Cooperative Control of Multi-Robot Systems with a Low-degree Formation

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Abstract. The utilization of team of robots working in a cooperative manner has huge benefits in moving large payloads. To perform such tasks, a multi robot structure or formation is necessary to coordinate the motions of the robots in a well planned manner. In this paper, the formation control problem of multi car-like mobile robots have been studied. The purpose is to get a swarm of mobile robots in a certain formation pattern to track a desired trajectory to accomplish a set objective. A set of artificial potential field functions is proposed using the Direct Method of Lyapunov for avoiding inter-robot, inter-formation and obstacle collisions and attraction to their designated targets. The effectiveness of the proposed nonlinear acceleration control laws is demonstrated through computer simulation results which prove the efficiency of our control technique and also demonstrates scalability for a large group of robots.

Keywords: Formation, Lyapunov; nonholonomic mobile robots, low-degree.

1 Introduction

Multi-Robot Systems (MRSs) have great advantages over single robot systems in terms of efficiency, redundancy, flexibility, robustness, cost benefits[1]. There are various areas which have benefited from the use of MRSs. Some of these include underwater and space exploration, hazardous environments, service robotics, entertainment, military rescue and reconnaissance, controlling formations of satellites and autonomous vehicle [2]. Cooperative control of MRSs have received considerable attention due to the fact that several collaborative tasks can be performed more efficiently and robustly using MRSs which may not be possible with individual robots acting alone. The benefits become apparent when considering distributed tasks, dangerous or hazardous tasks, tasks which require redundancy [3], and can also provide flexibility to task execution and robustness [1]. In many cases, the execution of the task requires formation control [4], and the accomplishment of the overall operation depends on each mobile robot operating in a desired manner. Cooperative Control problem discussed in this paper is referred to the problem of MRSs to coordinate their motions on executing tasks cooperatively in a given workspace. From recent literature, one of the most critical applications of cooperative behavior is in payload transportation [5]. In such applications, motion coordination with minimal error is required. Control strategies for formation or cooperative control of mobile robots can be roughly categorized into virtual structure, behavior-based and leader follower schemes [6]. Authors have proposed different schemes using different control methodologies for coordinating behaviors of MRSs. These control methods include centralized, completely decentralized or hybrid [7]. [8] proposed an adaptive scheme using graph theory together with Lyapunov based techniques to enclose a moving target and attain a desired inter robot formation. Hybrid systems comprising of distributed smooth-time varying feedback control laws together with a reactive control framework is proposed in [4] and [9] while synchronization in the context of Lagrangian systems control was proposed in [10] for cooperative robot control. [5] proposed a cluster space control to control a group of four nonholonomic wheeled robots for object transportation.

We propose a leader-follower scheme to develop a cooperative formation controller for coordinating groups of MRSs. In this control methodology, we assign a leader in each MRS, who takes the responsibility to specify the objective of the task and one who also dictates a geometric path from some initial configuration to some final configuration. The control strategy formulates a low degree formation which allows for slight distortions in the formation structure of the leader and follower. These distortions would normally appear if the group encounters an obstacle. As in [11], this low degree formation structure is strict but allows for slight but temporary distortions for the formation to avoid obstacles. Based on artificial potential fields, the Direct Method of Lyapunov is then used to derive continuous acceleration-based controllers which render our system stable. The remainder of this paper is structured as follows: in Section 2, the robot model is defined; in Section 3, the artificial potential field functions are defined under the influence of kinodynamic constraints; in Section 4, the acceleration-based control laws are derived, while in Section 5, stability analysis of the robotic system is carried out; in Section 6, we demonstrate the effectiveness of the proposed controllers via computer simulations which guide the follower robot to track the leaders reference path while maintaining a low-degree formation; and finally, Section 7 concludes the paper and outlines future work in the area.

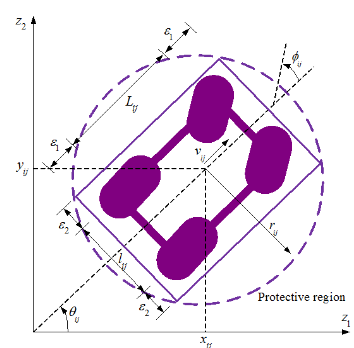

Fig. 1. Kinematic model of the agent $\mathcal{A}_{hi}$.

2 Vehicle Model

In this section, we derive a kinematic model for multiple formations of car-like mobile robots. We will consider h, $h \in \mathbb{N}$ formations with n, $n \in \mathbb{N}$, car-like mobile robots in the Euclidean plane, where the ith agent in the hth formation is denoted by $\mathcal{A}_{hi}$. We denote the hth formation as $\mathcal{A}_h$. Without loss of generalization, we let $\mathcal{A}_{h1}$ represent the leader in the hth formation structure while the others in $\mathcal{A}_h$ take the role of followers. With reference to Fig. 1 and for $h = 1, \ldots, M$, $i = 1, \ldots, n$, (x_{hi}, y_{hi}) represents the Cartesian coordinates and gives the reference point of $\mathcal{A}_{hi}$.

Moreover, θ_{hi} gives the orientation of $\mathcal{A}_{hi}$ with respect to the z_1-axis of the z_1z_2-plane while ϕ_{hi} gives the steering angle with respect to its longitudinal axis of $\mathcal{A}_{hi}$.

L_{hi} represents the distance between the centers of the front and rear axles and l_{hi} is the length of each axle of $\mathcal{A}_{hi}$.

The mobile robots have velocity level nonholonomic constraint. We assume no slippage (i.e., $\dot{x}_{hi} \sin\theta_{hi} - \dot{y}_{hi} \cos\theta_{hi} = 0$) and pure rolling (i.e., $\dot{x}_{hi} \cos\theta_{hi} + \dot{y}_{hi} \sin\theta_{hi} = v_{hi}$) of the car-like mobile robots.

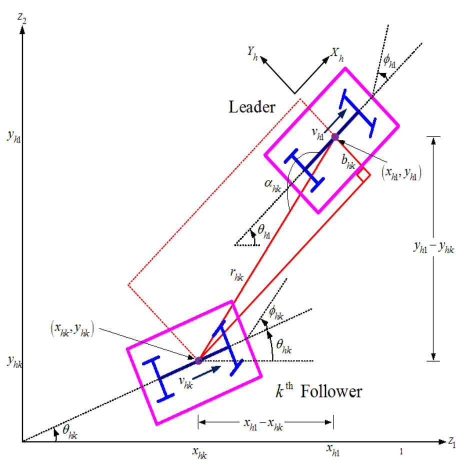

Fig. 2. The proposed scheme utilizing a rotation of axes with axis fixed at the leader.

Next, to ensure that each robot safely steers past an obstacle, we adopt the nomenclature of [12] and construct circular regions that protect the robot. With reference to Fig. 1, given the *clearance parameters* $\epsilon_1 > 0$ and $\epsilon_2 > 0$, we enclose the each vehicle by a protective circular region centered at (x_{hi}, y_{hi}) with radius $r_{hi} = \frac{1}{2}\sqrt{(L_{hi} + 2\epsilon_1)^2 + (l_{hi} + 2\epsilon_2)^2}$ for $h = 1, \ldots, M$, and $i = 1, \ldots, n$.

These generate the *nonholonomic constraints* on the system. The kinodynamic model of the system, adopted from [11] is

$$\left.\begin{aligned} &\dot{x}_{hi} = v_{hi}\cos\theta_{hi} - \tfrac{L_{hi}}{2}\omega_{hi}\sin\theta_{hi}, \quad \dot{y}_{hi} = v_{hi}\sin\theta_{hi} + \tfrac{L_{hi}}{2}\omega_{hi}\cos\theta_{hi}, \\ &\dot{\theta}_{hi} = \tfrac{v_{hi}}{L_{hi}}\tan\phi_{hi} =: \omega_{hi}, \quad \dot{v}_{hi} := \sigma_{hi1}, \qquad \dot{\omega}_{hi} := \sigma_{hi2}, \end{aligned}\right\} \tag{1}$$

for $h = 1, \ldots, M$, and $i = 2, \ldots, n$. Here, v_{hi} and ω_{hi} are, respectively, the instantaneous translational and rotational velocities of $\mathcal{A}_{hi}$, while σ_{hi1} and σ_{hi2} are the instantaneous translational and rotational accelerations of $\mathcal{A}_{hi}$. Without any loss of generality, we assume that $\phi_{hi} = \theta_{hi}$. Now, system (1) is a description of the instantaneous velocities and accelerations of $\mathcal{A}_{hi}$.

2.1 Leader-Follower based Formation Scheme

To desire a substantial degree of rigidness in our formation of $\mathcal{A}_h$, we define two reference frames: the body frame which is fixed with the rotating body of each leader $\mathcal{A}_{h1}$ in $\mathcal{A}_h$, and a space frame, the inertial frame similar to one proposed in [13]. We assign a Cartesian coordinate system $(X_h - Y_h)$ fixed on the leaders body of the $\mathcal{A}_h$, as shown in Fig. 2, based on the concept of an instantaneous co-rotating frame of reference. Thus, when the leader, $\mathcal{A}_{h1}$ rotates, we have a rotation of the $X_h - Y_h$ axes. To define the co-rotating frame of reference, first an origin is selected on the leader robot, $\mathcal{A}_{h1}$ in $\mathcal{A}_h$ at (x_{h1}, y_{h1}). An axis of rotation is then setup that is perpendicular to the plane of motion of the leader, $\mathcal{A}_{h1}$. Thus, at any selected moment t, the chosen rotating frame of reference rotates at an angular rate equal to the rate of rotation of the leader, $\mathcal{A}_{h1}$ about (x_{h1}, y_{h1}). Thus, given the leader's position and its orientation, as long as (r_{hk}, α_{hk}) for $h = 1, \ldots, M$, and $k = 1, \ldots, n$ is fixed, the kth follower robot's position in $\mathcal{A}_h$ will be unique. We define the shape of $\mathcal{A}_h$ as $\zeta_h = [\zeta_{h2}, \zeta_{h3}, \ldots, \zeta_{hn}]^T$, where for $\zeta_{hk} = [r_{hk}, \alpha_{hk}]^T$ for $h = 1, \ldots, M$, and $k = 2, \ldots, n$.

Definition 1. *Let $2r_{hk} < r^d_{hk} < 2r_{hk} + \xi_h$ where for $\xi_h > 0$, for $h = 1, \ldots, M$, and $k = 2, \ldots, n$, the group of mobile robots make a $\zeta^d_h = [\zeta^d_{h2}, \zeta^d_{h3}, \ldots, \zeta^d_{hn}]^T$ formation, if $\exists T > 0, \forall t > T$:*

$$r_{hk} = r^d_{hk} \quad \text{and} \quad \alpha_{hk} = \alpha^d_{hk}.$$

This gives then the polar coordinate representation of the follower's position relative to that of the leader in a given formation. However, such representations using polar coordinates lead to certain singularities in the controllers. To eliminate such singular points, we consider the position of the kth follower in $\mathcal{A}_h$ by considering the relative distances of $\mathcal{A}_{hk}$ in $\mathcal{A}_h$, from its leader, $\mathcal{A}_{h1}$ along the given X_h and Y_h directions. Thus, we have:

$$\begin{aligned} A_{hk} &= -(x_{h1} - x_{hk}) \cos\theta_{h1} - (y_{h1} - y_{hk}) \sin\theta_{h1}, \\ B_{hk} &= (x_{h1} - x_{hk}) \sin\theta_{h1} - (y_{h1} - y_{hk}) \cos\theta_{h1}, \end{aligned} \tag{2}$$

for $h = 1, \ldots, M$, and $k = 2, \ldots, n$ and A_{hk} and B_{hk} are the relative positions with respect to the X_h-Y_h coordinate system of the kth followers in A_h. If A_{hk} and B_{hk} are known and fixed, the follower's position will be distinctive. Thus, to obtain a desired formation, one needs to know distances a_{hk} and b_{hk}, the desired relative positions along the X_h-Y_h directions, such that the control objective would be to achieve $A_{hk} \longrightarrow a_{hk}$ and $B_{hk} \longrightarrow b_{hk}$.i.e., $r_{hk} \longrightarrow r^d_{hk}$ and $\alpha_{hk} \longrightarrow \alpha^d_{hk}$, where $r^d_{hk} = \sqrt{a^2_{hk} + b^2_{hk}}$ and $\alpha^d_{hk} = \tan\left(\frac{a_{hk}}{b_{hk}}\right)$ for $h = 1, \ldots, M$, and $k = 2, \ldots, n$.

3 Artificial Potential Field Function

This section formulates collision free trajectories of the robot system under kinodynamic constraints in a given workspace. We want to design the acceleration controllers, σ_{hi1} and σ_{hi2}, so that the group of robots moves safely towards the leaders target while maintaining a desired low-degree formation.

3.1 Attractive Potential Field Functions

Attraction to Target For the establishment and advancement of the group of M formations having n mobile robots, we incorporate the leader-follower scheme within the framework of the Lyapunov-based control scheme(LbCS) [11]. A target is assigned to the leader of each formation. The leader, $\mathcal{A}_{h1}$ for $h = 1, \dots, M$ will move towards its defined target with center (p_{h11}, p_{h12}), while the follower robots, $\mathcal{A}_{hk}$ move with their leader while maintaining a desired relative position to their leader. For the attraction of the leader, $\mathcal{A}_{h1}$ to its designated target, we consider an attractive potential function for $h = 1, \dots, M$

$$V_{h1}(\mathbf{x}) = \frac{1}{2}\left[(x_{h1} - p_{h11})^2 + (y_{h1} - p_{h12})^2 + v_{h1}^2 + \omega_{h1}^2\right]. \tag{3}$$

For $\mathcal{A}_{hi}$ to maintain its desired relative position with respect to the leader, $\mathcal{A}_{h1}$, we utilize the following potential function for $h = 1, \dots, M$ and $i = 2, \dots, n$

$$V_{hi}(\mathbf{x}) = \frac{1}{2}\left[(A_{hi} - a_{hi})^2 + (B_{hi} - b_{hi})^2 + +v_{hi}^2 + \omega_{hi}^2\right]. \tag{4}$$

Auxiliary Function To guarantee the convergence of the mobile robots to their designated targets, we design an auxiliary function as

$$G_{h1}(\mathbf{x}) = \frac{1}{2}\left[(x_{h1} - p_{h11})^2 + (y_{h1} - p_{h12})^2 + \rho_{h1}(\theta_{h1} - p_{13})^2\right], \tag{5}$$

where p_{h13} is the prescribed final orientation of the leader robot, $\mathcal{A}_{h1}$ and

$$G_{hi}(\mathbf{x}) = \frac{1}{2}\left[(A_{hi} - a_{hi})^2 + (B_{hi} - b_{hi})^2 + \rho_{hi}(\theta_{hi} - \theta_{h1})^2\right], \tag{6}$$

for $i = 2, \dots, n$ and $h = 1, \dots, M$. The constant ρ_{hi} is a binary constant which we shall call the *angle-gain parameter* and will take a value of one only if a final pre-defined orientation is warranted, else it takes the default value of zero [11]. This auxiliary function is then multiplied to the repulsive potential field functions to be designed in the following subsections.

3.2 Repulsive Potential Field Functions

We desire the leader, $\mathcal{A}_{h1}$ and its followers, $\mathcal{A}_{hk}$ avoid all fixed and moving obstacles intersecting their paths.

Fixed Obstacles in the Workspace Let us fix $q \in \mathbb{N}$ solid obstacles within the boundaries of the workspace. We assume that the lth obstacle is a circular disk with center (o_{l1}, o_{l2}) and radius ro_l. For $\mathcal{A}_{hi}$ to avoid the lth obstacle, we consider

$$FO_{hil}(\mathbf{x}) = \frac{1}{2}\left[(x_{hi} - o_{l1})^2 + (y_{hi} - o_{l2})^2 - (ro_l + r_{hi})^2\right], \tag{7}$$

as an avoidance function, where $h = 1, \dots, M$, $i = 1, \dots, n$, and $l = 1, \dots, q$. These obstacle avoidance functions will be combined with appropriate tuning parameters to generate repulsive potential field functions in the workspace.

Moving Obstacles To generate feasible trajectories, we consider moving obstacles of which the system has prior knowledge. Here, each mobile robot, has to be treated as a moving obstacle for all other mobile robots in the workspace.

Minimum Inter-robot Distance We desire to maintain a minimum inter robot separation distance between the robots. This prevents $\mathcal{A}_{hi}$ from getting very close to (or colliding with) $\mathcal{A}_{hj}$ [11], especially during the re-establishment of the prescribed formation when the system is distorted. We can consider the following obstacle avoidance function

$$MO_{hij}(\mathbf{x}) = \frac{1}{2}\left[(x_{hi} - x_{hj})^2 + (y_{hi} - y_{hj})^2 - (r_{hi} + r_{hj})^2\right], \tag{8}$$

for $h = 1, \ldots, M$, and $i, j = 1, \ldots, n$, with $j \neq i$.

Inter Formation Avoidance We also desire for each formation structure in the system to avoid any other formation structure in the workspace. For ith body of $\mathcal{A}_h$ to evade the uth body of $\mathcal{A}_m$, we adopt

$$DO_{himu}(\mathbf{x}) = \frac{1}{2}\left[(x_{hi} - x_{mu})^2 + (y_{hi} - y_{mu})^2 - (r_{hi} + r_{mu})^2\right], \tag{9}$$

for $i, u = 1, \ldots, n$ and $h, m = 1, \ldots, M$ with $m \neq h$.

Dynamic Constraints Practically, the steering angles of the mobile robots are limited due to mechanical singularities while the translational speed is restricted due to safety reasons. Subsequently, we have $|v_{hi}| < v_{\max}$, where $v_{\max}$ is the *maximal achievable speed* of the $\mathcal{A}_{hi}$ and $|\omega_{hi}| < \frac{v_{\max}}{|\rho_{\min}|}$, where $\rho_{\min} := \frac{L_{hi}}{\tan(\phi_{\max})}$. This condition arises due to the boundness of the steering angle ϕ_{hi}. That is, $|\phi_{hi}| \leq \phi_{\max} < \pi/2$, where $\phi_{\max}$ is the *maximal steering angle*. Hence, we consider the following avoidance functions:

$$U_{hi1}(\mathbf{x}) = \frac{1}{2}\left(v_{\max} - v_{hi}\right)\left(v_{\max} + v_{hi}\right), \tag{10}$$

$$U_{hi2}(\mathbf{x}) = \frac{1}{2}\left(\frac{v_{\max}}{|\rho_{\min}|} - \omega_{hi}\right)\left(\frac{v_{\max}}{|\rho_{\min}|} + \omega_{hi}\right), \tag{11}$$

for $h = 1, \ldots, M$ and $i = 1, \ldots, n$. These positive functions would guarantee the adherence to the limitations imposed upon the steering angle and the velocities of $\mathcal{A}_{hi}$ when encoded appropriately into the Lyapunov function.

4 Design of the Acceleration Controllers

The nonlinear acceleration control laws for system (1), will be designed using LbCS as proposed in [11].

4.1 Lyapunov Function

We now construct the total potentials, that is, a Lyapunov function for system (1).

$$
\begin{aligned}
L_{(1)}(\mathbf{x}) = & \sum_{h=1}^{M}\sum_{i=1}^{n}\left\{V_{hi}(\mathbf{x}) + G_{hi}(\mathbf{x})\left[\sum_{l=1}^{q}\frac{\alpha_{hil}}{FO_{hil}(\mathbf{x})} + \sum_{s=1}^{2}\frac{\beta_{his}}{U_{his}(\mathbf{x})}\right]\right\} \\
& + \sum_{h=1}^{M}\sum_{i=1}^{n} G_{hi}(\mathbf{x})\left\{\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{hij}}{MO_{hij}(\mathbf{x})} + \sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{himu}}{DO_{himu}(\mathbf{x})}\right\} \quad (12)
\end{aligned}
$$

4.2 Nonlinear Acceleration Controllers

The design of the feedback controllers begins by noting that the functions f_{hik} to g_{hij} for $h = 1, \ldots, M$, $i = 1, \ldots, n$, $j = 1, 2$ and $k = 1, \ldots, 3$ are defined as (on suppressing $\mathbf{x}$):

$$
\begin{aligned}
f_{h11} = & \left[1 + \sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}} + \sum_{s=1}^{2}\frac{\beta_{h1s}}{U_{h1s}}\right](x_{h1} - p_{h11}) - G_{h1}\sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}^{2}}(x_{h1} - o_{hl1}) \\
& + \left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}} + \sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right](x_{h1} - p_{h11}) \\
& - \sum_{i=2}^{n}\left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}} + \sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right](A_{hi} - a_{hi})\cos\theta_{h1} \\
& + \sum_{i=2}^{n}\left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}} + \sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right](B_{hi} - b_{hi})\sin\theta_{h1} \\
& + \sum_{i=2}^{n}\left[1 + \sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}} + \sum_{s=1}^{2}\frac{\beta_{h1s}}{U_{h1s}}\right][-(A_{hi} - a_{hi})\cos\theta_{h1} + (B_{hi} - b_{hi})\sin\theta_{h1}] \\
& - 2G_{h1}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}^{2}}(x_{h1} - x_{hj}) - 2G_{h1}\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}^{2}}(x_{h1} - x_{mu}), \\
f_{h12} = & \left[1 + \sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}} + \sum_{s=1}^{2}\frac{\beta_{h1s}}{U_{h1s}}\right](y_{h1} - p_{h12}) - G_{h1}\sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}^{2}}(y_{h1} - o_{hl1}) \\
& + \left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}} + \sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right](y_{h1} - p_{h12})
\end{aligned}
$$

$$-\sum_{i=2}^{n}\left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}}+\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right](A_{hi}-a_{hi})\sin\theta_{h1}$$

$$-\sum_{i=2}^{n}\left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}}+\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right](B_{hi}-b_{hi})\cos\theta_{h1}$$

$$-\sum_{i=2}^{n}\left[1+\sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}}+\sum_{s=1}^{2}\frac{\beta_{h1s}}{U_{h1s}}\right][(A_{hi}-a_{hi})\sin\theta_{h1}+(B_{hi}-b_{hi})\cos\theta_{h1}]$$

$$-2G_{h1}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}^2}(y_{h1}-y_{hj})-2G_{h1}\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}^2}(y_{h1}-y_{mu}),$$

$$f_{h13}=\left[\sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}}+\sum_{s=1}^{2}\frac{\beta_{h1s}}{U_{h1s}}+\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}}+\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right]\rho_{h1}(\theta_{h1}-p_{h13})$$

$$-\sum_{i=2}^{n}\left[\sum_{l=1}^{q}\frac{\alpha_{h1l}}{FO_{h1l}}+\sum_{s=1}^{2}\frac{\beta_{h1s}}{U_{h1s}}+\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{h1j}}{MO_{h1j}}+\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{h1mu}}{DO_{h1mu}}\right]\rho_{hi}(\theta_{hi}-\theta_{h1}),$$

$$g_{h11}=1+G_{h1}\frac{\beta_{h11}}{U_{h11}^2},\qquad g_{h12}=1+G_{h1}\frac{\beta_{h12}}{U_{h12}^2},$$

and for $i=2,\dots,n$

$$f_{hi1}=\left[1+\sum_{l=1}^{q}\frac{\alpha_{hil}}{FO_{hil}}+\sum_{s=1}^{2}\frac{\beta_{his}}{U_{his}}\right][(A_{hi}-a_{hi})\cos\theta_{h1}-(B_{hi}-b_{hi})\sin\theta_{h1}]$$

$$+\left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{hij}}{MO_{hij}}+\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{himu}}{DO_{himu}}\right](A_{hi}-a_{hi})\cos\theta_{h1}$$

$$-\left[\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{hij}}{MO_{hij}}+\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{himu}}{DO_{himu}}\right](B_{hi}-b_{hi})\sin\theta_{h1}$$

$$-2\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\eta_{hij}G_{hi}}{MO_{hij}^2}(x_{hi}-x_{hj})-2\sum_{u=1}^{n}\sum_{\substack{m=1\\m\neq h}}^{M}\frac{\gamma_{himu}G_{hi}}{DO_{himu}^2}(x_{hi}-x_{mu})$$

$$-\sum_{l=1}^{q}\frac{\alpha_{hil}G_{hi}}{FO_{hil}^2}(x_{hi}-o_{hl1}),$$

$$f_{hi2} = \left[1 + \sum_{l=1}^{q} \frac{\alpha_{h1l}}{FO_{h1l}} + \sum_{s=1}^{2} \frac{\beta_{his}}{U_{his}}\right] [(A_{hi} - a_{hi}) \sin\theta_{h1} + (B_{hi} - b_{hi}) \cos\theta_{h1}]$$

$$+ \left[\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\eta_{hij}}{MO_{hij}} + \sum_{u=1}^{n} \sum_{\substack{m=1\\ m\neq h}}^{M} \frac{\gamma_{himu}}{DO_{himu}}\right] (A_{hi} - a_{hi}) \sin\theta_{h1}$$

$$+ \left[\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\eta_{hij}}{MO_{hij}} + \sum_{u=1}^{n} \sum_{\substack{m=1\\ m\neq h}}^{M} \frac{\gamma_{himu}}{DO_{himu}}\right] (B_{hi} - b_{hi}) \cos\theta_{h1}$$

$$-2 \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\eta_{hij} G_{hi}}{MO_{hij}^2} (y_{hi} - y_{hj}) - 2 \sum_{u=1}^{n} \sum_{\substack{m=1\\ m\neq h}}^{M} \frac{\gamma_{himu} G_{hi}}{DO_{himu}^2} (y_{hi} - y_{mu})$$

$$- \sum_{l=1}^{q} \frac{\alpha_{hil} G_{hi}}{FO_{hil}^2} (y_{hi} - o_{hl1}),$$

$$f_{hi3} = \left[\sum_{l=1}^{q} \frac{\alpha_{hil}}{FO_{hil}} + \sum_{s=1}^{2} \frac{\beta_{his}}{U_{his}} + \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\eta_{hij}}{MO_{hij}} + \sum_{u=1}^{n} \sum_{\substack{m=1\\ m\neq h}}^{M} \frac{\gamma_{himu}}{DO_{himu}}\right] \rho_{hi}(\theta_{hi} - \theta_{h1}),$$

$$g_{hi1} = 1 + G_{hi} \frac{\beta_{hi1}}{U_{hi1}^2}, \qquad g_{hi2} = 1 + G_{hi} \frac{\beta_{hi2}}{U_{hi2}^2}.$$

So, we design the following theorem:

Theorem 1. *Consider a team of M formation structures each comprising of n car-like mobile robots whose motion is governed by the ODE's described in system (1). The principal goal is to establish and control each follower robot in each formation structure to track its designated leader in a cooperative manner and facilitate maneuvers within a constrained environment and reach the target configuration. The subtasks include; restrictions placed on the workspace, convergence to predefined targets, and consideration of kinodynamic constraints. Utilizing the attractive and repulsive potential field functions, the following continuous time-invariant acceleration control laws can be generated, that intrinsically guarantees stability,in the sense of Lyapunov, of system (1) as well:*

$$\left.\begin{aligned} \sigma_{hi1} &= -[\delta_{hi1} v_{hi} + f_{hi1} \cos\theta_{hi} + f_{hi2} \sin\theta_{hi}]/g_{hi1}, \\ \sigma_{hi2} &= -\left[\delta_{hi2}\omega_{hi} + \tfrac{L_{hi}}{2}(f_{hi2} \cos\theta_{hi} - f_{hi1} \sin\theta_{hi}) + f_{hi3}\right]/g_{hi2}, \end{aligned}\right\} \tag{13}$$

for $h = 1, \ldots, M$ and $i = 1, \ldots, n$, where $\delta_{hi1} > 0$, and $\delta_{hi2} > 0$ are constants commonly known as convergence parameters.

5 Stability Analysis

Theorem 2. *Let (p_{h11}, p_{h12}) be the position of the target of the leader in $\mathcal{A}_h$, and p_{hi3} for $i = 1, \ldots, n$, be the desired final orientations of the robots in each*

$\mathcal{A}_h$. *Given* a_{hi} *and* b_{hi} *in* $\mathcal{A}_h$*, let* p_{hi1} *and* p_{hi2} *satisfy*

$$\begin{aligned} a_{hi} &= -(p_{h11} - p_{hi1})\cos\theta_{h1} - (p_{h12} - p_{hi2})\sin\theta_{h1}, \\ b_{hi} &= (p_{h11} - p_{hi1})\sin\theta_{h1} - (p_{h12} - p_{hi2})\cos\theta_{h1}, \end{aligned}$$

for $i = 2, \ldots, n$ *and* $h = 1, \ldots, M$.

Given $\mathbf{x}_{hi}^* := (p_{hi1}, p_{hi2}, p_{hi3}, 0, 0) \in \mathbb{R}^5$*, if* $\mathbf{x}_e := (\mathbf{x}_{11}^*, \mathbf{x}_{12}^*, \ldots, \mathbf{x}_{hn}^*) \in \mathbb{R}^{5\times n\times M}$ *is an equilibrium point for (1), then* $\mathbf{x}_e \in D(L_{(1)}(\mathbf{x}))$ *is a stable equilibrium point of system (1).*

Proof. One can easily verify the following, for $i = 1, \ldots, n$ and $h = 1, \ldots, M$:

1. $L_{(1)}(\mathbf{x})$ is defined, continuous and positive over the domain $D(L_{(1)}(\mathbf{x})) = \{\mathbf{x} \in \mathbb{R}^{5\times M\times n} : FO_{hil}(\mathbf{x}) > 0, l = 1, \ldots, q; DO_{himu}(\mathbf{x}) > 0, m = 1, \ldots, M, u = 1, \ldots, n, m \neq h;\ MO_{hij}(\mathbf{x}) > 0, j = 1, \ldots, n, j \neq i;\ U_{his}(\mathbf{x}) > 0, s = 1, 2\}$;
2. $L_{(1)}(\mathbf{x}^*) = 0$;
3. $L_{(1)}(\mathbf{x}) > 0 \ \ \forall \mathbf{x} \in D(L_{(1)}(\mathbf{x}))/\mathbf{x}_e$.

Next, consider the time derivative of the candidate Lyapunov function along a particular trajectory of system (1):

$$\dot{L}_{(1)}(\mathbf{x}) = \sum_{i=1}^{n} \left[f_{hi1}\dot{x}_{hi} + f_{i2}\dot{y}_{hi} + f_{hi3}\dot{\theta}_{hi} + g_{hi1}v_{hi}\dot{v}_{hi} + g_{hi2}\omega_{hi}\dot{\omega}_{hi} \right].$$

Substituting the controllers given in (13) and the governing ODEs for system (1), we obtain the following semi-negative definite function

$$\dot{L}_{(1)}(\mathbf{x}) = -\sum_{i=1}^{n} \left(\delta_{hi1}v_{hi}^2 + \delta_{hi2}\omega_{hi}^2 \right) \leq 0.$$

Thus, $\dot{L}_{(1)}(\mathbf{x}) \leq 0 \ \forall \mathbf{x} \in D(L_{(1)}(\mathbf{x}))$ and $\dot{L}_{(1)}(\mathbf{x}_e) = 0$. Finally, it can be easily verified that $L_{(1)}(\mathbf{x}) \in C^1\left(D(L_{(1)}(\mathbf{x}))\right)$, which makes up the fifth and final criterion of a Lyapunov function. Hence, $L_{(1)}(\mathbf{x})$ is classified as a Lyapunov function for system (1) and $\mathbf{x}_e$ is a stable equilibrium point in the sense of Lyapunov. □

Remark 1. This result is in no contradiction with Brockett's Theorem [14] as we have not proven asymptotic stability.

6 Simulation Results

In this section, we illustrate the effectiveness of the proposed continuous time-invariant controllers within the framework of the Lyapunov-based control scheme by simulating a virtual scenario.

We consider the motion of a pair of 4 cars in a double platoon formation in a two dimensional space with static obstacles in its path. Each follower robot in each formation structure is assigned a unique position relative to its leader as seen in Fig. 3. This is achieved by assigning appropriate values to (a_{hk}, b_{hk}) to obtain a geometric formation structure. While a leaders $\mathcal{A}_{11}$ and $\mathcal{A}_{21}$ moves towards its intended target, the followers, $\mathcal{A}_{hk}$ for $h = 1, 2$ and $k = 2, \ldots, 4$ maintains the desired relative position to its designated leader, thus maintaining a desired formation structure. Assuming that the appropriate units have been accounted for, Table 1 provides the corresponding initial and final configurations of the two robots and other necessary parameters required to simulate the scenario. Fig. 4 shows the evolution of the Lyapunov function and its time derivative along the system trajectory while Fig. 5 depicts the time evolution of the nonlinear controllers of the leaders $\mathcal{A}_{11}$ and $\mathcal{A}_{21}$. Figures 4 and 5 show the boundness and convergence of the variables at the final state, respectively, implying the effectiveness of the controllers.

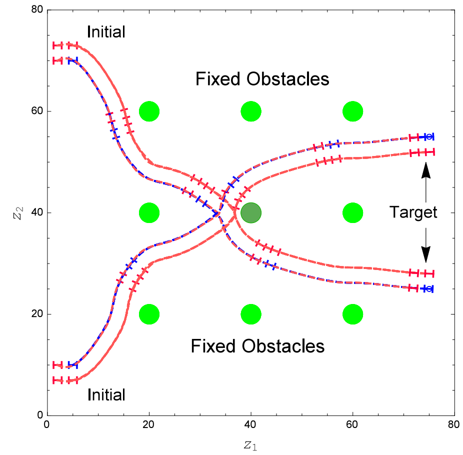

Fig. 3. The proposed scheme utilizing a rotation of axes with axis fixed at the leader.

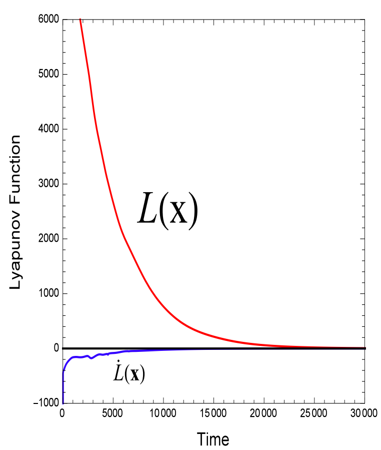

Fig. 4. Evolution of $L_1(\mathbf{x})$ and its time derivative $\dot{L}_{(1)}(\mathbf{x})$ for Scenario 1.

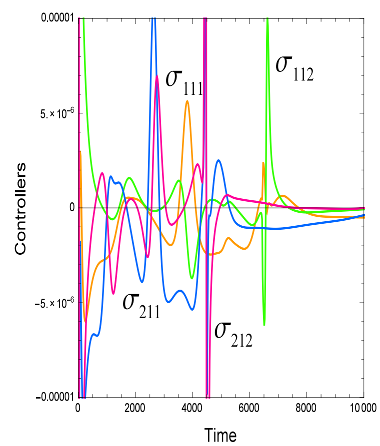

Fig. 5. Translational and Rotational accelerations of the leaders $\mathcal{A}_{11}$ and $\mathcal{A}_{21}$.

Table 1. Numerical values of initial and final states, constraints and parameters.

Initial Configuration	
Rectangular positions of leaders	$(x_{11}, y_{11}) = (5, 70), (x_{21}, y_{21}) = (5, 10)$
Desired relative distances of	$a_{12} = a_{23} = b_{14} = b_{22} = 0, b_{12} = b_{13} = -3$
followers	$a_{13} = a_{14} = a_{22} = a_{24} = b_{23} = b_{24} = 3$
Translational velocity	$v_{hi} = 0.5$ for $i = 1, \ldots, 4$, $h = 1, 2$
Rotational velocities	$\omega_{hi} = 0$, for $i = 1, \ldots, 4$, $h = 1, 2$
Angular positions	$\theta_{hi} = 0$, for $i = 1, \ldots, 4$, $h = 1, 2$
Constraints and Parameters	
Dimensions of robots	$L_{hi} = 1.6$, $l_{hi} = 1.2$ for $i = 1, \ldots, 4$, $h = 1, 2$
$\mathcal{A}_1$ leader's target :	$(p_{11}, p_{12}) = (79, 24.5)$, $rt_{11} = 0.5$,
$\mathcal{A}_2$ leader's target :	$(p_{21}, p_{22}) = (79, 56)$, $rt_{21} = 0.5$
Fixed Obstacles	$(o_{11}, o_{12}) = (20, 20), (o_{21}, o_{22}) = (20, 40)$
	$(o_{31}, o_{32}) = (20, 60), (o_{41}, o_{42}) = (40, 20)$
	$(o_{51}, o_{52}) = (40, 40), (o_{61}, o_{62}) = (40, 60)$
	$(o_{71}, o_{72}) = (60, 20), (o_{81}, o_{82}) = (60, 40)$
	$(o_{91}, o_{92}) = (60, 60)$, $ro_l = 2$ for $l = 1, \ldots, 9$
Max. translational velocity	$v_{\max} = 5$
Max. steering angle	$\phi_{\max} = \pi/2$
Clearance parameters	$\epsilon_1 = 0.1$, $\epsilon_2 = 0.05$
Control and Convergence Parameters	
Collision avoidance	$\alpha_{hil} = 1$, for $i = 1, \ldots, 4$, $h = 1, 2$, $l = 1, \ldots, 9$
	$\eta_{hij} = 0.01$ for $h = 1, 2$, $i, j = 1, \ldots, 4$, $j \neq i$,
	$\gamma_{himu} = 0.01$, for $h, m = 1, 2$, $i, j = 1, \ldots, 4$, $h \neq m$
Dynamics constraints	$\beta_{his} = 1$, for $i = 1, \ldots, 4$, $h, s = 1, 2$,
Convergence	$\delta_{11j} = 12000$, $\delta_{21j} = 8000$, for $j = 1, 2$
	$\delta_{hij} = 50$, for $i = 1, \ldots, 4$, $h, j = 1, 2$

7 Conclusion

In this paper we have proposed a leader follower scheme for the coordination of MRSs in an environment populated with obstacles. The contribution of this paper is the capability of the robots in a desired geometric formation to move from some initial configuration to some final state while maintaining a desired low degree formation. The advantage of the proposed scheme is that we can have multiple formations structures having different geometric shape. The approach also considers inter-robot and inter-formation collision avoidance. Collision free maneuvers with fixed obstacles are imbedded in the overall framework. The derived controllers produced feasible trajectories and ensured a nice convergence of the system to its equilibrium state while satisfying the necessary kinematic and dynamic constraints. The effectiveness of the proposed control laws were demonstrated via a computer simulation. The control system presented is also easily scalable for any number of agents in any geometric shape. Future research will address more general formation applications in 3D.

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