks to the above described structure of the linear controller). Correspondingly, the flux controller maintains the induced part flux constant, independently from the load. The $i_{sy}$ current component exhibits a step increase subsequent to the application of the load force, as expected, to cope for the application of the load force.
VII. CONCLUSION

This paper proposes the theoretical framework and the experimental application of the ADRC to a LIM. Such a nonlinear control method can be viewed as a particular kind of input–output linearization control technique, where the nonlinear transformation of the state is not a priori given by a model, but it is estimated online. Such an approach permits to cope with modeling errors as well as any uncertainty in the knowledge of the model parameters and load disturbances. All these benefits have been validated by means of numerical simulations, comparing the proposed ADRC with the classic FL. The effectiveness of the proposed ADRC has been verified experimentally on a suitably developed test setup, and the results clearly show the higher performance achieved.

REFERENCES


Fig. 18. (a) Linear speed of the drive, (b) induced part flux amplitude, and (c) currents $i_{r_1}$ and $i_{r_2}$ during a test at 0.3 ms$^{-1}$ with a load of 100 N.
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