

# Dynamical Compensation of the Load Torque in a High-Performance Electrical Drive with an Induction Motor

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**Abstract - This paper describes a new method for dynamical estimation of load disturbance in induction motors by using Nonlinear Unknown Input Observers (NUIO). This estimation is then used to compensate dynamically the load torque in a Field Oriented Control (FOC) induction motor drive to increase its load-rejection capability. The method has been verified both in simulation and experimentally on a experimental rig.**

## I. INTRODUCTION

High-performance electrical drives represent the backbone of manufacturing and transportation industry. Consequently big attention has been paid over the best control strategies to obtain speed tracking and load rejection capability. With this respect, one of the key factors that can affect heavily the dynamical performance of an electrical drive is the presence of load disturbance on the shaft. A first way to make the system robust to their effects is to predict such disturbance in order to obtain better rejection capabilities, thus resulting in improved performance via proper dynamical compensation. This problem has been addressed in the case of induction motor drives in [1], where load torque is estimated by using the mechanical equation and assuming load torque has a slow dynamics. Other subsequent works [2][3][4] have followed the same approach. Another strategy that has been considered is to use dynamical filters or observers, in particular the Kalman Filters or the Unscented Kalman Filters [5][6][7], but at the cost of increasing complexity and requiring assumptions on the dynamical equation of the load.

This work follows on the second kind of approach, but it addresses the load torque estimation problem in an innovative way, by using the theory of Nonlinear Unknown Input Observer (NUIO) [8],[9], whereby the system states are reconstructed without the knowledge of the unknown disturbance input, such as the load torque.

The theory of observers originated from the work of Luenberger in 1964 [11]. According to Luenberger, any system driven by the output and the knowledge of the systems state space can serve as an observer for that particular system. Wang later used Luenberger approach to design UIO by estimating the both the states of the system and extracting the known inputs [12]. Later on, several approaches for designing linear UIO for linearized systems were proposed using different techniques [13]-[15].

However from the last two decades, research has shifted from linear to nonlinear system observers. Earlier, class of non-linear Lipschitz design was based on the linear part of the system by imposing certain conditions on the nonlinearity. However, this approach inherited drawbacks that were related to observer convergence conditions which is considered difficult to satisfy for large value of Lipschitz constant [16]. One way to get around this is the use of a NUIO which makes use of the differential mean value theorem (DMVT) together with Linear Matrix Inequalities (LMI) [17], which however

has the drawback of requiring the solution of LMI of high order matrices.

This paper addresses the state estimation of an induction motor, including its flux components, by using a NUIO as in [17]. The structure of the dynamic model admits a solution that can be easily handled by LMI. After the computation of the rotor flux linkages, the unknown disturbance load is reconstructed and dynamically compensated both in simulation and experimentally on a suitably developed experimental rig.

## II. NONLINEAR UNKNOWN INPUT OBSERVER (NUIO) [17]

Let the following class of nonlinear systems be considered, where  $x(t)$  represent the state vector,  $y(t)$  the output,  $u(t)$  the known inputs and  $v(t)$  the unknown input vectors,  $A, B, C, D$  suitable matrices, and  $f(x(t))$  a vector nonlinear function:

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t)) + Gu(t) + Dv(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where  $A \in \mathbb{R}^{nxn}$ ,  $f(x(t)) \in R^n$ ,  $u \in R^m$ ,  $v \in R^d$ ,  $y \in R^p$ ,  $x \in R^n$ .  $u$  is the known input vector,  $v$  is the unknown input vector , and  $y$  is the vector of measured outputs.

The following observer is then proposed

$$\begin{aligned} \dot{\hat{x}}(t) &= Nz(t) + Ly(t) + MGu(t) + Mf(\hat{x}) \\ \hat{x}(t) &= z(t) - Ey(t) \end{aligned} \quad (2)$$

where  $N, L$  and  $M$  are defined as:

$$\begin{aligned} N &= MA - KC \\ L &= K(I_p + CE) - MAE \\ M &= I_n + EC \end{aligned} \quad (3)$$

For the problem studied, the matrices and the state vector are as follows, considering the dynamical model of the IM (see Appendix I):

- The measured outputs are the direct and quadrature stator currents in the stationary reference frame as well as the rotor speed  $\omega_r$  in electrical rad/s
- $T_L$  is the unknown load torque.
- The known inputs are the direct and quadrature voltages in the stationary reference frame.

The target is to estimate simultaneously the states of the fluxes  $\psi_\alpha$  and  $\psi_\beta$  in the stationary reference frame and then the unknown load torque  $T_L$ .

By introducing the error with the estimate and its derivative:

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) = Mx(t) - z(t) \\ \dot{e}(t) &= M\dot{x}(t) - \dot{z}(t) \end{aligned}$$

The dynamics of the error can be obtained as:

$$\dot{e}(t) = Ne(t) + (MA - LC - NM)x(t) + M(f(x(t)) - f(\hat{x}(t))) + MDv(t) \quad (4)$$

By taking in consideration equation 3, the term  $MA - LC - NM = 0$ , therefore:

$$\dot{e}(t) = Ne(t) + M(f(x(t)) - f(\hat{x}(t))) + MDv(t)$$

It is known that:

$$f(x) - f(\hat{x}) \cong \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_5} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_5}{\partial x_1} & \dots & \frac{\partial f_5}{\partial x_5} \end{bmatrix} (\hat{x})(x - \hat{x}) = J_f(\hat{x})(x - \hat{x}) \quad (5)$$

where  $J_f(\hat{x})$  is the Jacobian of the function  $f(x(t))$  at the point  $\hat{x}$ , moreover, by using the “Differential Mean Value” (DMVT) theorem, it results that:

$$f(x) - f(\hat{x}) = J_f(c)(x - \hat{x})$$

Where  $c(t) \in [x(t), \hat{x}(t)]$

Thus

$$M(f(x) - f(\hat{x})) = MJ_f(c)e$$

Now, if there exists an E matrix such that (see further below (12)):

$$ECD = -D \rightarrow (EC + I_n)D = 0 \rightarrow MD = 0 \quad (6)$$

Then (4) becomes

$$\dot{e} = Ne + MJ_f(c)e = (N + MJ_f(c))e$$

Since  $N = MA - KC$ , this yields:

$$\dot{e}(t) = (MA - KC + MJ_f(c))e(t) \quad (7)$$

Let  $A + J_f(c) = A(J_f(c))$  where  $c$  varies with time inside the interval  $[x(t), \hat{x}(t)]$  whose boundary values also vary with time.

Thus

$$\dot{e}(t) = (MA(J_f(c)) - KC)e(t) \quad (8)$$

To prove that the error tends to zero, let the following Lyapunov function be introduced, where  $P$  is a symmetric positive definite matrix:

$$V(t) = e^T Pe(t) \quad P^T = P > 0 \quad (9)$$

By differentiating (6.9) the following equation is obtained:

$$V'(t) = e^T [PMA(J_f(c)) - PKC + A^T(J_f(c))M^T P - C^T K^T P]e$$

which must be negative definite and thus requires that:

$$PMA(J_f(c)) - PKC + A^T(J_f(c))M^T P - C^T K^T P < 0 \quad (10)$$

If all the component  $h_{ij} = \frac{\partial f_i}{\partial x_j}(c(t))$  of the Jacobian matrix, for any value of t, are bounded, i.e.:

$$\left| \frac{\partial f_i}{\partial x_j}(c(t)) \right| < \infty,$$

then each  $h_{ij}$  has a minimum  $\underline{h}_{ij}$  and a maximum  $\overline{h}_{ij}$ , so that  $h_{ij} \in [\underline{h}_{ij}, \overline{h}_{ij}]$ . As a consequence all of the elements  $h_{ij}(t)$  of the Jacobian matrix are contained in a parallelepiped  $H$ , whose faces are limited by  $\underline{h}_{ij}$  and  $\overline{h}_{ij}$ . This parallelepiped  $H$  has  $2^{n \times n}$  vertices  $\alpha$  given by  $\{\alpha_{ij}\}$ . The matrix  $J_f(\alpha)$  must be computed as many times as the number of vertices of the parallelepiped  $H$  and this is in general a strong limitation of the method for high values of  $n$ . However, in the case of the IM, since the values of  $J_f(\alpha)$  depend of the values of the 5 components of the  $x$  state vector, and since each of the components is upper and lower bounded in the same way, in total there are only 32 different configurations to be dealt with in the LMI, instead of the  $2^{5 \times 5}$  cases, which would be too cumbersome computationally. One possible configuration is given in (11), showing  $A(\alpha)$  with all upper bounds.

$$A(\alpha) = \begin{bmatrix} a_{11} & 0 & \frac{a_{12}}{T_r} & a_{12}\bar{x}_5 & a_{12}\bar{x}_4 \\ 0 & a_{11} & a_{12}\bar{x}_5 & \frac{a_{12}}{T_r} & a_{12}\bar{x}_3 \\ a_{21} & 0 & \frac{a_{22}}{T_r} & a_{22}\bar{x}_5 & a_{22}\bar{x}_4 \\ 0 & a_{21} & a_{22}\bar{x}_5 & \frac{a_{22}}{T_r} & a_{22}\bar{x}_3 \\ -\gamma\bar{x}_4 & \gamma\bar{x}_3 & \gamma\bar{x}_2 & -\gamma\bar{x}_1 & \delta_1 \end{bmatrix} \quad (11)$$

The following theorem holds [17]:

### Theorem

If (10) is satisfied in the vertices  $\{\alpha\}$  of the parallelepiped  $H$ , then it is satisfied also for all values of  $J_f(c(t))$  lying inside the parallelepiped  $H$ .

It is, therefore, only necessary to prove that (10) is satisfied in the vertices  $\{\alpha_{ij}\}$  so that:

$$PMA(\alpha) - PKC + A^T(\alpha)M^T P - C^T K^T P < 0 \quad \alpha \in \{\alpha_{ij}\} \quad (12)$$

This is a nonlinear matrix inequality that must be satisfied in all of the  $2^{n \times n}$  vertices of the parallelepiped  $H$  and is not easy to solve for the matrices  $P, M, K$ . There is a way, however, to transform it in an LMI (Linear Matrix Inequalities) by some simple transformations, as described below,

In any case, if (11) is satisfied, then also (10) is satisfied and (6) is such that  $e(t) \rightarrow 0$  ( $t \rightarrow \infty$ ).

In order to give a sufficient condition for the existence of the observer (11) is, as hinted above, transformed into a set of Linear Matrix Inequalities (LMI), making it easier to solve and for which there are many software toolboxes available. At first  $E$  is found out so that  $MD = 0$ , which makes the unknown input to be decoupled, so as to satisfy (6)

$$(I_n + EC)D = 0 \quad (13)$$

This means  $ECD = -D$ . This matrix equation gives solutions if:

$$\text{rank} \left[ \frac{CD}{D} \right] = \text{rank}[CD] \quad (14)$$

In the particular case of the IM, where  $p=3$  and  $n=5$ , this condition is satisfied (see Appendix 1).

All possible solutions of  $E$  are therefore given as

$$E = -D(CD)^+ + S(I_p - (CD)(CD)^+) \quad (15)$$

where the symbol  $+$  denotes the pseudo-inverse of a matrix, defined as  $X^+ = (X^T X)^{-1} X^T$  for a matrix  $X$ , and  $S$  is an arbitrary real matrix  $p \times p$ .

Taking in consideration that  $X^T X = I$ , by multiplying by  $(CD)$  both terms of (14) yields:

$$ECD = -D \quad (16)$$

And the required equality is obtained. By introducing the following matrices

$$U = -D(CD)^+ \quad V = I_3 - (CD)(CD)^+ \quad (17)$$

The equation (6.14) can be re-written as:

$$E = U + SV \quad (18)$$

In the case of the dynamical model of the IM,  $S$  has dimension  $3 \times 3$ .

By substituting  $M = I_n + EC$  (see eq. (3) and  $E = U + SV$  in (11) yields (if  $n=5$  as in the case of the IM):

$$\begin{aligned} ((I_5 + VC)A(\alpha))^T P + P(I_5 + VC)A(\alpha) + (VCA(\alpha))^T S^T P \\ + PS(VCA(\alpha)) - C^T K^T P - PKC < 0 \end{aligned} \quad \alpha \in \{\alpha_{ij}\} \quad (19)$$

which is still a nonlinear matrix inequality in matrices  $S, K$ . If  $PS = \bar{S}$  and  $PK = \bar{K}$  then (13) becomes:

$$\begin{aligned} ((I_5 + VC)A(\alpha))^T P + P(I_5 + VC)A(\alpha) + (VCA(\alpha))^T \bar{S}^T + \\ \bar{S}(VCA(\alpha)) - C^T \bar{K}^T - \bar{K}C < 0 \end{aligned} \quad (20)$$

which is, this time, a linear matrix inequality (LMI) in matrices  $P, \bar{S}, \bar{K}$

Inequality (19) has, therefore, a solution in  $P, S, K$  if inequality (20) has a solution in  $P, \bar{S}, \bar{K}$ . Thus, if there exists  $\bar{S}, \bar{K}, P > 0$  such that (20) has a solution in the vertices  $\{\alpha_{ij}\}$  then the observer has  $\lim_{t \rightarrow \infty} e(t) \rightarrow 0$  for any initial condition  $e(0)$ .

In order to compute the unknown input  $v$  it is more convenient to use the discretised form of the observer [17], where it is shown that it is given by ( $k$  is the discrete time and  $A_d$  is the matrix after discretization of the IM continuous model):

$$\begin{aligned} v(k) = [D^T D]^{-1} D^T [\hat{x}(k+1) - A_d \hat{x}(k) - f(\hat{x}(k)) \\ - Gu(k)] \end{aligned} \quad (21)$$

In the case under study the unknown input is the load torque  $\tau_L$ . Remark that in (21) this estimation is made for each instant of time (or discrete time  $k$ ), making it possible to use it for the load compensation. The scheme adopted is shown in figure 1, where the estimated load torque, computed by (22) is  $\tau_{est}$

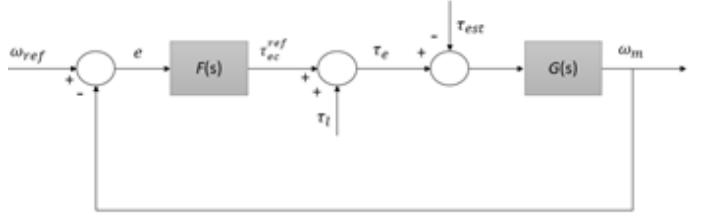


Figure 1: Closed loop speed control with load torque compensation

### III. RESULTS AND DISCUSSION

The NUIO for the induction motor has been included within a FOC (Field Oriented Control) induction motor drive where the rotor speed  $\omega_r$  is measured. The scheme is on figure 2.

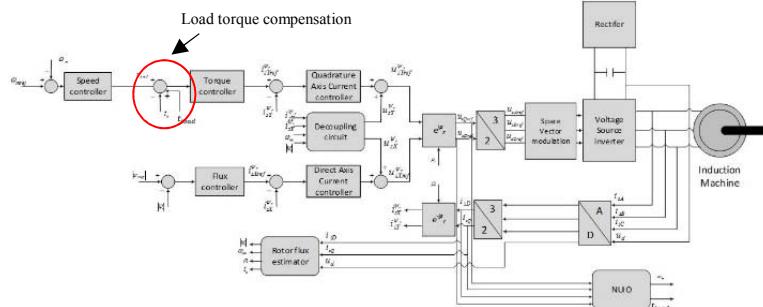


Figure 2: FOC scheme with NUIO and dynamic load torque compensation



Figure 3: Experimental rig of the IM electrical drive

The estimated load torque is used to improve the load-rejection capability of the electrical drive by compensating the actual load. As shown in figure 2, the estimated load torque is added to the summer, as indicated in the red circle, after the fashion proposed in figure 1.

The NUIO based dynamical load compensation has been tested in simulation and experimentally on a suitable experimental rig (Figure 3) with a 4-pole IM of 2.2 kW, rated voltage of 415 V, and rated speed of 1500 rpm. The load has been obtained by an electrical drive of a twin motor commanded in torque within a FOC strategy. This second drive is connected to the grid and a brake resistance is placed

in parallel to the DC-link and controlled by a chopper transistor.

The NUOI method is compared with the classical PI controller without torque compensation.

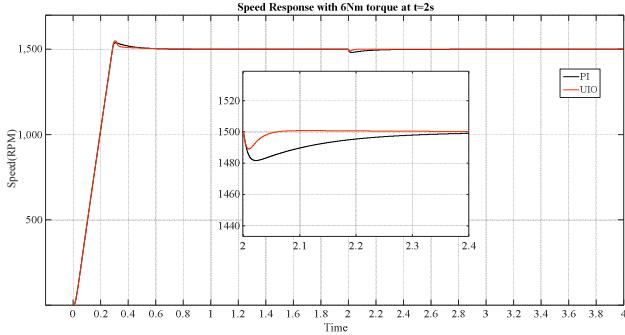


Figure 4: Rejection to load of 6 Nm at 2s (simulation)

Figure 4 shows the speed response in simulation with and without the dynamical load compensation obtained with the NUOI dynamical estimation of the load torque. The reference speed is 1500 rpm and at 2s a step load of 6 Nm has been given. It can be easily seen the significant improvement of the speed response as for the load-rejection capability in terms of reduced overshoot and tracking capability of the speed reference.

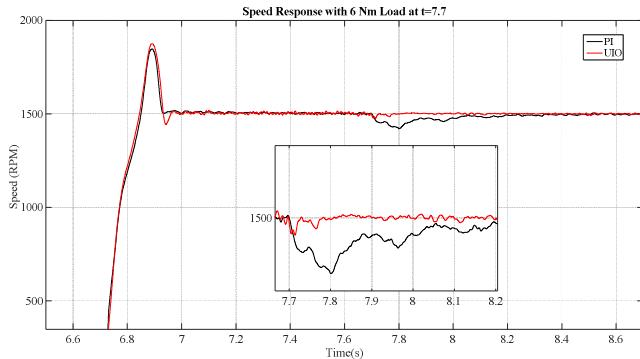


Figure 5: Rejection to load of 6 Nm at 7.7s (experimental)

Figure 5 shows the experimental results for the speed response with a speed reference of 1500 rpm when a load of 6 Nm is applied at 7.7s. The 2% steady state band is reached by the NUOI load compensated speed response well before the uncompensated speed response.

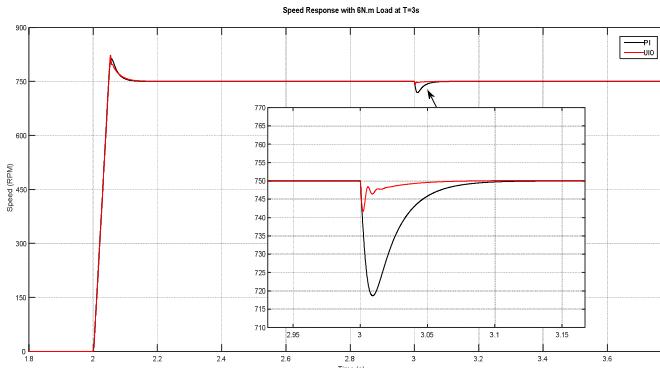


Figure 6: Rejection to load of 6 Nm at 3s (simulation)

Figure 6 shows the NUOI dynamical load torque compensation at reference speed of 750 rpm and at 3s a step load of 6 Nm has been given. It can be easily seen again the

improvement of the speed response as for the load-rejection capability of the drive in terms of reduced overshoot and tracking capability of the speed reference.

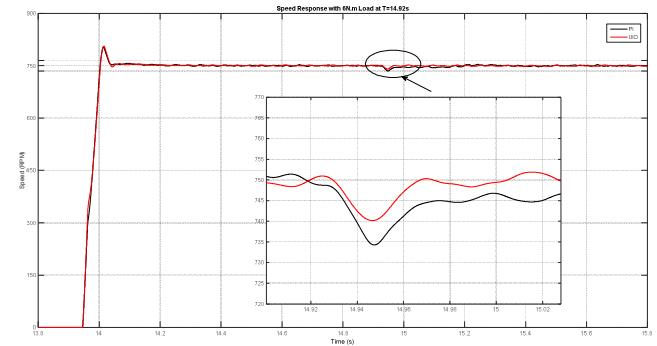


Figure 7: Rejection to load of 6 Nm at 14.92s (experimental)

Figure 7 shows the experimental results for the speed response when a speed reference of 750 rpm when a load of 6 Nm is applied at 14.92s. The 2% band for steady state is reached by the NUOI compensated speed much before the uncompensated speed and with less overshoot.

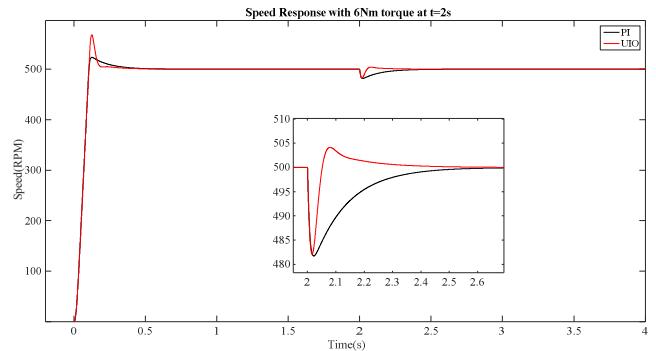


Figure 8: Rejection to load of 6 Nm at 2s (simulation)

Figure 8 shows in simulation the speed response when the reference speed is 500 rpm and when at 2s a step load of 6 Nm has been given. It can be easily seen the strong improvement of the load-rejection capability of the drive in terms of reduced overshoot and tracking capability of the speed reference.

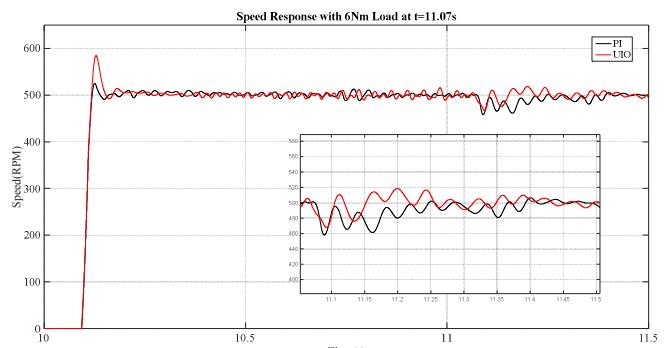


Figure 9: Rejection to load of 6 Nm at 11.07s (simulation)

Figure 9 shows the experimental results for the speed response with a speed reference of 500 rpm, when a load of 6 Nm is applied at 11.07s. The 2% band for steady state is reached by the NUOI compensated speed much before the uncompensated speed and with less overshoot.

#### IV. CONCLUSION

This paper presents the application of NUIO for the dynamical compensation of the load torque to improve the load-rejection capability of a FOC induction motor drive. The simulation and experimental results show much better performance for the NUIO compensated electrical drive with respect to classical one in terms of load-rejection and speed tracking.

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#### List of Symbols:

- $\sigma$  = Magnetic coupling coefficient
- $p$  = Number of Pole Pairs
- $R_s$  = Stator Resistance
- $R_r$  = Rotor Resistance

- $L_s$  = Stator Inductance
- $L_r$  = Rotor Inductance
- $L_m$  = Magnetizing Inductance
- $J$  = Motor Inertia
- $T_r$  = Rotor Time Constant
- $\delta_1 = -B/J$
- $\gamma = 3/2 p L_m / (L_r J)$
- $\delta = -1/J$
- $\tau_L$  = Load Torque

$$\begin{aligned} \mathbf{A}_{11} &= -(R_s / (\sigma L_s) + (1-\sigma) / (\sigma T_r))I = a_{11}I \\ \mathbf{A}_{12} &= L_m / (\sigma L_s L_r) \{(1/T_r)I - \omega_r J\} = a_{12} \{(1/T_r)I - \omega_r J\} \\ \mathbf{A}_{22} &= -(1/T_r)I - \omega_r J = a_{22} \{(1/T_r)I - \omega_r J\} \\ \mathbf{A}_{21} &= \{L_m / T_r\}I = a_{21}I \\ \mathbf{B}_1 &= 1 / (\sigma L_s)I = bI \end{aligned}$$

$$C = [I \ 0]$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

#### APPENDIX 1

The values of the matrices used in the NUIO are:

$$\begin{aligned} \begin{bmatrix} i_{sd} \\ i_{sq} \\ \dot{\psi}_{rd} \\ \dot{\psi}_{rq} \\ \dot{\omega}_r \end{bmatrix} &= \begin{bmatrix} a_{11} & 0 & \frac{a_{12}}{T_r} & 0 & 0 \\ 0 & a_{11} & 0 & \frac{a_{22}}{T_r} & 0 \\ a_{21} & 0 & \frac{a_{22}}{T_r} & 0 & 0 \\ 0 & a_{21} & 0 & \frac{a_{22}}{T_r} & 0 \\ 0 & 0 & 0 & 0 & \delta_1 \end{bmatrix} \begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \\ \dot{\psi}_{rd}(t) \\ \dot{\psi}_{rq}(t) \\ \omega_r(t) \end{bmatrix} + \begin{bmatrix} a_{12}\psi_{rq}(t)\omega_r(t) \\ a_{12}\psi_{rd}(t)\omega_r(t) \\ a_{22}\psi_{rq}(t)\omega_r(t) \\ a_{22}\psi_{rd}(t)\omega_r(t) \\ \gamma(\psi_{rd}(t)i_{sq}(t) - \psi_{rq}(t)i_{sd}(t)) \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \delta \end{bmatrix} \\ \mathbf{x} &= \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \\ \psi_\alpha(t) \\ \psi_\beta(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}, A = \begin{bmatrix} a_{11} & 0 & \frac{a_{12}}{T_r} & 0 & 0 \\ 0 & a_{11} & 0 & \frac{a_{12}}{T_r} & 0 \\ a_{21} & 0 & \frac{a_{22}}{T_r} & 0 & 0 \\ 0 & a_{21} & 0 & \frac{a_{22}}{T_r} & 0 \\ 0 & 0 & 0 & 0 & \delta_1 \end{bmatrix}, G = \begin{bmatrix} b & 0 \\ 0 & b \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ D &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \delta \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, f(x) = \begin{bmatrix} a_{12}x_4x_5 \\ a_{12}x_3x_5 \\ a_{22}x_4x_5 \\ a_{22}x_3x_5 \\ \gamma(x_3x_2 - x_4x_1) \end{bmatrix} \end{aligned}$$

Thus, n=5, p=3, m=2, d=1.

$$\begin{aligned} CD &= \begin{bmatrix} 0 \\ 0 \\ \delta \end{bmatrix} \text{ and } \begin{bmatrix} CD \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ which are both vectors and then their rank is 1} \\ (CD)^+ &= ((CD)^T(CD))^{-1}(CD)^T = \begin{bmatrix} 0 & 0 & \frac{1}{\delta} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ U &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

In the case of the IM, the matrices to be used are:

$$\begin{aligned} (I_5 + VC) &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \\ &\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

$$A(\alpha) = A + J_f = \begin{bmatrix} a_{11} & 0 & \frac{a_{12}}{T_r} & 0 & 0 \\ 0 & a_{11} & 0 & \frac{a_{12}}{T_r} & 0 \\ a_{21} & 0 & \frac{a_{22}}{T_r} & 0 & 0 \\ 0 & a_{21} & 0 & \frac{a_{22}}{T_r} & 0 \\ 0 & 0 & 0 & 0 & \delta_1 \end{bmatrix} + J_f$$

$$J_f = \begin{bmatrix} 0 & 0 & 0 & a_{12}x_5 & a_{12}x_4 \\ 0 & 0 & a_{12}x_5 & 0 & a_{12}x_3 \\ 0 & 0 & 0 & a_{22}x_5 & a_{22}x_4 \\ 0 & 0 & a_{22}x_5 & 0 & a_{22}x_3 \\ -\gamma x_4 & \gamma x_3 & \gamma x_2 & -\gamma x_1 & 0 \end{bmatrix}$$

$$VC = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$