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Practical test for closed-loop identification and control on Magnetic Levitation system: A fractional-order approach

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Abstract— **Magnetic levitation system is an interesting equipment to demonstrate an intricate control design problem. Because of its non-linearity and exhibit uncertain open-loop properties together critically unstable in nature, identifying such system and then designing an effective controller is very demanding. This paper proposes identification and control strategy for real-time magnetic levitation system based on fractional theory and an orthogonal series. Without removing the controller from the loop, the unknown system is identified in form of low-order fractional model accurately. Then, a new fractional controller namely FOPID (fractional-order-proportional-integral-derivative) is tuned to improve the system performance. Real-time experiment study clearly illustrates the effectiveness of the presented tuning method.**

I. INTRODUCTION

Magnetic levitation (MagLev) technology has been a concept of immense interest due to its minimum friction and low energy consumption characteristics. It has become very useful in the field of real-life engineering applications such as high-speed MagLev trains, levitation of wind turbine, frictionless bearings, vibration isolation tables, satellite launching etc. However, it is not so simple to illustrate such type of systems. The MagLev system is a typical non-linear open-loop unstable system and corresponding controller is required to stabilize the system in various input conditions [1], [2]. In general, it is always desired to perform the test in the closed-loop configuration to avoid drift in operating point induced by disturbances.

Various techniques have been developed for the modeling and control of MagLev systems in literature. Real-time MagLev system modeling and control based on SIMLAB [2], nonlinear six-state model of the system [3], neural networks based method [4], 2-DOF (degree of freedom) PID compensation [1], 2-DOF control and disturbance estimator method [5] have been discussed in literature. These methods are based on integer order model and classical PID control. Despite the conventional PID is widely used in industry, main issue with a highly unstable system is that it usually exhibits large overshoots at inconvenient positions.

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At present, fractional-order modeling and control have become popular among researchers due to ability of handling previously intractable problems related to complex dynamics representation [6]. Basically, fractional-order provides additional degrees of freedom in both system modeling and controller design. This is main reason to use fractional-order transfer function in the various systems of engineering and sciences. In particular, a magnetic levitation plant studied using FOPID controller based on the dominant pole placement method by Swain et al. [7]. The FOPID controller with inherent anti-windup capability for MagLev system is demonstrated in [8]. Altintas and Aydin [9] utilized big bang big crunch and genetic algorithms to optimize FOPID and IOPID (integer-order PID) parameters of the same system. Frequency domain method for fractional-order controller design using nonlinear adaptive seeker optimization algorithm was described in [10]. Modeling and control using FOPID based on particle swarm optimization and genetic algorithm was illustrated in [11]. The comparative study prevails how such fractional-order adoption performs better than conventional (integer-order) controller [12], [13]. It has been proven with better dynamic performance especially to the uncertain load and operating gap. The MagLev system was studied with FOPID by Majhi et al. [14] using iterative tuning method of firefly algorithm and by Verma et al. [15] using Nelders-Mead optimization algorithm. Recently, a simplified fractional order controller [16] was validated for a MagLev system without including an optimization routine.

Aiming in this paper is to identify the MagLev system under a real-time closed-loop normal operating condition. The technique can identify the present system without removing the controller from the loop and using the available input-output data. Haar orthogonal series is adopted in this work to estimate parameters of the fractional-order model without complex fractional derivatives in the procedure. Further, the method calculates the new FOPID values for better system performances to replace the classical PID control. The proposed method was subjected to a simulation based evaluation as well as real-time experimental tests on a MagLev system.

II. Basic concepts

A. Fractional order calculus

Fractional calculus is a generalization of non-integer (real) order integration and differentiation. There exist numerous definitions to characterize fractional integration and differentiation. In our work, we use the R-L (Riemann-Liouville) definition which can be written as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \quad (1) $$

where $n-1 < \alpha < n$, $n \in N$, and Γ denotes gamma function.

and its operator is generally defined as

$$ {}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \alpha > 0; \\ 1 & \alpha = 0; \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & \alpha < 0; \end{cases} \quad (2) $$

where a and t are the bounds of the operation and α ($\alpha \in R$) is the order of operation.

Suppose the FO derivative and integral are represented in the Laplace domain with zero initial. Then [6],

$$ L\left[{}_0D_t^\alpha f(t)\right] = s^\alpha F(s) \quad (3) $$

$$ L\left[I_0^\alpha f(t)\right] = \frac{1}{s^\alpha} F(s) \quad (4) $$

where s^α is a fractional Laplacian operator.

B. Integer to fractional-order PID

The core of magnetic levitation system is controller design as the linearization has to be carried out around a non-zero operating point, which is very challenging. Various techniques have been developed for the control of MagLev systems based on PID controllers. The closed-loop autotuning of classical PI(D) controller is simple to realize and generally exhibits the satisfactory outcomes [17]. But, FOPID control can be more complicated specially during online autotuning due to more parameters to be tuned at a time [18]. On other side, it offers more flexibility and extra degree of freedom for robust control design due to fractional (non-integer) orders of derivative and integral components [14], [19]. Most commonly used structures of IOPID and FOPID are, respectively

$$ C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s \quad (5) $$

$$ C_{FOPID}(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \quad (6) $$

where k_p, k_i and k_d are proportional, integral, and derivative gain and (λ, μ) are any positive real numbers.

C. Brief theory on Haar wavelet operational matrix

In this paper, fractional calculus and Haar wavelet based method [20], [21] is incorporated to identify the parameters of the unknown MagLev system. This method converts fractional derivative and integral into simple algebraic matrix multiplication and reduces computational complexity. The fractional order integration (FOI) of Haar wavelet for first M terms can be expressed as,

$$ \left(I^\alpha H_M\right)(t) \approx P^\alpha_{M\times M} H_M(t) \quad (7) $$

where H_M is the Haar function vector and $P_{M\times M}$ is called the Haar wavelet operational matrix of FOI. The Haar wavelet operational matrix of the FOI is obtained as,

$$ P^\alpha_{M\times M} \approx \Omega_{M\times M} F_\alpha \Omega^{-1}_{M\times M} \quad (8) $$

where F_α [22] is the block pulse functions operational matrix of FOI and $\Omega_{M\times M}$ [20] is the M-square Haar matrix.

III. Magnetic Levitation system

The MagLev system is a simple structure, composed of an LED light source, an electromagnet, an optoelectronic sensor, amplifier module, an analogue control module, data acquisition card, and a steel ball. The system configuration and experimental setup for GML1001 model (from Googol Technology Ltd.) is demonstrated in Figs. 1 and 2 respectively.

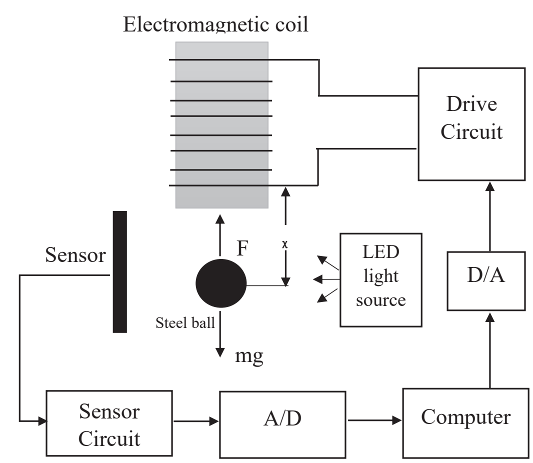

Fig. 1. Schematic diagram of MagLev system

The MagLev system consists of a ferromagnetic steel ball as a levitated object against gravity which is placed underneath the electromagnet. The position and displacement of steel ball is controlled using electromagnetic force of attraction. The current through electromagnetic

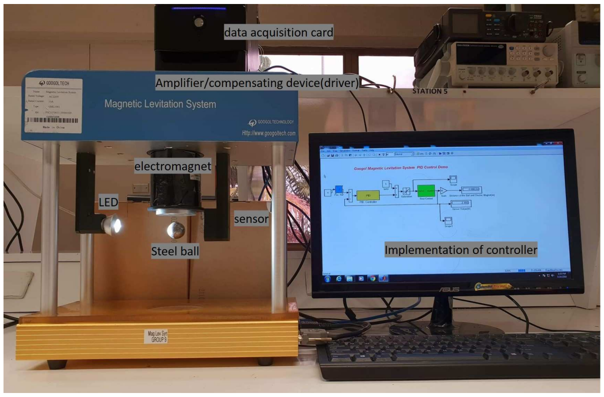

Fig. 2. Experimental setup for MagLev control system

TABLE I
THE PHYSICAL MAGLEV SYSTEM PARAMETERS [23]

Parameters	Value
m - mass of the steel ball	$22g$,$60g$
g - acceleration due to gravity	$9.81m/s^2$
i_0 - current at equilibrium	$0.6105A$
x_0 - position at equilibrium	$20.0mm$
K_a - power amplifier gain	5.8929
K_s - sensor gain	458.72

windings induces an electromagnetic force which can be regulated by adjusting current through drive circuit. The LED light source and sensor are used to detect the variation of distance between the steel ball and electromagnetic windings. When distance changes, the sensor measures the variation of light intensity and generates corresponding voltage signal which is proportional to illumination.

The Analog to Digital (A/D) interface converts this feedback sensor voltage to digital output which is then compared with the reference trajectory to produce the error signal. This error signal is finally fed to the controller to produce the controlled output. The output of the controller is fed to the Digital to Analog (D/A) converter which generates controlled analog electric current through the electromagnet. The flow of current in the coil produces the force generated by the magnetic field that counterbalances the gravitational pull of the ball. There is no mechanical contact between steel ball and electromagnet which eliminates frictional forces. By regulating the electric current in the circuit, the electromagnetic force can be adjusted to be equal to the weight of the steel ball, thus the ball will levitate in equilibrium state. Computer MATLAB/Simulink environment is used as a main control unit.

The simple nonlinear model for MagLev system in terms of ball position x and electromagnetic coil current i using Newton's second law of motion in vertical direction,

$$m\frac{d^2x}{dt^2} = mg + K\left(\frac{i}{x}\right)^2 \tag{9}$$

After simplification open loop MagLev system can be given [23],

$$L(s) = \frac{Y(s)}{U(s)} = \frac{K_s x(s)}{K_a i(s)} = \frac{-\left(\frac{K_s}{K_a}\right)}{\frac{i_0}{2g}s^2 - \frac{i_0}{x_0}} \tag{10}$$

The real system parameters are given in Table I [23]. Using system dynamics with ball weight $22g$, the MagLev system transfer function is given in [23] as,

$$L(s) = \frac{77.8421}{0.0311s^2 - 30.5250} \tag{11}$$

For our experiment, we have used steel ball with mass $60g$ which will change the system transfer function and our aim is to find new transfer function for the MagLev system and design robust controller using FOM and IOM. However, only gain varies with the ball change [24].

IV. PROPOSED IDENTIFICATION AND CONTROL TECHNIQUE

In this section, identification and control strategy is discussed. The first step is to identify the unstable MagLev system fractional order model. In second step, FOPID with five unknown parameters are tuned based on previously identified model. Although MagLev system is non-linear and unstable, it can be linearized at equilibrium point using controller in closed loop configuration. Therefore, it exhibits conditional stability and linearization property at equilibrium. The basic closed loop configuration for MagLev system is depicted in Fig. 3, where r, y, e represent setpoint, output and error signal respectively.

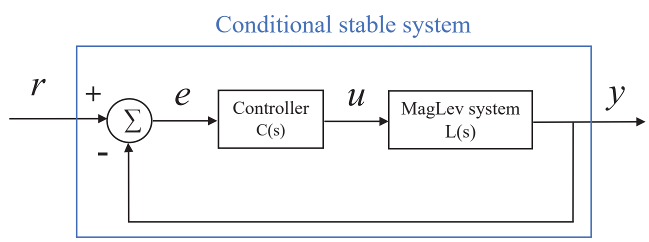

Fig. 3. Closed loop configuration for MagLev system

Consider MagLev system with fractional order model as,

$$L(s^\alpha) = \frac{b_0}{a_1 s^\alpha + a_0} \tag{12}$$

where a_1, a_0 and b_0 are arbitrary real numbers and α is the real positive number. As per (11), if $\alpha = 2$, it gives integer order model for MagLev system. Here, generalized

FOPID is considered in order to improve the performance of overall system and can be expressed as,

$$C_{FOPID}(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \tag{13}$$

With unity feedback, the closed-loop transfer function is given by,

$$G_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{L(s)C(s)}{1 + L(s)C(s)} \tag{14}$$

Therefore,

$$G_{cl}(s) = \frac{\left(\frac{b_0}{a_1 s^\alpha + a_0}\right)\left(k_p + \frac{k_i}{s^\lambda} + k_d s^\mu\right)}{1 + \left(\frac{b_0}{a_1 s^\alpha + a_0}\right)\left(k_p + \frac{k_i}{s^\lambda} + k_d s^\mu\right)} \tag{15}$$

After simplification,

$$G_{cl}(s) = \frac{k_p b_0 s^\lambda + k_i b_0 + k_d b_0 s^{\mu+\lambda}}{a_1 s^{\alpha+\lambda} + (a_0 + k_p b_0)\, s^\lambda + k_i b_0 + k_d b_0 s^{\mu+\lambda}} \tag{16}$$

Now, dividing numerator and denominator by $s^{\alpha+\lambda}$ gives,

$$G_{cl}(s) = \frac{k_p b_0 s^{-\alpha} + k_i b_0 s^{-\alpha-\lambda} + k_d b_0 s^{-\alpha+\mu}}{a_1 + (a_0 + k_p b_0)\, s^{-\alpha} + k_i b_0 s^{-\alpha-\lambda} + k_d b_0 s^{-\alpha+\mu}} \tag{17}$$

If the output and input of the system is expanded in terms of Haar wavelet, then

$$\begin{aligned} (I_0^\alpha y)(t) &\approx Y^T P^\alpha H_M(t) \\ (I_0^\alpha r)(t) &\approx R^T P^\alpha H_M(t) \end{aligned} \tag{18}$$

Therefore, (17) can be expressed as,

$$Y^T D H_M(t) = R^T N H_M(t) \tag{19}$$

where,

$$N = k_p b_0 P^\alpha + k_i b_0 P^{\alpha+\lambda} + k_d b_0 P^{\alpha-\mu} \tag{20}$$

$$D = a_1 I + (a_0 + k_p b_0)\, P^\alpha + k_i b_0 P^{\alpha+\lambda} + k_d b_0 P^{\alpha-\mu} \tag{21}$$

Finally, $y(t)$ can be obtained from (18) and (19) as,

$$y(t) = R^T N D^{-1} H_M(t) \tag{22}$$

Consider available system response datasets $\{rdata(k), ydata(k)\}_{k=1,\cdots M}$, the method estimates the unknown transfer function parameters. The proposed technique attempts to solve the problem :

$$ISTE = \min_\rho \sum_{k=1}^{M} [k(y(k) - ydata(k))]^2, \tag{23}$$

where ρ is the vector of unknown parameters $(b_0, a_0, a_1, \alpha, k_p, k_i, k_d, \lambda, \mu)$, $y(k)$ is the time domain response of (22) calculated using ρ, $ydata$ is the collected step response to fit to $y(k)$. In (23) $y(k)$ and $ydata(k)$ are the simulated response and collected response data at time t_k, and M is the total number of data points in the collected system response. This routine aims to find the model parameters that would ideally reduce the integral of squared-time-weighted-error (ISTE) to zero. The MATLAB function `fsolve` is used to calculate the best estimated parameters which satisfies the above objective function in (23).

A. Technique to identify MagLev system in closed loop configuration

The closed loop representation of unknown MagLev system with IOPID controller is shown in Fig. 4. With known IOPID parameters, the step response of the system is obtained.

The fractional order system $L(s^\alpha)$ and its parameters a_0, a_1, b_0 and α are calculated using $y(t)$ in (22) with $N = k_p b_0 P^\alpha + k_i b_0 P^{\alpha+1} + k_d b_0 P^{\alpha-1}$ and $D = a_1 I + (a_0 + k_p b_0)\, P^\alpha + k_i b_0 P^{\alpha+1} + k_d b_0 P^{\alpha-1}$.

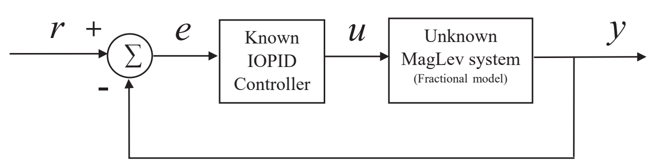

Fig. 4. Identification of MagLev system

B. FOPID tuning

After obtaining accurate system model in fractional-order, it is possible to extend the tuning for FOPID using obtained information. Bases on fractional order MagLev system identified in previous subsection, new FOPID parameters can be calculated. Fig. 5 shows the feedback control structure with FOPID.

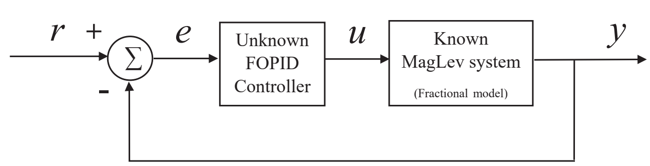

Fig. 5. FOPID tuning for MagLev system

In order to obtain the optimal values of new controller setting, we have adopted a well-known ISTE criteria as follows.

$$ISTE = \min_\rho \sum_{k=1}^{M} [k(e(k))]^2, \tag{24}$$

where $e(t) = r(t) - y(t)$. It is to note that $y(t)$ is now calculated again as per (22) with the unknown quantities $\rho = (k_p, k_i, k_d, \lambda, \mu)$.

V. Real-time validation and results

Following simple steps are followed as per previous discussion and obtained results to claim the applicability of the method.

Step (1): Using known IOPID parameters $k_p = 0.8$, $k_i = 0.45$, $k_d = 0.015$, $\lambda = 1$ and $\mu = 1$, $ydata$ and $rdata$ were obtained as shown in Fig. 6.

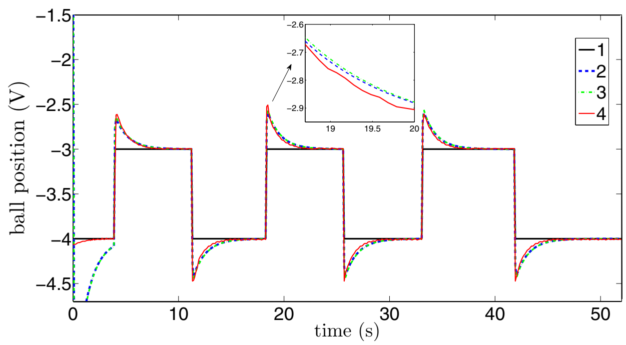

Fig. 6. Identified MagLev system : 1. setpoint (rdata) 2. identified fractional order model 3. identified integer order model and 4. real data (ydata)

Following the proposed technique of modeling, the integer order and fractional order models for MagLev system were obtained for the comparison.

$$L(s)\mid_{IOM} = \frac{120.8421}{0.0311s^2 - 31.5250} \tag{25}$$

$$L(s)\mid_{FOM} = \frac{120.8251}{0.0311s^{1.85} - 30.5251} \tag{26}$$

Step (2): Optimal new PID was obtained using integer model and objective function (24) as,

$$C_{IOPID}(s) = 2.312 + \frac{2.471}{s} + 0.191s \tag{27}$$

Similarly, a new FOPID was obtained using fractional model (26) as

$$C_{FOPID}(s) = 2.5 + \frac{6.01}{s^{0.875}} + 0.99s^{0.901} \tag{28}$$

The comparative results are shown in Fig. 7. It can be observed in simulation the effectiveness of FOPID over PID control. Results clearly show that FOPID improved the performance and response of the MagLev system. The improvement can be illustrated in terms of peak overshoot, steady state error and settling time.

VI. Conclusion

We have presented fractional-calculus based identification and controller design strategies for MagLev system to

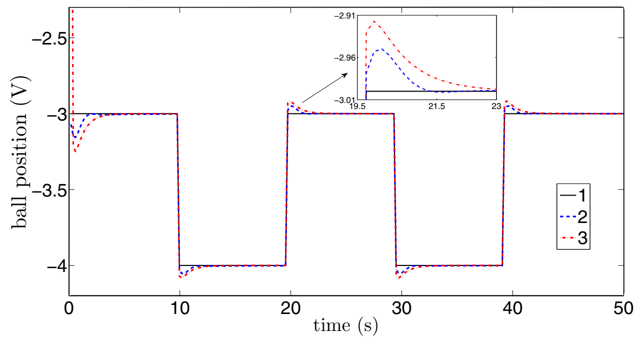

Fig. 7. Controller performance in simulation : 1. setpoint 2. FOPID performance 3. IOPID performance

regulate the position of the steel ball to a desired setpoint. Haar operational matrix converts fractional derivatives into simple algebraic matrix multiplication that eventually reduces computational complexity. The fractional order model represents the physical MagLev system more accurately than the conventional integer order model. The FOPID can give better performance in terms of less overshoot and quick settling time with observing same rise time with PID.

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