Feedback Linearizing Control of Induction Motor Considering Magnetic Saturation Effects

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***Abstract*—This paper presents a input-output Feedback Linearization (FL) control technique for rotating Induction Motors (IM), which takes into consideration the magnetic saturation of the iron core. Starting from a new formulation of the IM dynamic model, taking into consideration the magnetic saturation, expressed in a space-state form in the rotor flux oriented reference frame, the corresponding FL technique has been developed. To this aim, a particular care has been given to the choice of the non-linear functions interpolating the magnetic parameters vs the rotor magnetizing current curves and the corresponding magnetic characteristic. The proposed FL technique has been tested experimentally on a suitably developed test set-up and compared, under the same bandwidths of the speed and flux closed loops, with both the FL technique not taking into consideration the magnetic saturation and with the industrial standard in terms of high performance control of the IM: Field Oriented Control (FOC).**

***Index Terms*—Induction Motor (IM), input-output feedback linearization, magnetic saturation effects.**

Table I
LIST OF SYMBOLS

SYMBOLS	
u_{sx}, u_{sy}	stator voltages in the rotor flux oriented reference frame;
i_{sx}, i_{sy}	stator currents in the rotor flux oriented reference frame;
i_{mrx}, i_{mry}	magnetizing currents in the rotor flux oriented reference frame;
$\|\mathbf{\Psi}_r\| = L_m\|\mathbf{i}_{mr}\|$	rotor flux amplitude;
$L_s(L_r)$	stator (rotor) inductance;
L_m	3-phase magnetizing inductance;
$R_s(R_r)$	stator (rotor) resistance;
$L_{s\sigma} = L_m - L_s$	stator leakage inductance;
$L_{r\sigma} = L_m - L_r$	rotor leakage inductance;
$\sigma = 1 - \frac{L_m^2}{L_s L_r}$	total leakage factor;
$T_r = \frac{L_r}{R_r}$	rotor time constant;
ω_r	angular speed of the rotor (in electrical angles);
a	mechanical angular acceleration;
p	pole-pairs;
J_m	rotor inertia.

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I. INTRODUCTION

The control of induction machine has been an area of primary interest in academia and industry for many years. The design of high performance control techniques for IMs is, however, quite a difficult task. Actually two control inputs (amplitude and frequency of the stator voltages) and two outputs are available: the rotating speed and the rotor or stator or magnetizing flux amplitude. The fist control target is related to the high dynamic performance to be achieved in the IM motion, the second control target is related to the optimization of the IM efficiency (electrical losses minimization). This is an intrinsically non-linear problem, since the electromagnetic torque is proportional to the product between the rotor flux amplitude and the stator current. Starting from scalar control solution based on the steady-state model of the IM, the industrial standard in terms of high performance control of IMs has bee established as the field oriented control [1], [2], [3]. The control system theory, however, offers an important corpus of non-linear control methodologies for dealing with highly non-linear systems [4], [5]. Among these, one of the most promising is the so-called input-output Feedback Linearization (FL). Nevertheless, very few applications of non-linear control methods to electrical drives are present in the scientific literature. Indeed, few papers deal with the input-output feedback linearization of induction motors [6], [7], [8]. The input-output feedback linearization control technique is, however, a model based control and thus suffers primarily from two disadvantages: 1) the accuracy of the dynamic model on which the control law is based, 2) the corresponding correct knowledge of the model parameters. An inaccurate modelization of the plant under control can make the performance of the FL be even worse than those achievable with other less sophisticated techniques. As far as IM control is concerned, the best improvements in the adoption of the FL with respect to FOC are to expected in working conditions where simultaneous speed (torque) and rotor flux variations occur (optimal efficiency). In such conditions, the correctness of the classic IM dynamic model can fail since the magnetic saturation effects of the iron core, typically not accounted for in the classic model, become significant. In such conditions not

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only all the inductance and leakage factors of the IM model vary with the magnetization level of the machine with a highly non-linear behaviour, but also new terms in the dynamic model arise, not existing in the classic model. The non-linearity of the model increases thus consistently. Some dynamic models of IM taking into consideration the magnetic saturation have been developed in the scientific literature [9], [10], [11], [12], [13], each of which presents its peculiarity in representing the magnetic saturation. Many of the above models, however, do not present a space-state representation, and therefore are not particularly useful for developing control techniques. Starting from the above remarks, this paper proposes an input-output feedback linearization technique for induction machine drives which takes into consideration the IM saturation effects. From this standpoint, a dynamic model of the IM taking into consideration the magnetic saturation, which is inspired to [1, Chapter 6] has been formulated, rearranging the entire set of equations in space-state form, assuming as state variables the stator current and the rotor magnetizing current space-vectors. To the best of the authors knowledge, the only application of the FL to induction machine control taking into consideration the magnetic saturation is [9]. The approach followed by [9] is however different from that presented here at least in two aspects: 1) it is based on a different dynamic model taking into consideration the saturation effects developed in [14], leading to the formulation of a completely different control law, 2) the output vector adopted in [9] are the rotor position and the flux amplitude, while those adopted here are the stator current and the speed.

The proposed FL technique taking into consideration the magnetic saturation has been verified experimentally on a suitably developed test set-up. In any case, differently from [9] where no comparison is presented, to explicitly show the increase of dynamic performance of the control action achievable thanks to the adoption of the proposed technique, here the methodology has been compared with the corresponding FL technique based on the classic dynamic model of the IM [15] and with the FOC, under the assumption of same bandwidths of the speed and rotor flux closed loops.

II. Space-vector mathematical model of IM including saturation effects

In principle, the dynamic model taking into consideration the magnetic saturation of the iron core has been derived in the following using the same approach shown in [1, Section 6.1.1.1]. The model as presented in [1] is not written, in a space-state form, and is not therefore particularly suitable to be used for defining the input-output feedback linearization technique.

Starting from the current model of the IM, the rotor equations, obtained assuming the rotor magnetizing current i_{mr} as state variable, in a rotating reference frame (rotating at speed ω_g), can be written as [1, Equation 6.1-17]:

$$\frac{di_{mrx}}{dt} = \frac{1}{T_r^*} i_{sx} - \frac{1}{T_r^*} i_{mrx} + \frac{T_r}{T_r^*}(\omega_g - \omega) i_{mry}, \quad (1)$$

$$\frac{di_{mry}}{dt} = \frac{1}{T_r^*} i_{sy} - \frac{1}{T_r^*} i_{mry} - \frac{T_r}{T_r^*}(\omega_g - \omega) i_{mrx}, \quad (2)$$

where the modified rotor time constant T_r^* is:

$$T_r^* = T_r \frac{L}{L_m}, \quad (3)$$

while L is called dynamic magnetizing inductance and is equal to:

$$L = \frac{d|\mathbf{\Psi}_r|}{d|\mathbf{i}_{mr}|} L_m + |\mathbf{i}_{mr}| \frac{dL_m}{d|\mathbf{i}_{mr}|}. \quad (4)$$

Note that there is a linear relationship between the rotor flux space vector and the rotor magnetizing current: $|\mathbf{\Psi}_r| = L_m |\mathbf{i}_{mr}|$. Now let us define the following coefficients:

$$a_{11} = \frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma T_r}, \quad a_{12} = \frac{1}{\sigma L_s T_r}, \quad a_{21} = L_s \frac{1-\sigma}{T_r}, \quad a_{22} = \frac{1}{T_r}. \quad (5)$$

If the parameters a_{11}, a_{12}, a_{21} and a_{22} are replaced with a_{11}^*, a_{12}^*, a_{21}^* and a_{22}^*, where the rotor time constant T_r is substituted with the modified rotor time constant T_r^*, then the stator voltage equations [1, Equations 6.1-11, 6.1-12] can be manipulated as follow, in order to obtain the two scalar equations of the two stator current components in the state-space form. In particular the two stator current equations can be written in a generic rotating reference frame (rotating at speed ω_g) as:

$$\begin{aligned} \frac{di_{sx}}{dt} = & - c_1 i_{sx} + (\omega_g + c_2 T_r(\omega_g - \omega_r)) i_{sy} + c_3 i_{mrx} \\ & - ((c_3 T_r - a_{21}^* f_1 T_r^*) \omega_g - c_3 T_r \omega_r) i_{mry} \\ & - c_2 \frac{i_{sx}^2}{i_{mrx}} + f_1 u_{sx}, \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{di_{sy}}{dt} = & - c_1 i_{sy} - (\omega_g + c_2 T_r(\omega_g - \omega_r)) i_{sx} + c_3 i_{mry} \\ & + ((c_3 T_r - a_{21}^* f_1 T_r^*) \omega_g - c_3 T_r \omega_r) i_{mrx} \\ & - c_2 \frac{i_{sy}^2}{i_{mrx}} + f_1 u_{sy}, \end{aligned} \quad (7)$$

with:

$$c_1 = a_{11}^* + a_{12}^*(\Delta L - 2\Delta L^*), \quad (8)$$

$$c_2 = a_{12}^* \Delta L^*, \quad (9)$$

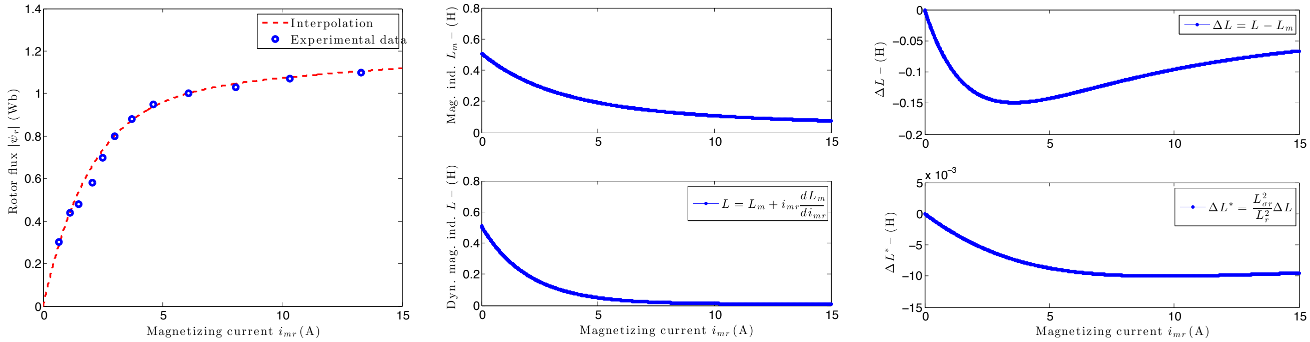
$$c_3 = a_{21}^* f_1 + a_{12}^*(\Delta L - \Delta L^*), \quad (10)$$

where $f_1 = \frac{1}{\sigma L_s}$, $\Delta L = L - L_m$ and $\Delta L^* = \frac{L_{\sigma r}^2}{L_r^2} \Delta L$. It should be noted that, not only all the coefficients a_{11}^*, a_{12}^*, a_{21}^*, a_{22}^*, c_1, c_2 and c_3 depend from the magnetic saturation of the iron core due to the variation with the rotor magnetizing current of the corresponding inductance and leakage factor terms, but also new terms arise not included in the classic dynamic model.

Now it is possible to fix the rotating speed ω_g of the reference frame to be equal to the rotor flux space vector rotating speed $\omega_g = \omega_{mr}$. Indeed, also for the saturated machine, the i_{mry} component of the magnetizing current in (2) is zero if:

$$\omega_g = \omega_r + a_{22} \frac{i_{sy}}{i_{mrx}}, \quad (11)$$

and $i_{mry}(0) = 0$. This last choice has the physical interpretation that the space vector of the magnetizing current is aligned with the x-axis of the rotating reference frame. It is



(a) (b) (c)

Figure 1. (a) Magnetizing curve of the IM, (b) Magnetizing inductance L_m and dynamic magnetizing inductance L curves, (c) Curves of ΔL and ΔL^*.

useful to note that the rotating speed ω_g is equal to the case of unsaturated machine. This means that the saturation effects modify only the amplitude or the rotor flux space-vector, while its angle remains unchanged.

Substituting (11) in (1)-(2) and in (6)-(7), and fixing $i_{mry}(0) = 0$, the electromagnetic part of the state-space model of the induction machine taking into account the saturation effects, in the rotor flux oriented reference frame is:

$$\frac{di_{sx}}{dt} = -c_1 i_{sx} + \omega_r i_{sy} + (a_{22} + c_2)\frac{i_{sy}^2}{i_{mrx}} + c_3 i_{mrx} - c_2\frac{i_{sx}^2}{i_{mrx}} + f_1 u_{sx}, \quad (12)$$

$$\frac{di_{sy}}{dt} = -(a_{11} - c_2) i_{sy} - \omega_r i_{sx} - (a_{22} + c_2)\frac{i_{sx} i_{sy}}{i_{mrx}} - \frac{f_1 a_{21}}{a_{22}}\omega_r i_{mrx} - c_2\frac{i_{sy}^2}{i_{mrx}} + f_1 u_{sy}, \quad (13)$$

$$\frac{di_{mrx}}{dt} = a_{22}^*(i_{sx} - i_{mrx}), \quad (14)$$

$$\frac{d\rho}{dt} = \omega_g = \omega_r + a_{22}\frac{i_{sy}}{i_{mrx}}. \quad (15)$$

In order to obtain the complete state-space model of the induction machine, the mechanical equation should be added. With this regards, it is well known [1] that the saturation does not affect the electromagnetic torque expression, but the value of the torque varies indirectly due to the variation of the stator current and rotor flux, in addition to the variation due to the magnetic parameters. Based on this consideration, the expression of the electromagnetic torque of a saturated IM can be written as: $t_e = \frac{2}{3}p\frac{L_m^2}{L_r}i_{mrx}i_{sy} = \frac{2}{3}p(L_s - \sigma L_s)i_{mrx}i_{sy}$. From this, and under the assumption of constant leakage inductance, it is observed that the saturation indirectly affects the electromagnetic torque by means of coefficient L_s, which varies because of L_m.The mechanical equation, applying the Newton's equation to a rotating mass with inertia moment J_m and viscous friction coefficient b_r, can be written as:

$$\dot{\omega}_r = -a_{33}\omega_r + f_3(L_s - \sigma L_s)i_{mrx}i_{sy} - g_5 t_L, \quad (16)$$

where $a_{33} = \frac{b_r}{J_m}$, $f_3 = \frac{2}{3}\frac{p^2}{J_m}$, $g_5 = \frac{p}{J_m}$ and t_L is the load torque.

A. Dependence of the magnetic parameters on the rotor magnetizing current

Some remarks are to be made about the dependance of the IM parameters from the magnetic saturation. Indeed with respect to the unsaturated model of the IM, these parameters are time–varying and a suitable procedure to identify the waveforms of the parameters is required [16]. This procedure has been applied in this work to obtain the magnetizing curve of the saturated model of IM (See Figure1.(a)). However, here a different interpolation procedure has been used: actually if the polynomial interpolation were used, then, at a certain level of $|\mathbf{i}_{mr}|$ the amplitude of several variables of the system (i.e. $|\mathbf{\Psi}_r|$, L_m, L_s etc...) would tend to infinity, leading up to an incorrect behavior of the observer. For this reason, in order to avoid numerical problems when the $|\mathbf{i}_{mr}|$ increases over the maximum value considered for the interpolation[1], a function based interpolation, working for all values of the magnetizing current, has been proposed. This last choice ensures a better fitting of the experimental data.

As can be see from Figure1.(a), the interpolating curve of the magnetizing characteristic can be written as a sum of an exponential function and a linear function, as follows:

$$|\mathbf{\Psi}_r| = \alpha\left(1 - e^{-\beta|\mathbf{i}_{mr}|}\right) + \gamma|\mathbf{i}_{mr}|. \quad (17)$$

The coefficients α, β and γ, after choosing the interpolating function, have been obtained by means of an optimization procedure (nonlinear least square), minimizing the distance of the curve $|\mathbf{\Psi}_r|$ from the experimental points of the characteristic. In particular the interpolating curve, shown by the dashed red line in Figure1.(a), has been obtained for $\alpha = 0.98$, $\beta = 0.47$ and $\gamma = 0.01$. Starting from equation (17) the expression of L_m can be analytically obtained as: $L_m = \frac{|\mathbf{\Psi}_r|}{|\mathbf{i}_{mr}|} = \alpha\frac{\left(1-e^{-\beta|\mathbf{i}_{mr}|}\right)}{|\mathbf{i}_{mr}|} + \gamma$. Replacing this expression into the equation of the dynamic magnetizing inductance L, the following expression for the dynamic magnetizing inductance is obtained: $L = \alpha\beta e^{-\beta|\mathbf{i}_{mr}|} + \gamma$. This approach permits the straightforward definition of all the inductance

[1]This situation could occurs when the controller is not tuned correctly and the $|\mathbf{i}_{mr}|$ assumes a high value during transient. In this case the flux observer, which itself takes into consideration the magnetic effects of the iron core, for high values of $|\mathbf{i}_{mr}|$ considers an increasing magnetizing inductance, that gives a completely wrong flux estimation.

terms expressions as well as the leakage factor expressions, which are implicitly parametrized once the magnetizing curve has been properly fitted.

At this point an interesting physical interpretation of the coefficients α, β and γ can be done. In fact considering that $\lim_{|\mathbf{i}_{mr}|\to 0} L_m = \alpha\beta+\gamma$ and $\lim_{|\mathbf{i}_{mr}|\to\infty} L_m = \gamma$, then γ can be interpreted as the magnetizing inductance when the machine is full saturated and the relation $\alpha\beta+\gamma$ as the tangent of the magnetizing curve to the origin, which represents the initial status of magnetization of the iron core. From the experimentally point of view it is a reasonable choice to fix α equal to the value of the rated rotor flux. All these considerations give an alternative procedure with respect to the optimization one in order to obtain the interpolating curve.

Figure 1.(b) shows the waveforms of L_m and L respectively. Using the assumption of constant leakage inductances, also the expressions of the modified rotor time constant T_r^*, the model coefficients ΔL and ΔL^* (shown in Figure 1.(c)), and the coefficients c_1, c_2 and c_3 can be obtained.

III. Input–output feedback linearization control

In the following the feedback linearization procedure based on the above presented model of saturated IM will be shown. The adopted linearization approach is inspired to [15] where it has been developed and applied to the unsaturated IM. Here, however, some issues arising from the suitable definition of the magnetizing saturation effects, will be focused on. This will lead to the definition of additional control terms with respect to the saturated IM case, due to the magnetizing saturation effects.

Looking at the equations (12)–(13), the two control inputs u_{sx} and u_{sy} are designed through a state feedback as follows:

$$u_{sx} = \frac{1}{f_1}\Bigg(-\omega_r i_{sy} - (a_{22}+c_2)\frac{i_{sy}^2}{i_{mrx}} - c_3 i_{mrx} + c_2\frac{i_{sx}^2}{i_{mrx}} + \nu_x \Bigg), \quad (18)$$

$$u_{sy} = \frac{1}{f_1}\Bigg(-c_2 i_{sy} + \omega_r i_{sx} + (a_{22}+c_2)\frac{i_{sx}i_{sy}}{i_{mrx}} + \frac{f_1 a_{21}}{a_{22}}\omega_r i_{mrx} + c_2\frac{i_{sy}^2}{i_{mrx}} + \nu_y \Bigg), \quad (19)$$

where ν_x and ν_y are additional control inputs that will be designed suitably. Replacing (18) and (19) in the model (12)–(14), and including the mechanical equation (16), the following model is obtained:

$$\frac{di_{sx}}{dt} = -c_1 i_{sx} + \nu_x, \quad (20)$$
$$\frac{di_{sy}}{dt} = -a_{11} i_{sy} + \nu_y, \quad (21)$$
$$\frac{di_{mrx}}{dt} = -a_{22}^* i_{mrx} + a_{22}^* i_{sx}, \quad (22)$$
$$\frac{d\omega_r}{dt} = -a_{33}\omega_r + f_3(L_s-\sigma L_s) i_{mrx} i_{sy} - g_5 t_L. \quad (23)$$

The model (20)–(23) is the basis of the field oriented control of the IM taking into consideration the saturation effect. Indeed with the feedback laws (18) and (19) the dynamics of the stator currents are made linear and decoupled between them (a variation of ν_x produces only a variation of i_{sx}, and a variation of ν_y produces only a variation of i_{sy}). Moreover the magnetizing current depends only from i_{sx} and if the machine works at constant flux the torque and thus the speed depends only on i_{sx}.

Remark 1: Feedback laws (18) and (19) hold only if $i_{mrx} \neq 0$, otherwise $\{u_{sx}, u_{sy}\} \to \infty$ when $i_{mrx} \to 0$. This is a common feature with the unsaturated IM [15], where one existence condition for the linearizability of the model is that the flux amplitude was different from zero, in order to ensure the existence diffeomorphic map, i.e. it is an invertible function that maps one differentiable manifold to another, such that both the function and its inverse are smooth.

As can be see from model (20)–(23), the speed and flux dynamics are not decoupled in each working condition. Indeed the decoupling between speed and flux works only if the machine works at constant flux, otherwise the speed dynamic presents a nonlinearity with respect to the input, and further important coupling between speed and magnetizing current comes from the dependance of the model coefficients on $\mathbf{i}_{mr}$ due to the saturation effects. This motivates the following study that will permit the exact input–output feedback linearization of the model.

Let us define a new state variable "a" called *angular acceleration*, in place of i_{sy}:

$$a = -a_{33}\omega_r + f_3(L_s-\sigma L_s) i_{mrx} i_{sy} - g_5 t_L. \quad (24)$$

If the load torque variation is assumed to be sufficiently slow, i.e. $\dot{t}_L \approx 0$, then the derivate of a can be written as follows:

$$\frac{da}{dt} = -a_{33}a + f_3(L_s-\sigma L_s)a_{22}^*(i_{sx}-i_{mrx})i_{sy} - f_3(L_s-\sigma L_s)a_{11}i_{mrx}i_{sy} + f_3(L_s-\sigma L_s)i_{mrx}\nu_y + f_3 L_m' a_{22}^*\big(i_{sx}-i_{mrx}\big)i_{sy}i_{mrx}, \quad (25)$$

where L_m' is the derivative of L_m with respect to i_{mrx}, and the fact that $\frac{d}{di_{mrx}}(\sigma L_s) = 0$ and $\frac{d}{di_{mrx}}L_{\sigma s} = 0$ has been used, that is one of the initial hypothesis on the basis of which the model has been deduced. Looking at (25), it is easy to see that the feedback term that linearizes the speed dynamics can be defined as follow:

$$\nu_y = \frac{a_{33}}{f_3(L_s-\sigma L_s)i_{mrx}}a + (a_{11}+a_{22}^*)i_{sy} - a_{22}^*\frac{i_{sx}i_{sy}}{i_{mrx}} - \frac{L_m'}{(L_s-\sigma L_s)}a_{22}^*(i_{sx}-i_{mrx})i_{sy} + \frac{1}{f_3(L_s-\sigma L_s)i_{mrx}}\nu_y'. \quad (26)$$

Replacing (26) into (25) and writing the model (20)–(23) in term of the new state variable (24), the magnetizing current and speed dynamic appear to be linear and decoupled and can be controlled through the inputs ν_x and ν_y'. Indeed speed does not depend on the magnetizing current transient even during its variations. Finally the last step to linearize the model of the motor is the following. Let the following state variable be defined as

$$\nu_{i_{mrx}} = a_{22}^*(i_{sx} - i_{mrx}). \tag{27}$$

Using (27) the time derivative of the magnetizing current is $\frac{di_{mrx}}{dt} = \nu_{i_{mrx}}$, and choosing as state variable $\nu_{i_{mrx}}$ instead of i_{sx} results in:

$$\frac{d\nu_{i_{mrx}}}{dt} = a_{22}^*\Bigg(a_{22}^{*\prime}(i_{sx} - i_{mrx})^2 - (c_1 + a_{22}^*)i_{sx} + a_{22}^* i_{mrx} + \nu_x\Bigg), \tag{28}$$

where $a_{22}^{*\prime}$ is the derivative of a_{22}^* with respect to i_{mrx}. Looking at the (28), the feedback term that linearizes the magnetizing current dynamics can be defined as follow:

$$\nu_x = -a_{22}^{*\prime}(i_{sx}-i_{mrx})^2+(c_1+a_{22}^*)i_{sx}-a_{22}^* i_{mrx}+\nu_x'. \tag{29}$$

Replacing (29) into (28) and writing the model (20)–(23) in term of the new state variables (27) and (24) the model can be finally written as:

$$\frac{di_{mrx}}{dt} = \nu_{i_{mrx}}, \tag{30}$$

$$\frac{d\nu_{i_{mrx}}}{dt} = \nu_x', \tag{31}$$

$$\frac{d\omega_r}{dt} = a, \tag{32}$$

$$\frac{da}{dt} = \nu_y'. \tag{33}$$

Model (30)–(33) is the linearized model of the induction motor with decoupled speed and magnetizing current dynamics taking into consideration the saturation effects. In summary to achieve the input-output feedback linearizing control of IM, considering saturation effects, the inputs ν_x' and ν_y' have to be chosen to fix the magnetizing current and speed dynamic of the model (30)–(33). Then through a first state feedback ν_x and ν_y are obtained starting from ν_x' and ν_y' by (29) and (26). Finally through a second state feedback given by (18) and (19) the voltage source u_{sx} and u_{sy} are obtained starting from ν_x and ν_y. The only condition to ensure the existence of this type of control is that the magnetizing current i_{mrx} is different from zero. This constraint is coherent with the physical constraint that the machine can correctly work only if magnetized.

Remark 2: The state feedback laws presented in this paper are not presented in other works in literature to the best of the Author's knowledge. They are derived starting from the motor model considering the saturation effects. Usually the models used in literature when this type of control is used consider the linearity of the magnetic circuit.

It is not too hard to verify that the state feedback laws in the case of saturated machine coincide with the state feedback laws of unsaturated machine if the saturation effects are not considered [15]. This equivalence can be deduced from the fact that, if the saturation effects are not considered, then $L = L_m$, i.e. the magnetizing inductance does not vary with the magnetizing current. This leads to the fact that $T_r^* = T_r$ and $\Delta L = \Delta L^* = 0$, and consequently $c_1 = a_{11}$, $c_2 = 0$ and $c_3 = a_{21}f_1$. Moreover all parameters are constant, and all the derivatives with respect to the i_{mrx} are zero.

Only a couple of clarifications are needed at this point:

- The state feedback laws in the case of saturated machine are more complicated, but covers the machine dynamics better with respect to the simplified feedback laws of the unsaturated machine. This is particular important since it is well known that the feedback linearization are not robust with resect to the parameter variations and for this reason a more detailed model will give a more performing control system.
- To build the equations (26), for both saturated and unsaturated machine, the acceleration signal is needed. However in the experimental implementation this signal is not usually available and it is particularly noisy. For this reason it could be a good approximation to neglect the term contained the acceleration, since it is due to the viscous friction. It corresponds to consider the friction as a term of the unknown load torque, which does not reduce the goodness of the proposal approach.

The discussion about the controller design is not shown here since it is the same procedure applied in [17], [18] for the linear IM case. The values of bandwidth and phase margin used here are:

- Bandwidth: $B_{-3db} = 140$ rad/s for the speed loop and $B_{-3db} = 1180$ rad/s for the magnetizing current loop;
- Phase margins: $m_\phi = 91°$ for the speed loop and $m_\phi = 44°$ for the magnetizing current loop.

IV. Experimental setup

A test setup has been suitably built to validate the proposed FL control technique. The machine under test is a 2.2 kW IM SEIMEC model HF 100LA 4 B5. The IM has been equipped with an incremental encoder. The employed test set up consists of:

- A three-phase 2.2 kW induction motor; with parameters shown in Table II;
- A frequency converter which consists of a three-phase diode rectifier and a 7.5 kVA, three-phase VSI;
- One electronic card with voltage sensors (model LEM LV 25-P) and current sensors (model LEM LA 55-P) for monitoring the instantaneous values of the stator phase voltages and currents; and one voltage sensor (Model LEM CV3-1000) for monitoring the instantaneous value of the DC link voltage;
- A dSPACE card (DS1103) with a PowerPC 604e at 400 MHz and a floating-point DSP TMS320F240.

The test set-up is equipped also with a torque controlled PMSM (Permanent Magnets Synchronous Motor) model Emerson Unimotor FM mechanically coupled to the IM, to implement an active load for the IM. The electromagnetic torque is measured on the shaft by a torquemeter model Himmelstein 59003V(4-2)-N-F-N-L-K.

A photo of the employed test set-up is shown in Fig. 2.

Table II
RATED DATA OF THE MOTOR

Rated power	2.2 kW	Rated speed	1425 rpm
Rated voltage	380 V	Rated torque	14.9 Nm
Rated frequency	50 Hz	Pole pairs	2
$\cos\phi$	0.75	Inercia moment	0.0067 kgm^2

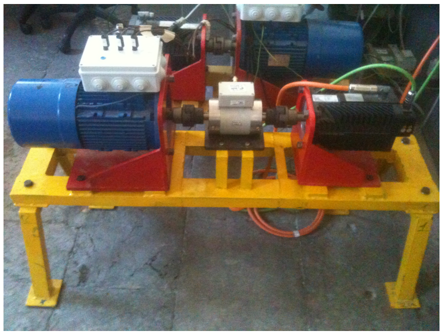


Figure 2. Photo of the experimental set-up.

V. EXPERIMENTAL RESULTS

The proposed technique (in the following "FL with sat.") has been compared experimentally with both the FL which does not take into consideration the magnetic saturation ("FL without sat.") and with the industrial standard in terms of IM control: the field oriented control (FOC). With this regard, to make a meaningful comparison, all of the 3 controllers have been tuned so as to ensure the same bandwidth of the speed and flux closed loops. Moreover, in all cases the rotor flux amplitude and phase have been obtained by adopting the current model of the IM expressed in the rotor flux oriented reference frame (to avoid open-loop integration problems). Coherently with the respective controllers, however, while the "FL with sat." adopts the current model taking into consideration the magnetic saturation, both the "FL without sat." and the FOC adopt the classic current model of the IM. Finally, the "FL without sat." adopts fixed magnetic parameters computed in correspondence to the knee of the magnetizing curve, at the rated magnetization level of the IM.

As a first test, to highlight the improvements achievable with the proposed FL technique, a simultaneously speed and flux reference step variations at no load has been given to the IM drive, of the type $\omega_{rref} = 0 \rightarrow 100$ rad/s, $|\mathbf{\Psi}_r|_{ref} = 0.2 \rightarrow 0.8$ Wb. Fig. 3.(a) shows the reference and measured IM speed during such a transient, obtained respectively with the "FL with sat.", the "FL without sat.", and the FOC. It can be easily observed that the rise time obtained with "FL with sat." is about 0.04 s, which is much lower than that obtained with "FL without sat." equal to 0.3 s, and that obtained with FOC equal to 0.08 s. This is further confirmed by computation of IAE (integral absolute error) and ITAE (integral time absolute error) provided in Tab. III. It can easily observed that IAE obtained with the "FL with sat." is 3.4 times lower than that obtained with the "FL without sat." and 2.9 times lower than that obtained with the FOC. It should be further remarked, that even if during the first instants of the transient the performance of the "FL without sat." is better than those of the FOC, while approaching the steady-state the performance of FOC improves. This is coherent with the well-known dependence of the FL performance from the correct knowledge of the IM parameters, which can even make the adoption of the FL useless with respect to FOC in conditions of not perfect tuning. This is confirmed by the performance indexes in Tab. III, which in practice show a better behaviour of the FOC with respect to the "FL without sat." On the contrary, improving the model on which the FL is based by taking into consideration the saturation permits the performance to be significantly improved. Fig. 3.(b) shows the corresponding waveforms of the rotor flux amplitudes, obtained with all the three control techniques. Even this figure shows that the "FL with sat." permits a much faster flux control than both "FL without sat" and the FOC. This is clearly observable looking at the time rises and IAE and ITAE indexes provided in Tab. III. It should be further noted that, while the two FL present almost the same rise time, the "FL without sat" present a huge overshoot and a much bigger steady-state error at 0.2 Wb. This is coherent with the above considerations about the degradation of the FL performance in presence of incorrect knowledge of the model (or its parameters) adopted for the FL control law definition. Fig.s 3.(d), 3.(e) and 3.(f) show the corresponding stator current i_{sx}, i_{sy} components obtained with all the 3 control techniques. It can be observed that these waveforms present their typical shape with i_{sx} constant at steady-state corresponding to the constant magnetization level of the machine and i_{sy} with a stepwise shape. The main difference between the two FLs and FOC is that, while with FL the current components are governed by the dynamics imposed by FL, and thus do not present a saturation effect, FOC present a classic saturation effect due to the anti wind-up of the PI controllers. Fig. 3.(c) shows the corresponding measured electromagnetic torque.

As a second test, the drive has been operated at the constant speed of 60 rad/s and, at $t = 0$ s, a simultaneous step of the load torque and reference flux of the type, $t_L = 0 \rightarrow 15$ Nm, $|\mathbf{\Psi}_r|_{ref} = 0.2 \rightarrow 0.8$ Wb, has been given the drive. This test reproduces a typical transient of a drive with integrated a minimum losses (maximum efficiency) technique, where a simultaneous load torque and flux variation occur. Fig.s 4.(a)-4.(f) show the same kind of waveforms, as shown for the first test, obtained with the three controllers. The speed waveform clearly shows a significant improvement of the load torque rejection achievable with the "FL with sat.", followed by the "FL without sat." and finally by the FOC. The flux waveforms show that the "FL with sat." permits a far better control than "FL without sat.", with respect to which presents the same dynamic behaviour without any overshoot and steady-state oscillations, as well as than FOC, with respect to which it present a far better dynamic performance. In particular, "FL with sat." presents a better steady-state tracking error than "FL without sat.", especially for low values of the flux, thanks to the more accurate magnetic modelization of the IM. These results are confirmed by the i_{sx}, i_{sy} current component and torque waveforms, highlighting the better dynamic performance achievable with the "FL with sat.". Finally Tab. IV

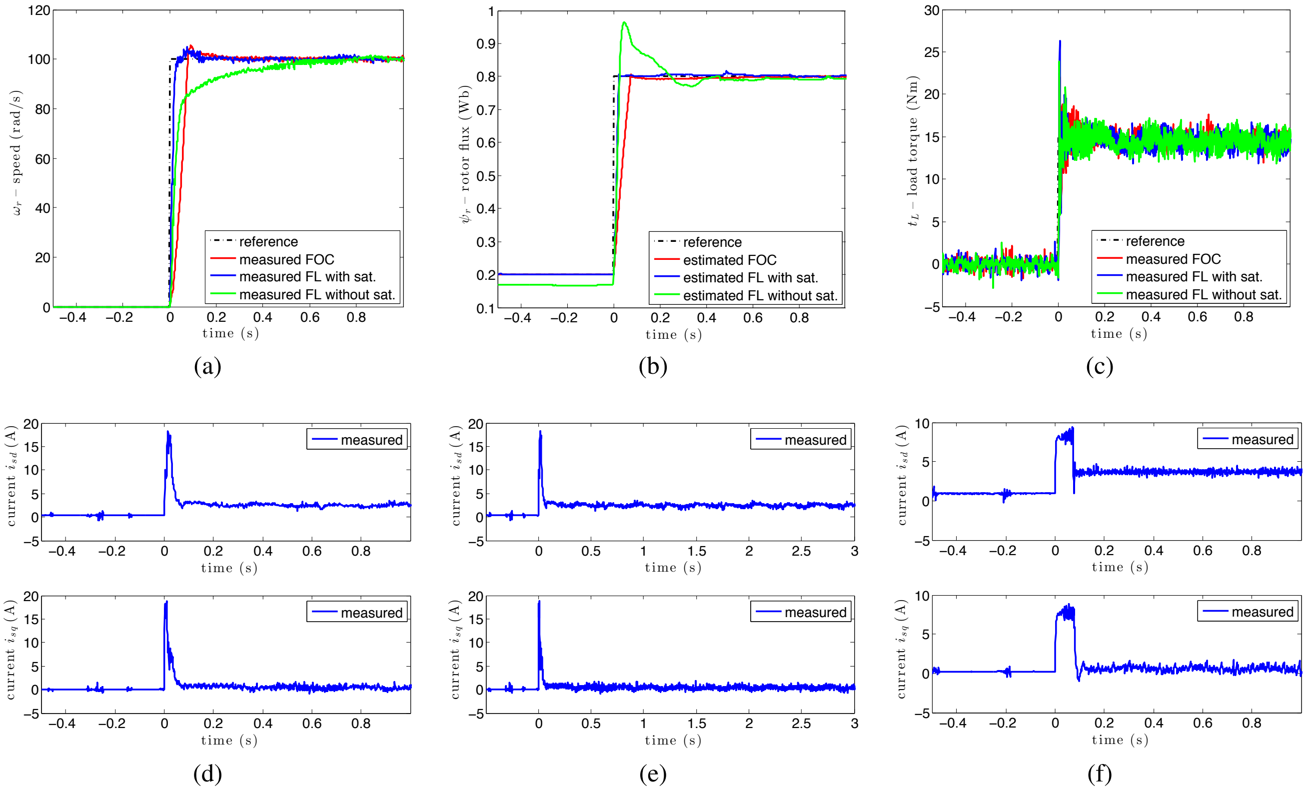


Figure 3. Experimental results during a contemporary speed and flux step reference, $\omega_{rref} = 0 \rightarrow 100$ rad/s, $|\mathbf{\Psi}_r|_{ref} = 0.2 \rightarrow 0.8$ Wb: (a) rotor speed, (b) rotor flux, (c) load torque, (d) direct and quadrature current components ("FL with sat" case) (e), direct and quadrature current components ("FL without sat" case), (f) direct and quadrature current components (FOC case).

shows IAE and ITAE indexes for this last test, confirming the best dynamic performance of the proposed control technique.

Table III
PERFORMANCE INDICATORS DURING A CONTEMPORARY SPEED AND FLUX STEP REFERENCE, $\omega_{rref} = 0 \rightarrow 100$ RAD/S, $|\mathbf{\Psi}_r|_{ref} = 0.2 \rightarrow 0.8$ WB.

	$e_\omega = \omega_{rref} - \omega_r$	$e_\psi = \|\mathbf{\Psi}_r\|_{ref} - \|\mathbf{\Psi}_r\|$
-"FL with sat."	IAE=1.8337	IAE=0.0114
	ITAE=2.0906	ITAE=0.0129
-"FL without sat"	IAE=6.2710	IAE=0.0325
	ITAE=7.2796	ITAE=0.0384
-FOC	IAE=5.3448	IAE=0.0244
	ITAE=5.7227	ITAE=0.0260

Table IV
PERFORMANCE INDICATORS DURING A CONTEMPORARY LOAD AND FLUX STEP REFERENCE, $\omega_{rref} = 0 \rightarrow 100$ RAD/S, $|\mathbf{\Psi}_r|_{ref} = 0.2 \rightarrow 0.8$ WB.

	$e_\omega = \omega_{rref} - \omega_r$	$e_\psi = \|\mathbf{\Psi}_r\|_{ref} - \|\mathbf{\Psi}_r\|$
-"FL with sat."	IAE=0.5316	IAE=0.0106
	ITAE=0.7740	ITAE=0.0120
-"FL without sat"	IAE=2.1313	IAE=0.0263
	ITAE=2.5755	ITAE=0.0338
-FOC	IAE=1.8244	IAE=0.0224
	ITAE=2.0681	ITAE=0.0233

VI. CONCLUSION

This paper presents an input-output feedback linearization control technique for induction motors, which takes into consideration the magnetic saturation. Starting from a new formulation of the IM dynamic model, taking into consideration the magnetic saturation, expressed in a space-state form in the rotor flux oriented reference frame, the corresponding FL technique has been developed. With this aim, particular attention has been paid to the choice of the non-linear functions interpolating the magnetic parameters vs the rotor magnetizing current curves and the magnetic characteristic. The proposed FL technique has been tested experimentally on a suitably developed test set-up and compared, under the same bandwidths of the speed and flux closed loops, with both the FL technique not taking into consideration the magnetic saturation and with the industrial standard in terms of high performance control of the IM: field oriented control. Results clearly show thus the proposed "FL with sat." significantly improves the dynamic performance of both "FL without sat." and FOC, with a reduction of the IAE indexes in the two tests. Finally, results clearly show the opportunity to include the magnetic saturation on the IM modelling on the basis of the which the FL controller is developed, because more accurate modelling permits a significant improvement of the FL controller.

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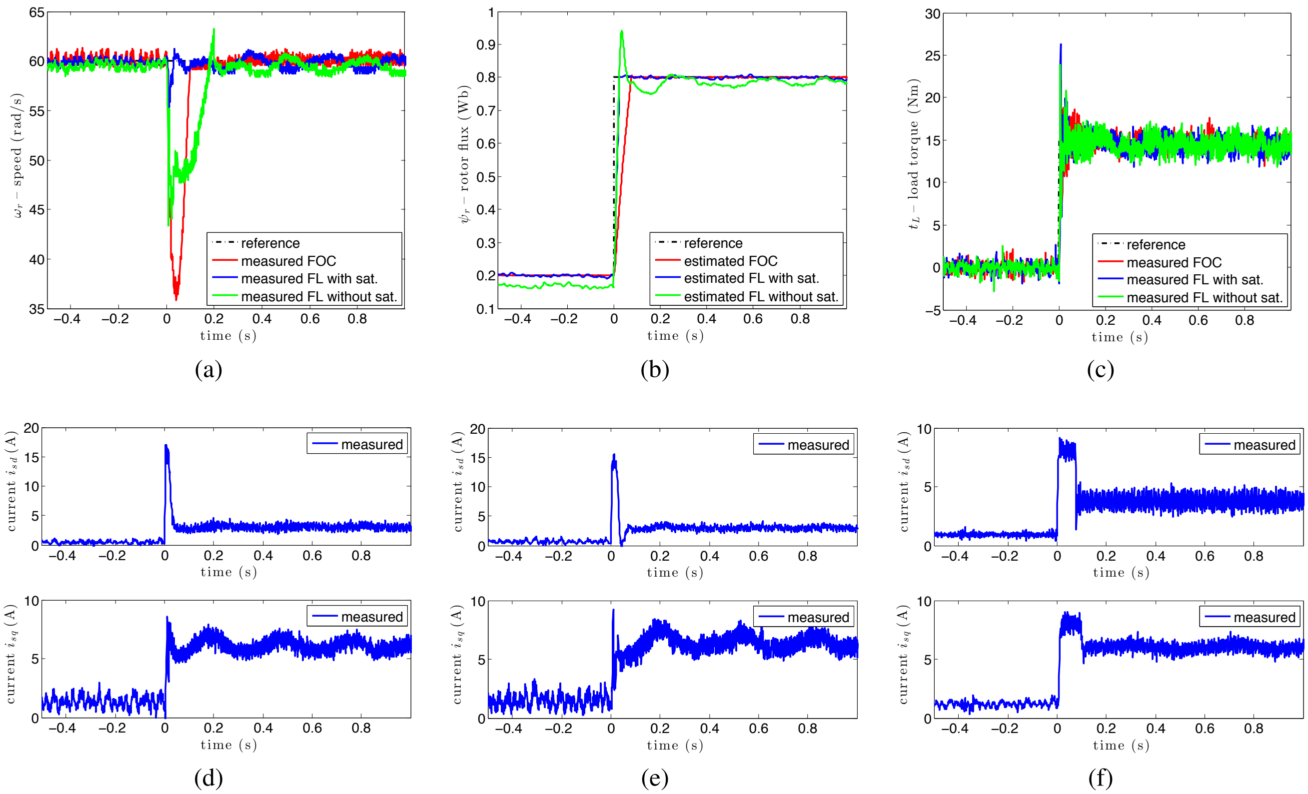

Figure 4. Experimental results during a contemporary load and flux step reference, $t_L = 0 \longrightarrow 15$ Nm, $|\mathbf{\Psi}_r|_{ref} = 0.2 \longrightarrow 0.8$ Wb: (a) rotor speed, (b) rotor flux, (c) load torque, (d) direct and quadrature current components ("FL with sat" case) (e), direct and quadrature current components ("FL without sat" case), (f) direct and quadrature current components (FOC case).

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