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Integer-valued polynomials on the Hurwitz ring of integral quaternions. (English summary)
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Let $\mathbb{H}$ denote the ring of Hurwitz quaternions, $\mathbb{H}(\mathbb{Q})$ the division algebra of rational quaternions, and $\mathrm{Int}(\mathbb{H})=\{f(x)\in\mathbb{H}(\mathbb{Q})[x]: f(\mathbb{H})\subseteq\mathbb{H}\}$ the $\mathbb{H}$-algebra of quaternionic polynomials taking values in $\mathbb{H}$ at elements of $\mathbb{H}$. For a fixed nonnegative integer n, let c_n denote a generator for the left ideal consisting of elements $c\in\mathbb{H}$ for which $c\cdot f(x)\in\mathbb{H}[x]$ if $f(x)\in\mathrm{Int}(\mathbb{H})$ is of degree $\le n$. There exists a unique integer k_n for which $c_n=(1+i)^{k_n}w_n$ and $w_n\in\mathbb{H}$ is not left divisible by the prime element $1+i$. The paper under review provides lower and upper bounds for this k_n. The authors establish that $k_n\ge\frac{2}{5}(n-\sum n_i)+\sum\overline{n}_i$ where $n=\sum n_i6^i$ is the expression of n in base 6 and $\overline{n}_i=0$ if $n_i<4$ and $\overline{n}_i=1$ if $n_i=4$ or 5. A computational method is provided for producing an upper bound for k_n, and it is shown that the upper and lower bounds thus produced coincide for $n\le 6$.

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