

A Nonlinear Observer for Rotor Flux Estimation Considering Magnetic Saturation Effects in Induction Motor Drives

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Abstract—This paper proposes a non-linear observer for Induction Machine (IM) drives which takes into consideration the saturation effects. The non-linear observer is based on an original formulation of the dynamic model of the IM taking into consideration the magnetic saturation of the iron core. A Lyapunov based convergence analysis is proposed in order to suitably compute the observer gain guaranteeing the stability of the observer. The proposed non-linear observer has been tested in numerical simulation and experimentally on a suitably developed test set-up. Its behaviour has been compared to that obtained with a classic Full-Order Luenberger Observer (FOLO) in variable flux working conditions, in terms of accuracy of the amplitude and phase of both the rotor flux linkage and the stator currents space vectors. Results have shown the capability of the proposed non-linear observer to correctly estimate the rotor flux linkage amplitude and phase under flux varying conditions including strong variation of the saturation of the iron path with accuracy in the flux estimation much higher than that obtained with the classic FOLO.

Index Terms—Induction Motor (IM), nonlinear observer, magnetic saturation effects.

Table I
LIST OF SYMBOLS

SYMBOLS	
u_{sD}, u_{sQ}	inductor voltages in stator reference frame;
i_{sD}, i_{sQ}	inductor currents in stator reference frame;
i_{mrD}, i_{mrQ}	magnetizing currents in stator reference frame;
$ \psi_r = L_m \mathbf{i}_{mr} $	rotor flux amplitude;
$L_s (L_r)$	stator (rotor) inductance;
L_m	3-phase magnetizing inductance;
$R_s (R_r)$	stator (rotor) resistance;

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$L_{s\sigma} = L_m - L_s$	stator leakage inductance;
$L_{r\sigma} = L_m - L_r$	rotor leakage inductance;
$\sigma = 1 - \frac{L_m^2}{L_s L_r}$	total leakage factor;
$T_r = \frac{L_r}{R_r}$	rotor time constant;
ω_r	angular speed of the rotor (in electrical angles);
p	pole-pairs;
J_m	rotor inertia.
\mathbf{I}_n	identity matrix $n \times n$

I. INTRODUCTION

Control of induction machine has been a challenging research subject for many years. Starting from scalar control solution based on the steady-state model of the IM, the industrial standard in terms of high performance control of IMs has been established as the so-called Field Oriented Control (FOC) [1], [2], [3]. Direct field oriented control, whatever is the flux space vector on which field orientation is performed (rotor, stator or magnetizing), requires the correct knowledge of the amplitude and phase of the flux vector itself. This is generally performed adopting the so-called flux models, which can be straightforward divided into estimators and observers. There are basically two forms of implementation of an estimator: open-loop and closed-loop, the distinction between the two being whether or not a correction term, involving an estimation error term, is exploited to adjust on-line the response of the estimator [1]. A closed-loop estimator is referred to as an observer. In general observers are preferable to open-loop estimators, since they permit the robustness against parameter variations and noise to be improved. If the machine is considered a deterministic plant, correspondingly the observer is a deterministic observer, like the full-order Luenberger observer [4], [5] and the Reduced Order Observer (ROO) [6], [7]; scientific literature is full of solutions for the gain matrix choice for both observers.

On the contrary, if the machine is considered a stochastic system, the Kalman Filter (KF) can be used, respectively in its linear or non-linear version (Extended Kalman Filter (EKF)) [8], [9]. Another approach for the analysis of the observer has been proposed by [10], integrated with a detailed sensitivity analysis versus the parameters variations [11]. Finally, [12] proposes an analysis of flux observers from a perspective of the control system theory. Apart from the EKF, whose complexity and computational requirement is high besides the difficulty of its tuning, all the above observers are linear (Not

considering the speed as a state variable). Moreover, it should be noted that the EKF treats only the non-linearity of the IM due to the dependence of the electromagnetic torque on the product between two state variables (the rotor flux and the stator current for example).

There are, however, working conditions where contemporary speed (torque) and rotor flux variations occur (techniques for the on-line electric losses minimisation). In such conditions, the correctness of the state-space IM dynamic model can fail, since the magnetic saturation effects of the iron core, typically not accounted for in the classic model, become significant. In such working conditions not only do all of the inductance and leakage factors of the IM model vary with the magnetization level of the machine with highly non-linear laws, but also new terms in the dynamic model arise, not existing in the classical model. The non-linearity of the model increases therefore consistently. Some dynamic models of IM taking into consideration the magnetic saturation have been developed in the scientific literature [13], [14], [15], each of which presents its peculiarity in representing the magnetic saturation. Many of the above models, however, do not present a space-state representation, and therefore do not reveal particularly useful for the development neither of control techniques nor of state observers.

The control system theory has proposed methodologies for the definition and tuning of non-linear observers, under the hypothesis of a guaranteed convergence [16], [17], [18]. For a nonlinear system, the structure of the observers is not at all as obvious as it is for a linear system, and they prove interesting tools for a correct flux estimation in working conditions of variable flux. To the best of the authors' knowledge none of these observers has been applied to the IM flux estimation accounting for the magnetic saturation.

Starting from the above considerations, this paper proposes a non-linear observer for induction machine drives which takes into consideration the IM saturation effects. The non-linear observer is based on an original formulation of the dynamic model of the IM with the magnetic saturation, which is inspired to [1, Chapter 6], but entirely reformulated rearranging it in space-state form, after assuming as state variables the stator current and the rotor magnetizing current space-vectors. Moreover, it has been expressed in the stator reference frame differently from [1]. The observer is inspired to [16], [17], where the choice of the observer gain may be aided through the use of Lyapunov's method. Alternatively another possibility for the choice of the gain is shown in [18]. The proposed non-linear observer has been tested in numerical simulation and experimentally on a suitably developed test set-up. Its behaviour has been compared to that of a classic full-order Luenberger observer (not taking into consideration the saturation effects) in variable flux working conditions. Such a non-linear observer reveals particularly useful to be used in the deep field weakening region and in presence of electrical losses minimization techniques.

Notation:

- $A \otimes B$ denotes the Kronecker product between matrices A and B .
- $\|\cdot\|$ denotes the standard euclidian norm.

II. SPACE-VECTOR DYNAMIC MODEL OF IM INCLUDING MAGNETIC SATURATION EFFECTS

In principle, the dynamic model taking into consideration the magnetic saturation of the iron core has been derived in the following using the same approach shown in [1, Section 6.1.1.1].

Differently from the model in [1], which is not written in a space-state form, and is not therefore particularly suitable for being used to define a nonlinear observer, the proposed model has been deduced in a space-state form. Moreover it has been developed in the stationary reference frame and thus does not require any vector rotation, implying complexity and computational demand in its implementation on a programmable hardware.

Let the following coefficients of the classical state-space model [4] be considered:

$$\begin{aligned} a_{11} &= \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma T_r}, & a_{12} &= \frac{1}{\sigma L_s T_r}, \\ a_{21} &= L_s \frac{1 - \sigma}{T_r}, & a_{22} &= \frac{1}{T_r}. \end{aligned} \quad (1)$$

Let the coefficients in equation (1) be rewritten replacing the rotor time constant T_r with the modified rotor time constant T_r^* as follows:

$$\begin{aligned} a_{11}^* &= \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma T_r^*}, & a_{12}^* &= \frac{1}{\sigma L_s T_r^*}, \\ a_{21}^* &= L_s \frac{1 - \sigma}{T_r^*}, & a_{22}^* &= \frac{1}{T_r^*}, \end{aligned} \quad (2)$$

where the modified rotor time constant is defined as:

$$T_r^* = T_r \frac{L}{L_m}, \quad (3)$$

where L is called dynamic magnetizing inductance and is equal to [1]:

$$L = \frac{d|\Psi_r|}{d|\mathbf{i}_{mr}|} = L_m + |\mathbf{i}_{mr}| \frac{dL_m}{d|\mathbf{i}_{mr}|}. \quad (4)$$

Using the above time-varying coefficients, the equations shown in [1, Section 6.1.1.1] can be manipulated in order to obtain the following state-space model in a stationary reference frame:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}(|\mathbf{i}_{mr}|)\mathbf{x} + \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases} \quad (5)$$

where \mathbf{x} , \mathbf{u} and \mathbf{y} are the state, the input and the output vector respectively defined as:

$$\begin{aligned} \mathbf{x} &= [i_{sD} \quad i_{sQ} \quad i_{mrd} \quad i_{mrq}]^T, & \mathbf{u} &= [u_{sD} \quad u_{sQ} \quad \omega_r]^T, \\ \mathbf{y} &= [i_{sD} \quad i_{sQ}]^T \end{aligned} \quad (6)$$

and $\mathbf{A}(|\mathbf{i}_{mr}|)$, $\mathbf{f}(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$ and \mathbf{C} are defined in (7) (see top of next page), with:

$$\begin{aligned} c_1 &= a_{11}^* + a_{12}^*(\Delta L - 2\Delta L^*), & c_2 &= a_{12}^*\Delta L^*, \\ c_3 &= a_{21}^*f_1 + a_{12}^*(\Delta L - \Delta L^*), \end{aligned} \quad (8)$$

where $f_1 = \frac{1}{\sigma L_s}$, $\Delta L = L - L_m$ and $\Delta L^* = \frac{L_m^2}{L_s^2} \Delta L$. In the case under study, the speed ω_r is assumed to be known in the

$$\mathbf{A}(|\mathbf{i}_{mr}|) = \begin{bmatrix} -c_1 & 0 & c_3 & 0 \\ 0 & -c_1 & 0 & c_3 \\ a_{22}^* & 0 & -a_{22}^* & 0 \\ 0 & a_{22}^* & 0 & -a_{22}^* \end{bmatrix}, \quad \mathbf{g}(\mathbf{x}) = \begin{bmatrix} f_1 & 0 & a_{21}T_r f_1 i_{mrq} \\ 0 & f_1 & -a_{21}T_r f_1 i_{mrd} \\ 0 & 0 & -i_{mrq} \\ 0 & 0 & i_{mrd} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \frac{c_2}{i_{mrq}^2 + i_{mrd}^2} (2i_{sQ}^2 i_{mrd} - i_{sD}^2 i_{mrd} - 3i_{sD} i_{sQ} i_{mrq}) + \frac{c_3 - a_{21}f_1}{i_{mrq}^2 + i_{mrd}^2} (i_{sD} i_{mrq}^2 - i_{sQ} i_{mrd} i_{mrq}) \\ \frac{c_2}{i_{mrd}^2 + i_{mrq}^2} (2i_{sD}^2 i_{mrq} - i_{sQ}^2 i_{mrq} - 3i_{sD} i_{sQ} i_{mrd}) + \frac{c_3 - a_{21}f_1}{i_{mrd}^2 + i_{mrq}^2} (i_{sQ} i_{mrd}^2 - i_{sD} i_{mrq} i_{mrd}) \\ \frac{c_2}{i_{mrd}^2 + i_{mrq}^2} (i_{sD} i_{mrq}^2 - i_{sQ} i_{mrd} i_{mrq}) \\ \frac{c_2}{i_{mrd}^2 + i_{mrq}^2} (i_{sQ} i_{mrd}^2 - i_{sD} i_{mrq} i_{mrd}) \end{bmatrix} \quad (7)$$

design of the proposed observer. The speed can therefore be considered as an input [19]. It should be noted that the space-state dynamic model has been purposely formulated in such a way that the linear and non-linear contribution are separated. Such a formulation of the model is very important to develop the non-linear observer, as clearly described in the following.

The effects of the saturation are clearly shown from the time-varying characteristic of matrix \mathbf{A} , whose coefficients vary with the rotor magnetizing current, and from the non-linear term $\mathbf{f}(\mathbf{x})$. With this regard it is useful to note that, if the saturation effects are not considered, then $L = L_m$, i.e. the magnetizing inductance does not vary with the magnetizing current. This leads to the fact that $T_r^* = T_r$ and $\Delta L = \Delta L^* = 0$, and accordingly $c_1 = a_{11}$, $c_2 = 0$ and $c_3 = a_{21}f_1$. Consequently matrix \mathbf{A} became time-invariant and equal to the standard IM model with the assumption of linearity in the magnetizing circuit, and $\mathbf{f}(\mathbf{x}) = 0$ because $c_2 = 0$ and $c_3 - a_{21}f_1 = 0$. The corresponding full order observer becomes that in [4].

A. Dependence of the magnetic parameters on the rotor magnetizing current

Some remarks are to be made about the dependance of the IM parameters from the magnetic saturation. Indeed with respect to the unsaturated model of the IM, these parameters are time-varying and a suitable procedure to identify the waveforms of the parameters is required [20]. This procedure has been applied in this work to obtain the magnetizing curve of the saturated model of IM (See Figure 1.(a)). However, here a different interpolation procedure has been used: actually if the polynomial interpolation were used, then, at a certain level of $|\mathbf{i}_{mr}|$ the amplitude of several variables of the system (i.e. $|\Psi_r|$, L_m , L_s etc...) would tend to infinity, leading up to an incorrect behavior of the observer. For this reason, in order to avoid numerical problems when the $|\mathbf{i}_{mr}|$ increases over the maximum value considered for the interpolation¹, a function based interpolation, working for all values of the magnetizing current, has been proposed. This last choice ensures a better fitting of the experimental data. As can be seen from Figure 1.(a), the interpolating curve of the magnetizing

characteristic can be written as a sum of an exponential function and a linear function, as follows:

$$|\Psi_r| = \alpha (1 - e^{-\beta |\mathbf{i}_{mr}|}) + \gamma |\mathbf{i}_{mr}|. \quad (9)$$

The coefficients α , β and γ , after choosing the interpolating function, have been obtained by means of an optimization procedure (nonlinear least square), minimizing the distance of the curve $|\Psi_r|$ from the experimental points of the characteristic. In particular the interpolating curve, shown by the dashed red line in Figure 1.(a), has been obtained for $\alpha = 0.98$, $\beta = 0.47$ and $\gamma = 0.01$. Starting from equation (9) the expression of L_m can be analytically obtained as: $L_m = \frac{|\Psi_r|}{|\mathbf{i}_{mr}|} = \alpha \frac{(1 - e^{-\beta |\mathbf{i}_{mr}|})}{|\mathbf{i}_{mr}|} + \gamma$. Replacing this expression into the equation of the dynamic magnetizing inductance L , the following expression for the dynamic magnetizing inductance is obtained: $L = \alpha \beta e^{-\beta |\mathbf{i}_{mr}|} + \gamma$. This approach permits the straightforward definition of all the inductance terms expressions as well as the leakage factor expressions, which are implicitly parametrized once the magnetizing curve has been properly fitted.

At this point an interesting physical interpretation of the coefficients α , β and γ can be done. In fact considering that $\lim_{|\mathbf{i}_{mr}| \rightarrow 0} L_m = \alpha\beta + \gamma$ and $\lim_{|\mathbf{i}_{mr}| \rightarrow \infty} L_m = \gamma$, then γ can be interpreted as the magnetizing inductance when the machine is full saturated and the relation $\alpha\beta + \gamma$ as the tangent of the magnetizing curve to the origin, which represents the initial status of magnetization of the iron core. From the experimentally point of view it is a reasonable choice to fix α equal to the value of the rated rotor flux. All these considerations give an alternative procedure with respect to the optimization one in order to obtain the interpolating curve.

Figure 1.(b) shows the waveforms of L_m and L respectively. Using the assumption of constant leakage inductances, also the expressions of the modified rotor time constant T_r^* , the model coefficients ΔL and ΔL^* (shown in Figure 1.(c)), and the coefficients c_1 , c_2 and c_3 can be obtained.

III. NONLINEAR OBSERVER

One way for obtaining an observer is to imitate the procedure used in a linear system, namely to construct a model of the original system (5) and force it with the “residual”:

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{C}\hat{\mathbf{x}}, \quad (10)$$

¹This situation could occurs when the controller is not tuned correctly and the $|\mathbf{i}_{mr}|$ assumes a high value during transient. In this case the flux observer, which itself takes into consideration the magnetic effects of the iron core, for high values of $|\mathbf{i}_{mr}|$ considers an increasing magnetizing inductance, that gives a completely wrong flux estimation.

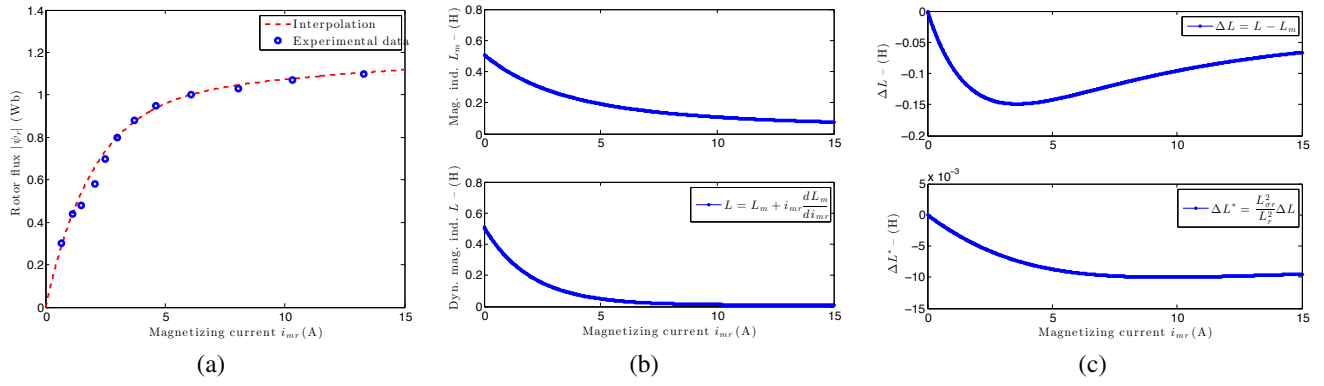


Figure 1. (a) Magnetizing curve of the IM, (b) Magnetizing inductance L_m and dynamic magnetizing inductance L curves, (c) Curves of ΔL and ΔL^* .

where with the symbol " $\hat{\cdot}$ " the estimated variables are indicated. The equation of the observer thus becomes

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}(|\hat{\mathbf{i}}_{mr}|)\hat{\mathbf{x}} + \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{g}(\hat{\mathbf{x}})\mathbf{u} + \mathbf{k}e. \quad (11)$$

where k is a suitably chosen function.

Taking inspiration from [16], the choice of k may be aided through the use of Lyapunov's second method. Using this method, [16] shows that k can be simply related to constant gain matrix \mathbf{K} chosen to stabilize the portion of the system without $\mathbf{f}(\mathbf{x})$, under suitable condition. Indeed if the gain is chosen such that matrix $\hat{\mathbf{A}} - \mathbf{K}\mathbf{C}$ has all eigenvalues with negative real part for each values of $|\mathbf{i}_{mr}|$, then asymptotic stability of the observer is ensured if $\mathbf{f}(\mathbf{x})$ satisfies a Lipschitz condition, i.e. if $\forall \mathbf{x}_1, \mathbf{x}_2$ in the state space there exists a positive constant l such that:

$$\|\mathbf{f}(\mathbf{x}_1) - \mathbf{f}(\mathbf{x}_2)\| \leq l\|\mathbf{x}_1 - \mathbf{x}_2\|, \quad (12)$$

and if there exist two positive definite matrices \mathbf{P} and \mathbf{Q} , and a positive constant λ_0 such that the following inequality is satisfied:

$$\mathbf{P}(\hat{\mathbf{A}} - \mathbf{K}\mathbf{C}) + (\hat{\mathbf{A}} - \mathbf{K}\mathbf{C})^T \mathbf{P} = -\mathbf{Q} \leq -\lambda_0 \mathbf{I}. \quad (13)$$

In this case, asymptotic stability of the observer is assured for:

$$\frac{\lambda_0}{2l\|\mathbf{P}\|} > 1, \quad (14)$$

where l is the Lipschitz constant in (12). This analysis was substantially extended by [17].

In the case under studied, it is easy to verify that $\mathbf{f}(\mathbf{x})$ satisfies the locally Lipschitz condition. Indeed $\mathbf{f}(\mathbf{x})$ is always derivable except when $i_{mrd} = i_{mrq} = 0$. So $\mathbf{f}(\mathbf{x})$ is locally Lipschitz for all \mathbf{x} such that $i_{mrd}^2 + i_{mrq}^2 > \epsilon$, where ϵ is an arbitrary positive constant. However this constraint is coherent with the physical constraint that the machine can correctly work only if magnetized, so (12) is always satisfied for any proper working condition.

In order to obtain the gain \mathbf{K} such that (13) is satisfied, let matrix \mathbf{P} be defined as:

$$\mathbf{P} = \begin{bmatrix} 1 & \frac{c_3}{(1+\chi)a_{22}^*} \\ \frac{c_3}{(1+\chi)a_{22}^*} & \frac{c_3^2}{(1+\chi)a_{22}^{*2}} + \chi \end{bmatrix} \otimes \mathbf{I}_2 = \mathbf{P}_1 \otimes \mathbf{I}_2, \quad (15)$$

where χ is an arbitrary positive constant. Matrix \mathbf{P} is symmetric and positive definite, indeed it is easy to verify that $c_1 > 0$,

$c_2 > 0$, $c_3 > 0$ and $a_{22}^* > 0$ for all values of the magnetizing current, so the coefficient $p_{22} > 0$ if $\chi > 0$, moreover:

$$\det(\mathbf{P}) = \det(\mathbf{P}_1)^2 \det(\mathbf{I}_2)^2 = \left(\chi(a_{22}^{*2} + a_{22}^{*2}\chi^2 + 2a_{22}^{*2}\chi + \chi^2) \right)^2 > 0 \quad \forall \chi. \quad (16)$$

Noting that matrix \mathbf{A} can be written as: $\mathbf{A} = \begin{bmatrix} -c_1 & c_3 \\ a_{22}^* & -a_{22}^* \end{bmatrix} \otimes \mathbf{I}_2$, fixing $\mathbf{K} = [k_1 \ k_2]^T \otimes \mathbf{I}_2$, we can take advantage from the associative properties of the kronecker product in order to compute $\mathbf{P}(\hat{\mathbf{A}} - \mathbf{K}\mathbf{C}) + (\hat{\mathbf{A}} - \mathbf{K}\mathbf{C})^T \mathbf{P}$. Indeed we have that:

$$\begin{aligned} & \mathbf{P}(\hat{\mathbf{A}} - \mathbf{K}\mathbf{C}) + (\hat{\mathbf{A}} - \mathbf{K}\mathbf{C})^T \mathbf{P} \\ &= \left(\begin{bmatrix} 1 & \frac{c_3}{(1+\chi)a_{22}^*} \\ \frac{c_3}{(1+\chi)a_{22}^*} & \frac{c_3^2}{(1+\chi)a_{22}^{*2}} + \chi \end{bmatrix} \begin{bmatrix} -c_1 - k_1 & c_3 \\ a_{22}^* - k_2 & -a_{22}^* \end{bmatrix} \right. \\ &+ \left. \begin{bmatrix} -c_1 - k_1 & c_3 \\ a_{22}^* - k_2 & -a_{22}^* \end{bmatrix}^T \begin{bmatrix} 1 & \frac{c_3}{(1+\chi)a_{22}^*} \\ \frac{c_3}{(1+\chi)a_{22}^*} & \frac{c_3^2}{(1+\chi)a_{22}^{*2}} + \chi \end{bmatrix} \right) \otimes \mathbf{I}_2 \\ &= \begin{bmatrix} -2\left(c_1 + k_1 - \frac{(a_{22}^* - k_2)c_3}{(1+\chi)a_{22}^*}\right) & \\ \left(\frac{1+\chi}{a_{22}^*} + c_3\right)(a_{22}^* - k_2) - c_3a_{22}^*(c_1 + k_1 - a_{22}^*\chi) & \\ & -2\chi a_{22}^* \end{bmatrix} \otimes \mathbf{I}_2. \quad (17) \end{aligned}$$

Choosing $k_2 = a_{22}^*$ and $k_1 = \chi a_{22}^* - c_1$, equation (17) simplifies in:

$$\begin{aligned} & \mathbf{P}(\hat{\mathbf{A}} - \mathbf{K}\mathbf{C}) + (\hat{\mathbf{A}} - \mathbf{K}\mathbf{C})^T \mathbf{P} = \\ &= \begin{bmatrix} -2\chi a_{22}^* & 0 \\ 0 & -2\chi a_{22}^* \end{bmatrix} \otimes \mathbf{I}_2 = -2\chi a_{22}^* \mathbf{I}_4. \quad (18) \end{aligned}$$

At the end, if the following gain matrix has been chosen: $\mathbf{K} = [k_1 \ k_2]^T \otimes \mathbf{I}_2 = [\chi a_{22}^* - c_1 \ a_{22}^*]^T \otimes \mathbf{I}_2$, then, for any $\chi > 0$, condition (13) is satisfied for $\lambda_0 = 2\chi a_{22}^*$.

Note that λ_0 can be chosen arbitrarily, since χ is arbitrary. It implies that there always exists a χ such that also (14) is satisfied.

IV. EXPERIMENTAL SETUP

A test setup has been suitably built to validate proposed non-linear observer. The machine under test is a 2.2 kW



Figure 2. Photo of the experimental set-up.

IM SEIMEC model HF 100LA 4 B5, equipped with an incremental encoder. The employed test set up consists of:

- A three-phase 2.2 kW induction motor; with parameters shown in Table II;
- A frequency converter which consists of a three-phase diode rectifier and a 7.5 kVA, three-phase VSI;
- One electronic card with voltage sensors (model LEM LV 25-P) and current sensors (model LEM LA 55-P) for monitoring the values of the stator phase voltages and currents; and one voltage sensor (Model LEM CV3-1000) for monitoring the value of the DC link voltage;
- A dSPACE card (DS1103) with a PowerPC 604e at 400 MHz and a floating-point DSP TMS320F240.

The test set-up is equipped also with a torque controlled PMSM (Permanent Magnets Synchronous Motor) model Emerson Unimotor FM mechanically coupled to the IM, to implement an active load for the IM. The electromagnetic torque is measured on the shaft by a torquemeter model Himmelstein 59003V(4-2)-N-F-N-L-K.

A photo of the employed test set-up is shown in Fig. 2.

Table II
RATED DATA OF THE MOTOR

Rated power	2.2 kW	Rated speed	1425 rpm
Rated voltage	380 V	Rated torque	14.9 Nm
Rated frequency	50 Hz	Pole pairs	2
$\cos \phi$	0.75	Inertia moment	0.0067 kgm ²

V. SIMULATION AND EXPERIMENTAL RESULTS

Numerical simulations have been performed in Matlab[®]-Simulink[®] environment. With regard to simulations, as machine under test the dynamic model of the IM taking into consideration the magnetic saturation has been adopted. It is basically the same dynamic model adopted for implementing the non-linear observer. The simulated test has been performed twice, adopting the proposed non-linear observer taking into consideration the magnetic saturation, and adopting the classic FOLO without take into consideration the saturation effects. The classic FOLO has been tuned assuming constant electrical parameters of the IM, corresponding to the knee of the magnetization curve. With regard to the simulation test, a contemporary step variation of the IM reference speed, rotor

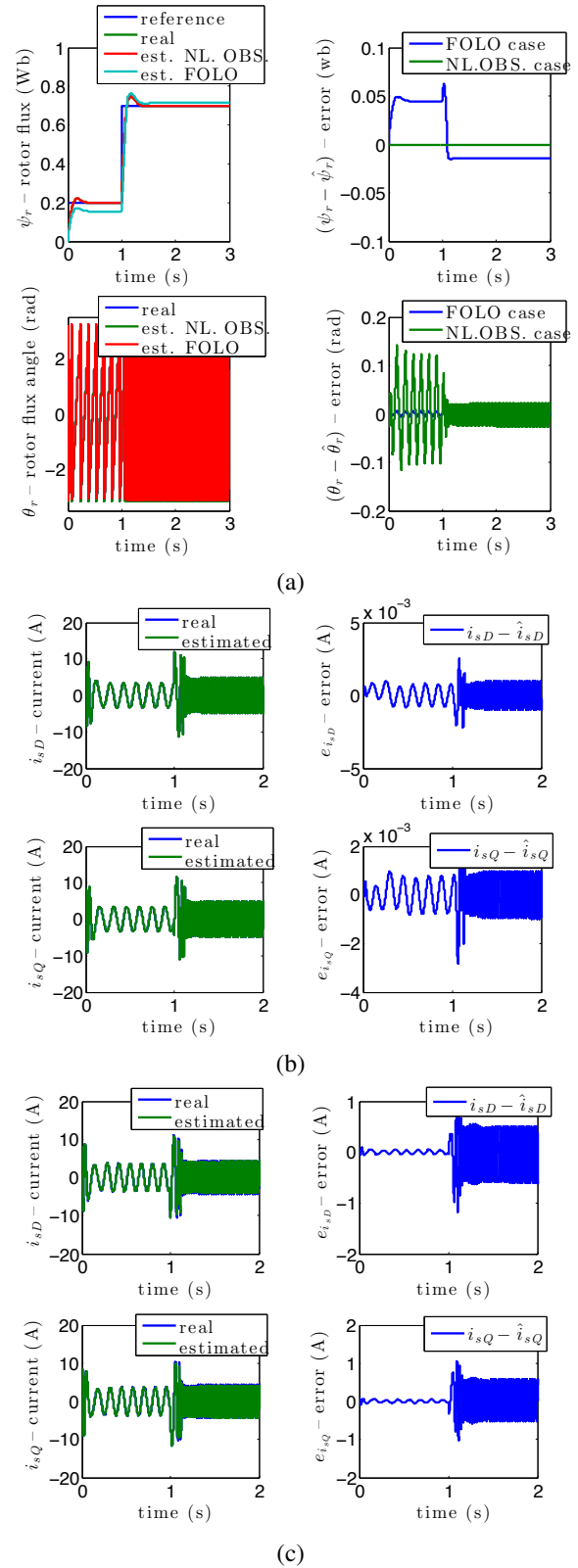


Figure 3. Simulation results during a contemporary speed, flux and torque step reference, $\omega_{ref} = 20 \rightarrow 100$ rad/s, $|\Psi_{ref}| = 0.2 \rightarrow 0.7$ Wb $t_L = 2 \rightarrow 10$: (a) estimated and real rotor flux, (b) estimated and real i_{sD} , i_{sQ} stator current components with nonlinear observer, (c) estimated and real i_{sD} , i_{sQ} stator current components with FOLO.

flux amplitude and load torque has been given the drive, of the type: $\omega_{ref} = 20 \rightarrow 100$ rad/s, $|\Psi_{ref}| = 0.2 \rightarrow 0.7$ Wb $t_L = 2 \rightarrow 10$ Nm. With such a test, the drive works at different speeds with different load torques and rotor flux levels: such a working condition emulates the behaviour of the drive in optimal efficiency conditions. To show the behaviour of the observers, independently from the control action, both the non-linear observer and the FOLO have been tested in parallel with respect to the control system, whereas the real flux has been feedback to close the flux control loop. Fig. 3.(a) shows the real rotor flux amplitude and the estimated one as well as its phase position, obtained with both the observers, as well as the corresponding estimation errors. It can be seen that, approaching the rated flux of the IM, in correspondence to which the parameters of the FOLO have been tuned, both observers work correctly with the estimated fluxes correctly tracking the real ones. On the contrary, at rotor flux equal to 0.2 Wb, while the proposed non-linear observer is able to correctly estimate the real flux, the classic FOLO presents a very high estimation error, equal to about the 7% of the real flux. Fig.s 3.(b)-(c) show respectively the real and estimated stator current components i_{sD} and i_{sQ} in the stator reference frame, as well as the instantaneous estimation error of both the observers. These figures clearly show that, while at the rated flux of the IM, in correspondence to which the parameters of the FOLO have been tuned, the observers tracking errors are very close to each other (as expected), for the lower value of the reference flux, the non-linear observer significantly overcomes the FOLO in terms of estimation accuracy. In particular it can be observed that the stator current estimation error is of the order 10^{-3} for the non-linear observer, while it is of the order unity for the FOLO, confirming the goodness of the proposed approach. This is to be expected, since the non-linear observer has embedded the knowledge of the magnetic working condition of the IM.

With regard to the experimental test, a set of contemporary step variation of the IM reference speed, rotor flux amplitude and load torque has been given the drive, of the type: $\omega_{ref} = 20 \rightarrow 40 \rightarrow 60 \rightarrow 80$ rad/s, $|\Psi_{ref}| = 0.2 \rightarrow 0.4 \rightarrow 0.6 \rightarrow 0.8$ Wb $t_L = 2 \rightarrow 4 \rightarrow 6 \rightarrow 8$ Nm. Fig. 4.(a) shows the reference and measured speed under the above described test. Fig.s 4.(b)-(c) show respectively the i_{sD} , i_{sQ} measured and estimated stator current components in the stator reference frame, as well as the instantaneous estimation error of both the observers. Like in the numerical simulation case, the test has been performed twice, adopting the proposed non-linear observer, and the FOLO. These figures confirm the simulation results and clearly show that, while approaching the rated flux of the IM, in correspondence to which the parameters of the FOLO have been tuned, the observers tracking errors are very close to each other (as expected), for low values of the reference flux (particularly 0.2 and 0.4 Wb), the non-linear observer significantly overcomes the FOLO in terms of estimation accuracy. The peak estimation error with the nonlinear observer is about 0.5 A, while it overcomes 2 A (4 times) with the FOLO. Correspondingly, Fig. 4.(d) shows the waveform of the rotor magnetizing current amplitude, as obtained with both the observers, which is proportional to

the rotor flux amplitude by $L_m(|i_{mr}|)$. It can be observed that, while approaching the rated flux of the IM, the $|i_{mr}|$ estimated by the two observers are very close to each other (as expected), for low values of the reference flux (particularly 0.2 and 0.4 Wb) they become quite different. In particular, at 0.2 Wb the $|i_{mr}|$ estimated by the non-linear observer is much lower than that estimated with the FOLO, coherently with the fact that in the linear region of the magnetizing curve the static magnetizing inductance is much higher. Since there are no flux sensors embedded in the IM, no direct comparison could be made between the estimated and the measured value of the rotor flux. Nevertheless, an indirect confirmation of the accuracy of the flux estimation has been performed here comparing, with both the observers, the measured torque on the IM shaft (with the above described torque meter) with the estimated torque. The torque has been estimated, with both observers, on the basis of the torque equation. Since it depends on the estimated rotor flux and the measured stator current, the verification of the accuracy of the torque estimation is an indirect verification of the accuracy of the flux estimation. It should be borne in mind that, coherently with the adopted modelization of the two observers, the torque estimation based on the non-linear observer takes into consideration the variable parameters, while that based on the FOLO assumes constant parameters. Fig.s 4.(e)-(f) show the electromagnetic and load torques, respectively estimated with both the observers and measured. It can be observed that, while approaching the rated flux of the IM, the torque errors are very close to each other (as expected), for low values of the reference flux (particularly 0.2 and 0.4 Wb), the non-linear observer significantly overcomes the FOLO observer in terms of estimation accuracy. In particular, at 0.2 Wb the torque error becomes even about 25% with the FOLO, while it is almost null with the proposed non-linear observer. This is an indirect experimental confirmation of the better accuracy achievable in the rotor flux estimation with the proposed observer.

VI. CONCLUSION

This paper proposes a non-linear observer for induction machine drives which takes into consideration the IM saturation effects. The non-linear observer is based on an original formulation of the dynamic model of the IM taking into consideration the magnetic saturation, suitably written in a state space form and expressed in the stationary reference frame. It belongs to the category of the non-linear observer characterized by a Lyapunov based convergence analysis. The proposed non-linear observer has been tested in numerical simulation and experimentally on a suitably developed test set-up. Its behaviour has been compared to that of a classic full-order Luenberger observer (not taking into consideration the saturation effects) in variable flux working conditions, as well as the stator current components. Results clearly show the capability of such a non-linear observer to correctly estimate the rotor flux linkage amplitude and phase under flux varying conditions including strong variation of the saturation of the iron path.

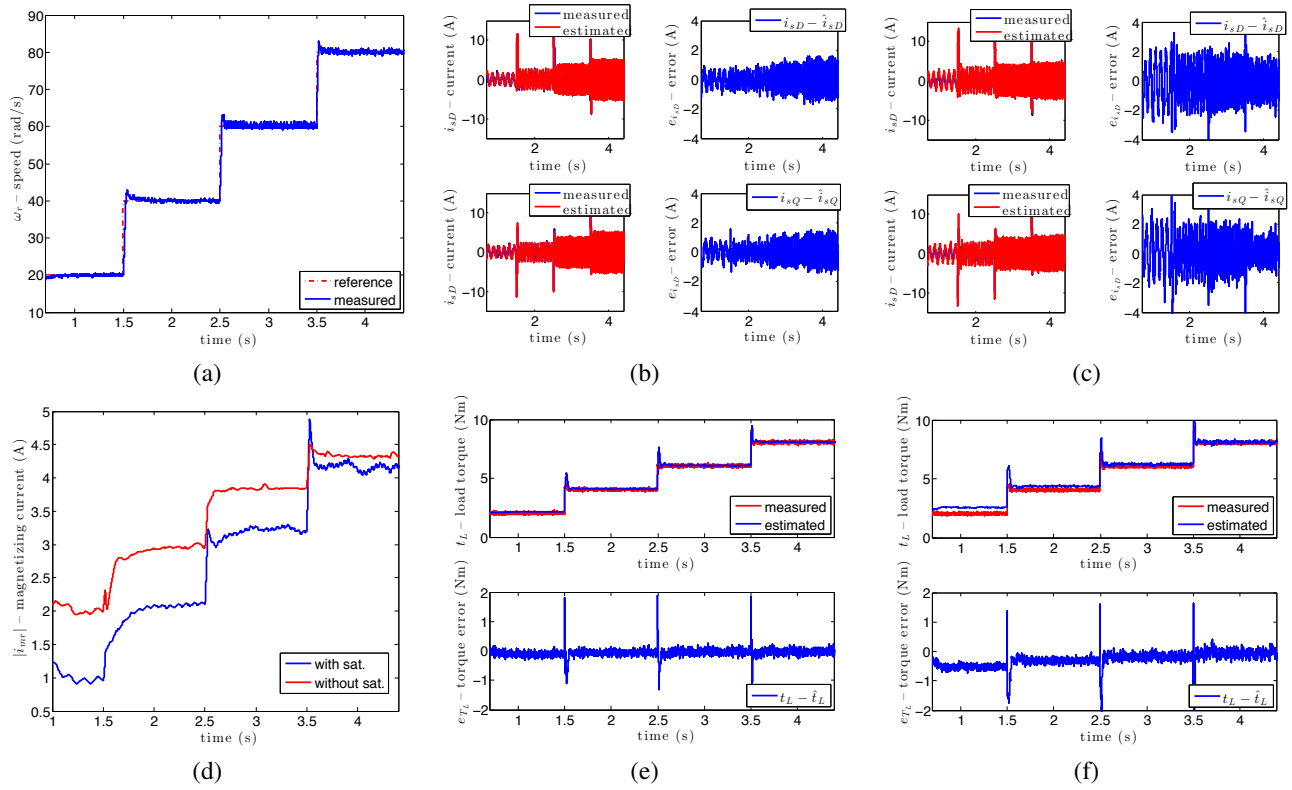


Figure 4. Experimental results during a contemporary speed, torque and flux step reference, $\omega_{rref} = 20 \rightarrow 40 \rightarrow 60 \rightarrow 80$ rad/s, $|\Psi_{rref}| = 0.2 \rightarrow 0.4 \rightarrow 0.6 \rightarrow 0.8$ Wb $t_L = 2 \rightarrow 4 \rightarrow 6 \rightarrow 8$ Nm: (a) rotor speed, (b) estimated and measured i_{sD} , i_{sQ} stator current components with nonlinear observer, (c) estimated and measured i_{sD} , i_{sQ} stator current components with FOLO, (d) rotor magnetizing current (e), estimated and measured electromagnetic torque with nonlinear observer, (f) estimated and measured electromagnetic torque with FOLO.

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