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The constraint optimization approach for robust PID design in AVR system

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A constraint optimization approach is discussed for estimating the proportional-integral-derivative (PID) controller gains used for an AVR system. A new evaluation function is given to ensure the less variation in the control signal input to the system without compromising the overall dynamic responses. The robustness of the system is guaranteed by imposing the maximum sensitivity in solving the optimizing problem. The proposed method ensures the better performance even in the presence of the uncertainties in plant parameters. The simulation studies are presented to validate the design method.

Keywords: AVR system, control efforts, robust control, particle swarm optimization

1. INTRODUCTION

The problem of maintaining a terminal voltage magnitude of a synchronous generator at a specified level due to increase in the reactive power load of the generator is known as automatic voltage regulator (AVR). In this AVR system the voltage output is senses through a potential transformer on one phase is rectified and compared to a dc setpoint signal [1]. The error signal controls the exciter field through proper setting of the amplifier gain which therefore increases the exciter terminal voltage. This results in an increase in the generator voltage. The crude method of setting the amplifier gain to obtain the desired terminal voltage always results in an unstable system. One way to protect this is to add a rate feedback controller in the AVR system; however the result shows with some steady-state errors in the output.

The achievement of a desired response in addition to a low-cost setting-up is essential requirements for any closed loop controlled system. Proportional-integral-derivative (PID) controllers are most common adopted in industry because of their simplicity, robustness, and successful practical applications. They have few tuning parameters and provide good control performance in most situations. The controller parameters can be determined from the identified transfer function models by using one of the tuning methods devised in the last 60 years [2, 3]. Some methods of PID tuning proposed in the literature are the Ziegler-Nichols method, the refined Ziegler-Nichols method, the Cohen-Coon method, the internal model control, the gain and phase margin method [4]. One of the methods for obtaining the optimal PID parameters is by minimizing an integral performance criterion (integral square error, integral absolute error etc.) [5].

Despite the many collection of tuning rules on PID, many industrial control loops operate poorly. In testing thousands of control loops in hundreds of operating plants, Techmation Inc. and others have found that more than 30% of the control loops actually increase variability over manual control [6]. In addition, pulp and paper mill auditing has shown that a large percentage of control loops actually de-stabilize product uniformity as a result of either valve stiction or poor controller tuning [7]. In general, it is difficult to obtain general tuning rules which provide an optimal or near optimal solution for PID gains. For this reasons, many literature have already been reported the optimal design of PID controller parameters using varies evolutionary algorithms like genetic algorithm (GA) [8], modified particle swarm optimization (PSO) [9–14]. Though the GA approach has been employed successfully to solve complex optimization problem, it sometimes degrades its performance in search capability due to the premature convergence [15].

Kennedy and Eberhart [16] were first to introduce the PSO algorithm in 1965. The idea behind this algorithm is inspired by the social behavior of flock of birds (called particles). The most important features of the PSO algorithm that make it a preferred algorithm for optimization by many researchers are that it is developed over a simple theoretical framework and is relatively easy to program and implement. It is also shown [24] that the PSO is a computationally inexpensive and has low memory requirements. In addition to this, the PSO has relatively small number (3–5) of user-defined parameters and they are not very sensitive to the convergence and final accuracy of the algorithm. In [17] it is shown that the PSO algorithm is robust in solving many continuous nonlinear optimization problems.

The above features and merits of the PSO algorithm have been the main reason for its selection as an optimization algorithm for optimal design and tuning of the parameters of the PID controller in this paper. Details about the new proposed version of the PSO with constraints is given further on in Section 4.

Efforts are made in the literature to solve some complex power system operation problems using the PSO [11–13, 17 – 21]. It has been presented that the optimization using the PSO method gave an improved solution in tuning of PID gains. In most study the controllers were optimized by minimizing integral performance criteria [5] or a criterion based on time domain specification of step responses [11]. However, the optimal controller by the reported methods may not be sufficient to obtain the optimum control input variations. In most PID control loops valves are the only components with moving parts and more variation in control signal has high costs in terms of valve wear and maintenance programs. In this context, less attention has been paid, although in [22, 23] a different kind of criterion has been defined to minimize the variations in the out of the controller.

In this paper, a new performance index is given in order to reduce the control input variation which will protect the actuator from the high maintenance cost and also guarantee the robustness of the closed-loop system against model discrepancy. It is therefore important that the controller design problem is formulated into two level of performance measured: the robustness of the system is obtained by constraints on the maximum sensitivity and the smoothness of the control actions is optimized via the time derivative of the error signal. We have adopted a practical high-order AVR system with a PID controller to evaluate the performance of the new measuring index.

2. SYSTEM MODELING AND PROBLEM FORMULATION

A closed-loop controlled system in this paper is composed of a linearized model of AVR system and PID controller as shown in Fig. 1, where the actual terminal voltage is V_T, the desired reference voltage is R and the controller output is U. Primary goal of an AVR system is to hold the terminal voltage amplitude of a synchronous generator at a specified level. A simplified system comprises four main blocks, namely amplifier, exciter, generator, and sensor. The linearized models of each function blocks have taken into account the gains and time constants and ignored any type of nonlinearities. In this article we have adopted the model parameters given in [1] to test the proposed constraint optimization scheme. Table 1 shows the AVR system of a generator with constant parameter values.

Table 1 Parameter values for an AVR system.

	Gain	Time constant
Amplifier	$K_A = 10$	$\tau_A = 0.10$
Exciter	$K_E = 1$	$\tau_E = 0.40$
Generator	$K_G = 1$	$\tau_G = 1.00$
Sensor	$K_R = 1$	$\tau_R = 0.05$

A structure of the PID controller is considered by a transfer function

$$C(s) = k_p + k_i s^{-1} + k_d s \quad (1)$$

where, k_p is proportional gain, k_i is integral gain and k_d is derivative gain of the controller.

In most research, the parameters $(k_p, k_i, k_d)$of the controller are based on known mathematical models. However, a modelling error usually causes unexpected conditions at the time of real-time applications of the PID. Therefore it is always desired to obtain the PID gains so that it provides robust stability against modelling errors and variations in plant dynamics. Åström and Hägglund [4] define the sensitivity function M_s to capture the sensitivity to small variations in plant dynamics and its maximum thus serves as a significant representative of closed-loop stability and robustness.

The maximum sensitivity (M_s) is the inverse of the shortest distance from the Nyquist curve to the critical point -1 and is given by

$$M_s = ||[1 + P(j\omega)C(j\omega)]^{-1}|| \quad (2)$$

where $P(j\omega)$ is the plant frequency response. The reasonable values of the maximum sensitivity, M_s are in the range of 1.3 to 2 [4], for the robust stability of the closed loop controlled system.

3. PROBLEM FORMULATION

A controller is designed to minimize the error $E(t)$ between the reference R(t) and the controlled variable $V_T(t)$. Hence, a criterion worth characterizing the time response of a system is usually given as an integral function of the error or its weighted products [5]. It was proved that the integral squared time error (ISTE) index gives good step response results with less overshoot and short settling time. Mathematically, the performance criterion ISTE can be expressed as

$$J_{ISTE} = \int_0^\infty t^2 E(\eta, t)^2 \mathrm{dt} \quad (3)$$

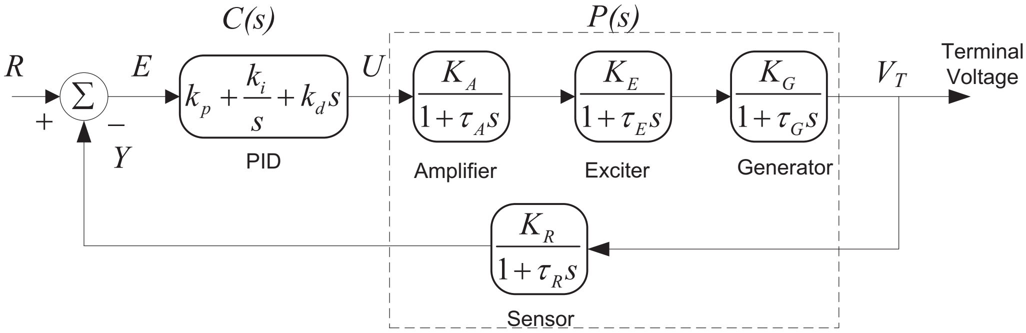

Figure 1 AVR system with PID controller [1].

However, the optimal controller by this index may not be sufficient to obtain the optimum control input variations. In most PI(D) control loops valves are the only components with moving parts and more variation in control signal has high costs in terms of valve wear and maintenance programs. In this work, the integral of $t^2\dot{E}(t)$ squared is used as a new objective function to penalize excessive excursion of control effort. The modified objective function therefore, becomes

$$J_{\min} = \int_0^\infty t^4 \dot{E}(\eta, t)^2 \mathrm{dt} \qquad (4)$$

In the process of optimization of PID values, first the transient response is optimized to obtain the minimal value of the index (4) that will give minimum total variation in the control variable. A constraint on the index (4) is introduced in the upper level to ensure that the resultant system is guaranteed stable against modeling error and variations in plant dynamics.

In order to set a good step response together with optimum control efforts and to guarantee the robustness of the system, the controller design scheme is formulated as follows

$$\left.\begin{array}{l} \eta^\dagger = \arg\min\limits_{\eta} \int_0^\infty t^4 \dot{E}(t)^2 \mathrm{dt} \\ \text{subject to: } M_s < M_s^* \end{array}\right\} \text{optimization problem} \qquad (5)$$

where $\eta = (k_p, k_i, k_d)$ is the vector of controller parameters and M_s^* denotes the specified value of maximum sensitivity. The robustness of the system is guaranteed by a constraint on the maximum sensitivity in this problem.

4. PROPOSED CONSTRAINED PSO ALGORITHM FOR TUNING CONTROLLER GAINS

The original PSO algorithm in [16] has gone through several modifications [25] with the aim of improving its convergence speed or decreasing the number of the user-defined parameters. One of the most popular versions is the PSO with inertia weight, as described in [25].

$$\begin{aligned} \mathbf{v_i} &\leftarrow \omega\mathbf{v_i} + U(0, \varphi_1) \\ &\quad \otimes (\mathbf{p}_i - \mathbf{x}_i) + U(0, \varphi_2) \otimes (\mathbf{p}_g - \mathbf{x}_i), \\ \mathbf{x_i} &\leftarrow \mathbf{x_i} + \mathbf{v_i} \end{aligned} \qquad (6)$$

where $i = 1, 2, ..., N$; ω is the inertia weight and N is the number of particles (usually $N <= 40$). The other parameters are, as follows:

$\mathbf{x}_i$ is the current location of the particle and $\mathbf{v}_i$ is the velocity (step) of the particle. The parameters ϕ_1 and ϕ_2 in (6) determine the magnitude of the random forces in the direction of personal best $\mathbf{p_i}$ and neighborhood best $\mathbf{p}_g$. These are often called acceleration coefficients.

$U(0,\phi_j)$, j=1,2; represents a vector of random numbers uniformly distributed in $[0,\phi_j]$ which is randomly generated at each iteration and for each particle; $\otimes$ is component-wise multiplication.

In the original version of PSO, each component of the step $\mathbf{v}_i$ is kept within the range $[-\mathrm{v}_{\max},+\mathrm{v}_{\max}]$ in order to represent the physical "power" of the birds. As for the inertia weight ω, it is experimentally found that the best performance could be obtained by initially setting ω to some relatively high value (e.g., 0.9 or even higher than 1.0), which corresponds to a system where particles move in a low viscosity medium and perform extensive exploration, and gradually reducing ω to a much lower value (e.g., 0.4 – 0.2), where the system would be more dissipative and exploitative and would be better at homing into local optima [25].

The above described original PSO algorithm is unconstrained. This means that it starts the search for the optimum at a given (user-defined) point or a small area within the parameter space and the trajectory of the birds flock (the particles in the swarm) during the optimization is unlimited within the space. As a result, the optimum could be found in area that lies extremely far from the initial starting point. While it could be theoretically correct, in many real applications such solution would be not practical. This will be definitely the case when applying the standard version of the PSO for tuning the parameters of the PID controller. All three parameters (k_p, k_i, k_d) have their own practical boundaries [min, max] and any value outside will be infeasible.

The idea of our proposed constrained PSO algorithm is very simple, namely: at each case (iteration) when a given boundary is violated by any of the particles, the particle i is returned to its previous position $\mathbf{x}_i$ and the step $\mathbf{v}_i$ is reversed with the same magnitude, but in the opposite direction, i.e. $\mathbf{v}_i = -\mathbf{v}_i$.

This simple heuristics has been tested on many simulated examples and it has been proven to work very stable. Some results on a two-dimensional test function are graphically shown in the following 3 figures.

Figure 2 shows the performance of the constrained PSO algorithm in the case when the global optimum is within the pre-determined boundaries (constraints). The following two figures Figs. 3 and 4 illustrate such typical cases when the global optimum is outside the fixed boundaries. It is seen that the algorithm tries to find the global optimum, but because it has no access beyond the constraints, the swarm of particles is constantly moving close to the boundary. As a result the swarm finds the so called conditional optimum.

5. SIMULATION AND RESULT DISCUSSION

M_s^* The design method has been tested on practical high-order AVR system with typical plant model parameters as shown in Table 1. In the study, is chosen as 1.6 to ensure the feasibility of the problem, which also allows 'faster' and 'robust' controller to be obtained from the proposed method. In order to examine the control performance by optimizing the index in (5), we measure the total variation in $U(t)$ which gives the performance index of the controller. If we discretize the input signal as a sequence, $U(KT) = [u_1, u_2, ..., u_i, ..]$then the total variation is

$$\text{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i| \tag{7}$$

which should be as small as possible to minimize variations in $U(t)$. The proposed method has been tried to obtain the optimal values of controller gains so that the desired transient responses are obtained with smooth input usage. In order to evaluate the merits of the proposed constraint optimization tuning, we also implemented the PSO method using the evaluation function given in (3) based on ISTE performance index [5]. We have also compared our results with the PID values depicted in Hadi [1] for the same AVR system. The following PSO parameters are utilized in the simulation after many trials in searching the best results by the proposed method.

Initial setting of PID controller is $(k_p, k_i, k_d) = (0.5, 0.2, 0.2)$

Population size=25

Inertia weight factor ω is set according to the following equation

$$\omega = \omega_{\max} - (\text{it - 1})\left(\frac{\omega_{\max} - \omega_{\min}}{\text{it}_{\max} - 1}\right)$$

where
$\omega_{\max} = 0.9$
$\omega_{\min} = 0.4$
it = current iteration number
$\text{it}_{\max}$ = maximum iteration number

$\varphi_1$and $\varphi_2$of (6) are set to 0.2

The algorithm has been developed in MATLAB 7.6 on Windows 7 core i5 Intel 4 GB RAM. The simulation results are plotted for terminal voltage outputs and respective control signal inputs for three different approaches. The performances are compared with respect to settling time (T_s) and most importantly total variations in control signal. The index given in (7) reflects the control efforts required by the method to obtain the setpoint input. The minimum value of TV with superior performance of the controlled system proves the best optimal setting of the controller gains. Table 2 shows the comparison results with other approaches. The last column in this

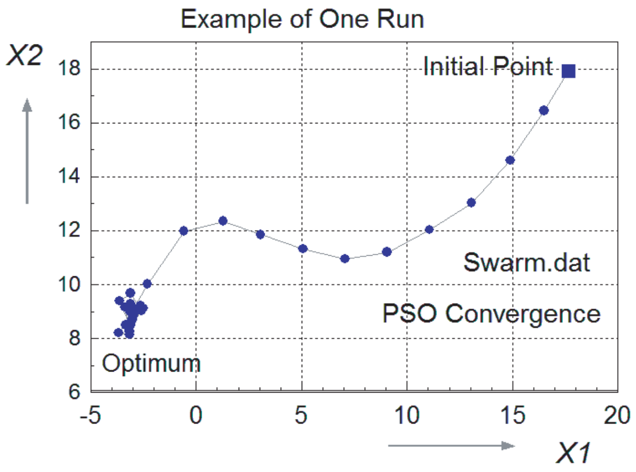

Figure 2 An example result when global optimum value is within boundaries.

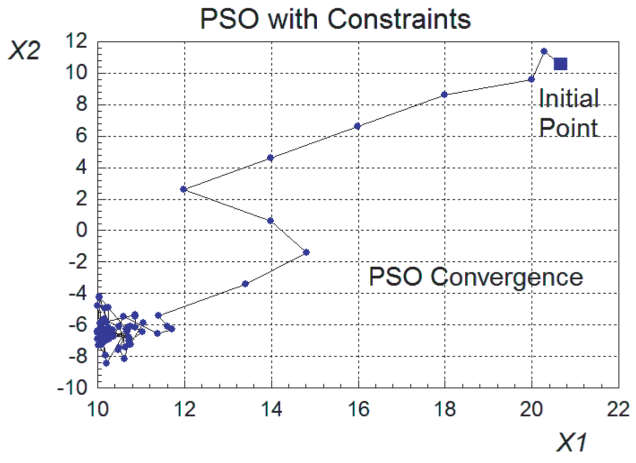

Figure 3 An example result when global optimum value is outside boundaries.

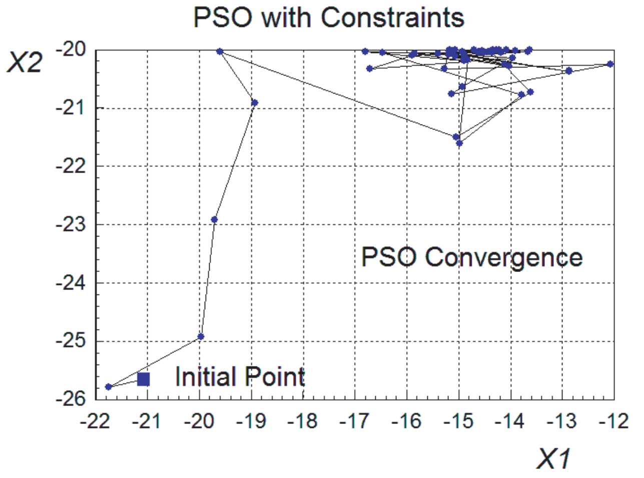

Figure 4 Other example result when global optimum value is outside boundaries.

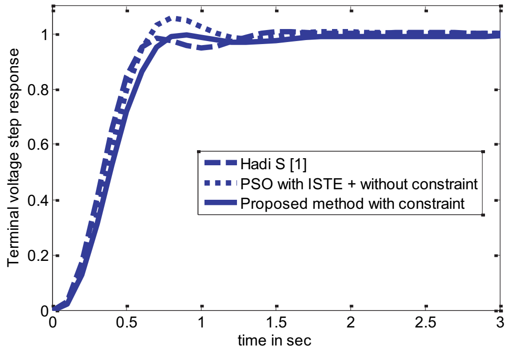

Figure 5 Terminal voltage responses

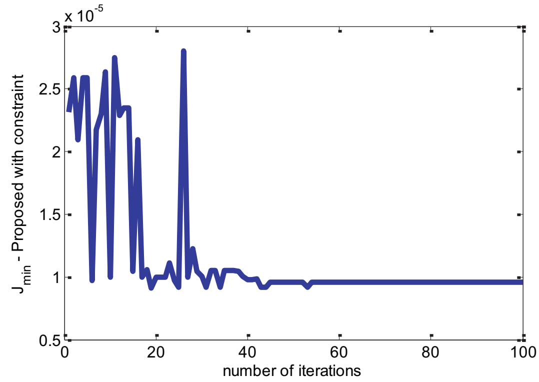

Figure 8 Best values of J from proposed scheme

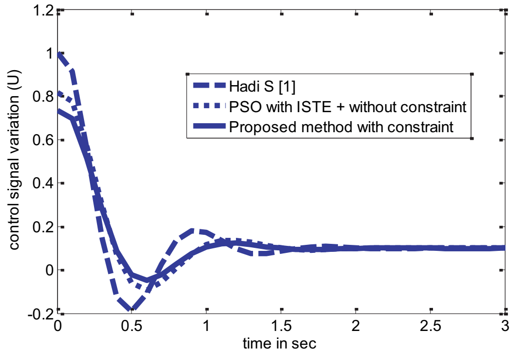

Figure 6 Control efforts for nominal system

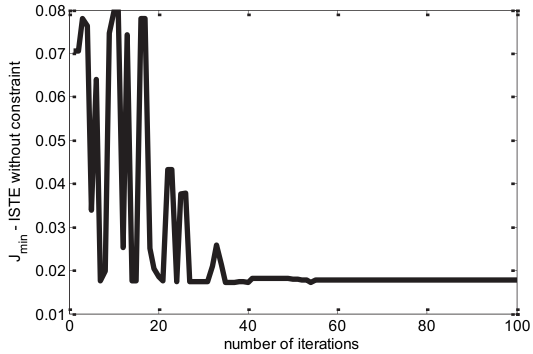

Figure 9 Best values of J from ISTE

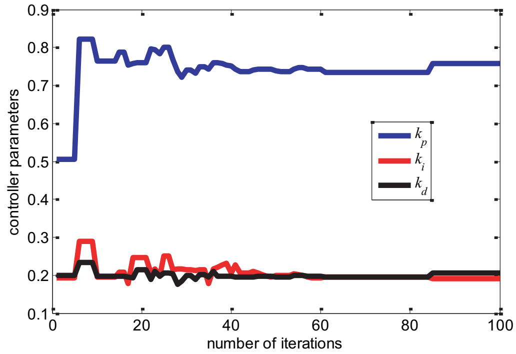

Figure 7 Controller parameters at the end

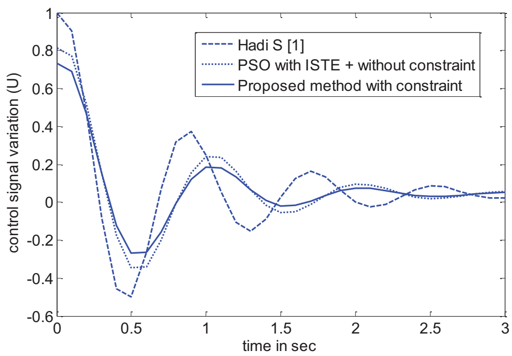

Figure 10 Robust responses for perturbation

table indicates the proposed optimization approach with constraint has better efficiency with respect to less setting time and control signal variations. Fig 3 and 4 show the enhanced performance of the proposed method with good subsequent control performance. It can be seen from the Table 2 that the optimization problem solved with (5) gives less control signal variation with TV=1.721, while in the case of ISTE in (3) and Hadi [1], the values are 2.009 and 2.716, respectively. The controller parameters corresponding to the best individual after each iteration of the constraint PSO optimization is shown in Fig. 4. The best values of performance index (J) from the new index in (5) and the ISTE index in (3) are plotted with each iteration number in Fig. 5 and 6, respectively.
of each generation

To show the robust performance of the control design, perturbations of ±50% in (K_A, K_G, τ_A, τ_G) of the nominal system are assumed. The robust responses for control signal variations are compared with those methods by Hadi [1] and PSO ISTE without constraint in Fig. 7. It is observed that the closed-loop performances are very robust by the given method mainly due to less variation of control signal or efforts in the condition when it is influenced by the parameter uncertainties. The impact of control performances during parameter variations are given in Table 3.

6. CONCLUSION

A unified performance index with constraint of maximum sensitivity was studied for the AVR system to hold the terminal voltage magnitude of a synchronous generator at a specified level. A new performance criterion was used for evaluating

Table 2 Comparison of the proposed method for nominal system

Method	**PID setting** (k_p, k_i, k_d)	**Robustness** M_s	**Setting time** T_s **(in sec)**	**Total variation in** $U(t)$ **i.e. control effort TV**
Hadi S [1]	(1.000, 0.250, 0.280)	1.853	1.025	2.716
PSO with ISTE and without constraint	(0.815, 0.254, 0.196)	1.676	0.886	2.009
Proposed method with constraint optimization	(0.734, 0.195,0.195)	1.597	0.699	1.721

Table 3 Impact of control performances during parameter variations.

Method	**Total variation in** $U(t)$ **(+50% variations in** K_A, K_G, τ_A, τ_G**)**	**Total variation in** $U(t)$ **(-50% variations in** K_A, K_G, τ_A, τ_G**)**
Hadi S [1]	4.665	1.760
PSO with ISTE and without constraint	3.165	1.381
Proposed method with constraint optimization	2.565	1.226

the PID gains to perform the optimum usage of input control efforts. Simulation results show that the proposed constraint optimization technique is capable to obtain the desirable responses with smooth input usage. Therefore, this will help to reduce high maintenance cost of wear and tear on valve parts. Also it can be seen that despite such large parameter variations, the system responses do not show notable differences from the nominal values. Therefore, the simulation results demonstrate the robustness of the controller against system parameter variations.

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