

## Design of a Fractional Order PI (FOPI) for the Speed Control of a High-Performance Electrical Drive with an Induction Motor

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**Abstract** - This paper describes the application of the Fractional Order PIs (FOPI) in the speed loop of a high performance induction motor electrical drive. In particular the speed tracking and load rejection capability of FOPI controller has been investigated and compared with both an integer-order PI and an IP both in simulation and experimentally with constant settling time. Illustrative study proves the simplicity and efficiency of the presented design method over integer controllers.

### I. INTRODUCTION

It is well-known that PI controllers are used in most control algorithms in industry since they can be easily understood and implemented in practice [1]. This is obviously true in high-performance electrical drives where many loops of current, flux and speed are present. As for this last one interest has recently been risen in the attempt to find schemes to achieve simultaneously both load rejection and speed tracking despite the possible nonlinearity of the mechanical dynamics, where traditional PIs can still be inadequate [2][3]. Actually only use of PIs can result in excessive overshoot and this can be detrimental in some applications, as in flywheels, where this can result in excessive stress of bearing and consequent reduction in life duration. One way to overcome this is either increasing the complexity of the controller by adding poles and zeroes, or by using 2 DOF (Degree of Freedom) controllers, like IPs. An idea is to employ the 2 DOF PI controller for speed control of electrical drive since the zero of the PI is absent in the case and the overshoot problems can be decreased.

Recently, fractional-order PI (FOPI) controllers have gained considerable importance both in research and industry. Podlubny [4] demonstrated the necessity of the fractional order controller as more efficient control actions can be obtained if the integral and derivative orders can vary in real value. Moreover a fractional controller can be approximated by a product of zeroes and poles [5] and with this respect it can easily replace a lag-lead controller, but with fewer parameters since only three parameters require tuning. At present, FOPI controllers have been applied successfully in many practical systems such as control of hard disk drive servo systems, control of power electronic converters [6]-[8].

In this work a FOPI is presented and compared to 1-DOF PI and 2-DOF IP controllers in terms of speed tracking and disturbance torque rejection. Moreover a simple tuning approach is also described which uses the optimum values of the PI and only tuning of  $\lambda$ . This is summarized in a few steps and then assessed both in simulation and experimental verification is on a suitable developed test bed.

### II. SPEED RESPONSE 1-DOF AND 2-DOF

It is well known that the speed loop for an electrical drive with induction motor with rotational inertia  $J$  and damping

coefficient  $B$ , can be described by the simplified scheme in figure 1, where  $J$  is the rotational inertia and  $B$  is the damping coefficient,  $\omega^*$  is the reference speed,  $\omega$  is the measured speed and  $K=1$ .

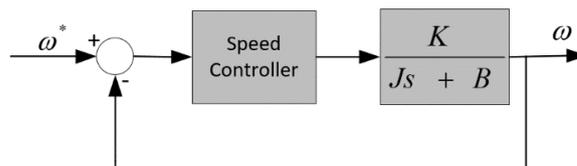


Figure 1: Simplified speed loop

Since the speed loop has the lowest bandwidth within the electrical drive, it is important to address its dynamical performance. Essentially, a high performance electrical drive must satisfy requirements of speed tracking with minimum overshoot as well as good load-torque rejection.

To achieve the above requirement the natural way is to use a conventional Proportional Integrator Derivative (PID) controller for its simplicity and flexibility [9]. The PI control is characterized by the following transfer function:

$$F(s) = K_p + \frac{1}{K_i} \quad (1)$$

And the corresponding control scheme is shown in Figure 2, where:

$$G(s) = \frac{1}{Js+B} \quad (2)$$

In this figure  $e$  represents the error,  $d$  the load torque disturbance to the speed-loop,  $u$  the actuating signal, and  $n$  the measurement error (here considered null).

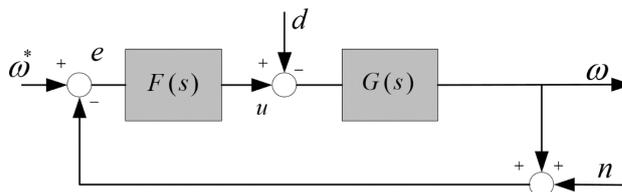


Figure 2: Speed controller with PI

Since it has only one loop that can be independently tuned, this scheme is a 1-DOF controller. In [2] and [10, 11, 12] it is shown how this PI cannot simultaneously achieve speed tracking and load rejection. For instance to meet the load-torque rejection with small speed dip and short restore time is noted that a large overshoot and longer settling time is observed in speed tracking due to the presence of the zero in the control loop [3, 4].

One way to overcome this problem is the use of a 2-DOF control scheme, like the IP (Integral-Control) one shown in figure 2. The overall effect of the IP scheme is the removal of

the zero and the closed loop transfer function is a 2<sup>nd</sup> order system and the overshoot problems can be diminished. In [13, 14, 15] both good speed tracking and load-toque rejection are shown to be achieved. In [2] a method for tuning an IP is presented.

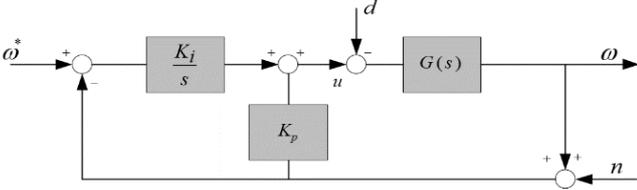


Figure 3: 2-DOF IP scheme

However, even by using a 1-DOF scheme, better performance can be still be achieved, in parity of bandwidth, by using a fractional order PI, thanks to the presence of an additional tunable parameter.

It is noteworthy to remark [17] that both the Pi and the IP the closed loop transfer function for the disturbance input is:

$$W_d(s) = -\frac{1}{J} \frac{s}{s^2 + \frac{B+K_p}{J}s + \frac{K_i}{J}} \quad (3)$$

Which means that the evolution of the system as a consequence of a disturbance is the same for IP and PI.

### III. SPEED RESPONSE: FOPI CONTROLLER

The idea is to use fractional order controller since, existing evidence confirm the fact that fractional order controllers outperform the integer order controller [16]. Actually, fractional-order controller can be used even when the integer-order controller is well performing [17, 18]. The fractional-order proportional integrator (FOPI) controller is a modified version of the existing fraction-order PID and a FOPI used in speed loop of the IM drive is shown in Figure 4.

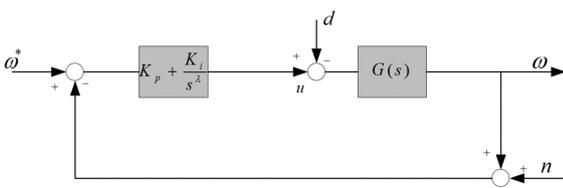


Figure 4: FOPI Controller

A FOPI controller has the following transfer function:

$$C(s) = K_p + K_i s^{-\lambda} \quad (4)$$

where  $\lambda$  is positive real parameter between 0 and 1. There are different ways to tune the parameters of a FOPI [19][20]. Here, however, a simpler method is chosen, better viable by industrialists and practitioners. At first, the values of  $K_p$  and  $K_i$  are tuned for a certain settling time or constant  $\zeta\omega_n$  by following the traditional PI control techniques. Then, by trial and error the value of the real number  $\lambda$  is selected that gives the best performance in terms of overshoot and other dynamical performance specifications, so as to overcome the PI. In the case of the speed response the step response has been analyzed. For this purpose the step response of the  $G(s)$  system with FOPI has been plotted in Figure for several values of  $\lambda$ .

From several tests, it has been observed that  $\lambda < 0.5$  gives an

aggressive response of speed with too active control action. A trade-off value of 0.67 has been chosen for  $\lambda$ .

### IV. FOPI SPEED RESPONSE: COMPARISON WITH PI AND IP

In the following the PI, the IP and the FOPI have been tuned for the speed controller of the FOC, whose scheme is shown in figure 6.

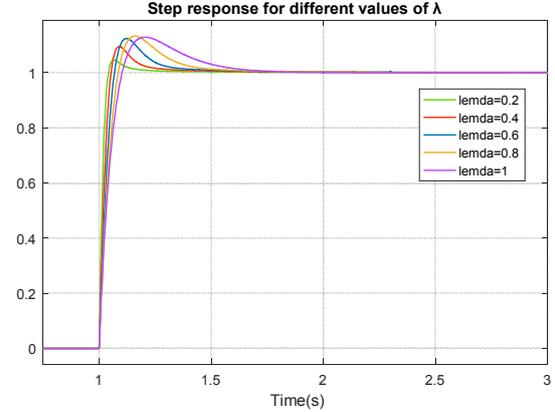


Figure 5: Step response of a FOPI controlled 1<sup>st</sup> order system

The criterion followed for the comparison was to find the best parameters of the PI, IP and FOPI in terms of dynamical response with the same settling time and imposing the damping ratio. In particular, the 2% setting time has been chosen as  $T_s=0.2s$  and then a damping ratio of  $\zeta=1$ . This last choice permits to better appreciate the appearance of overshoot due to the presence of zeros during comparisons

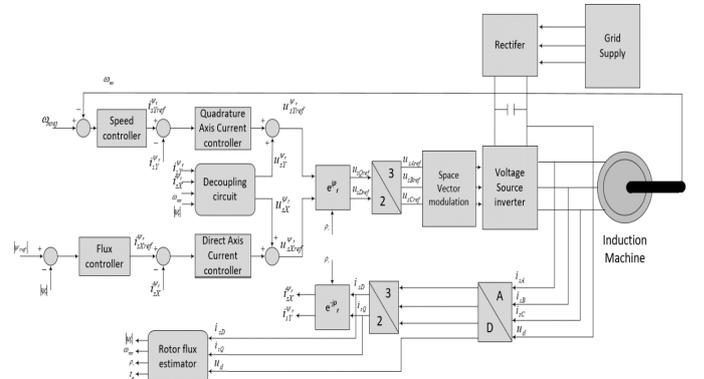


Figure 6: FOC scheme

Since the dynamics of the electrical loops is by far faster than the mechanical dynamics, the FOPI has been applied only to this loop, since no significant difference occurs if a FOPI or a PI is applied to the electrical (flux or current) loops. The parameters of the motor adopted in simulation and in the experimental results are shown in table I.

The PI compensator has been computed by using the mechanical equation of the motor, and by using a classical frequency response analysis [21]. The IP has been tuned following the guidelines of [2]-[3] keeping the same 2% settling time as the PI. The values are the following:

$$K_p = \alpha_s J - B \quad (5)$$

$$K_i = J \left( \frac{\alpha_s}{2\zeta} \right)^2 = \omega_n^2 J \quad (6)$$

where  $\alpha_s = \frac{B+K_p}{J} = 2\zeta\omega_n$ .

Thus, for a chosen overshoot, the choice of  $\alpha_s$ , which depends on the choice of  $K_p$ , determines the bandwidth of the system.

Table I Induction Motor Parameters

$P_N$ [kW]	$U_N$ [V]	$f_N$ [Hz]	$p$	$R_s$ [ $\Omega$ ]	$R_r$ [ $\Omega$ ]	$L_s$ [h]	$L_r$ [h]	$L_m$ [h]	$J$ [kg.m <sup>2</sup> ]
2.2	415	50	2	2.9	1.52	223e-3	229e-3	217e-3	8.3e-3

## V. EXPERIMENTAL SET UP

An experimental rig has been suitably developed in order to assess the FOPI in a high performing electrical drive with induction motor. Fig. 7 shows the experimental rig and highlights its main components.

The experimental rig consists of the following components:

- Two 3 phase induction machines, each working either as a motor or a generator.
- Two 3 phase VSI (7.5kVA Semikron IGBT inverter) supplying the motors.
- Sensors: for the voltage the LEM LV 25-P/SP5; for the current the LEM LA 55-P, and for the speed a WDG 58B incremental encoder.
- Two 3 phase Variac of 20 kVA for supplying the rectifiers connected with the inverters.
- One dSPACE autobox DS1007.



Figure 7: Experimental rig

The schematics of the experimental rig is described in Fig.8, which shows the feedback signals to the dSPACE autobox (via sensors) and the gate signals that are sent to the VSI switches. The VSI DC-link voltage is supplied by the 3 phase variac that is connected to the ac-side of the rectifier. The VSI is driven by a Space Vector-PWM.

The whole control strategy of the drive has been firstly developed in Matlab-Simulink<sup>®</sup> in simulation and then interfaced with dSPACE Board channels for input and output (I/O).

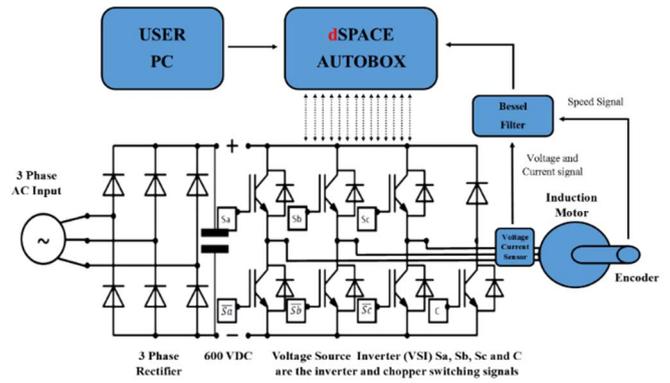


Figure 8: Schematics of the experimental rig

## VI. SIMULATION AND EXPERIMENTAL RESULTS

In the following both simulation and experimental tests have been carried out to compare the performance of FOPI with respect to PI and IP in the speed controller. The damping ratio has been imposed to be  $\zeta=1$ , and the 2% settling time  $T_s=0.2$ . Figure 9 shows the speed response in simulation when using FOPI, PI and IP controller and with no load during start-up with a reference of 1500 rpm. It can be observed that the speed tracks the reference of 1500 rpm, with a quicker response and lower overshoot with FOPI than PI, while of course IP has no overshoot because of the unit damping ratio. A torque load of 6 Nm (half rated torque) has been applied at  $t = 2.5s$ , and also it can be seen that FOPI has the fastest disturbance rejection compared to classical PI and IP, which, as expected theoretically [17] have practically the same.

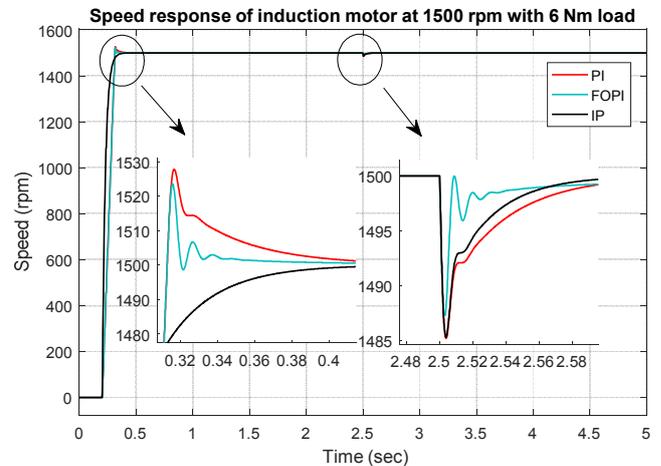


Figure 9 Simulation results of PI, FOPI and IP at 1500 rpm with 6 Nm

Figure 9 illustrates the speed response of IM at 1500rpm with a 6 Nm load applied at 2.5 seconds with PI, IP and FOPI controllers. This test verifies that a PI cannot have a good disturbance rejection and in the same time a fast speed tracking, which is not the case for the FOPI, which is tuned with a unit damping ratio.

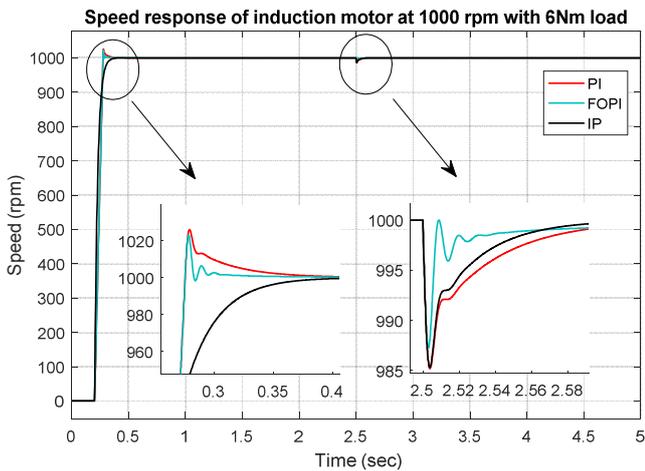


Figure 10 Simulation results of PI, FOPI and IP at 1000 rpm with 6 Nm

Figure 10 illustrates the speed response of IM at 1000rpm with a 6 Nm load applied at 2.5 seconds with PI, IP and FOPI controllers. It can be observed that FOPI controller again gives a quicker 2 percent speed tracking and a better load rejection at 2/3 rated speed of the IM.

Figure 11 illustrates the speed response of IM at 500 rpm with a 6 Nm load applied at 2.5 seconds with PI, IP and FOPI controllers. Once again FOPI controller gives a quicker 2 percent speed tracking and a better load rejection at 1/3 rated speed of the IM.

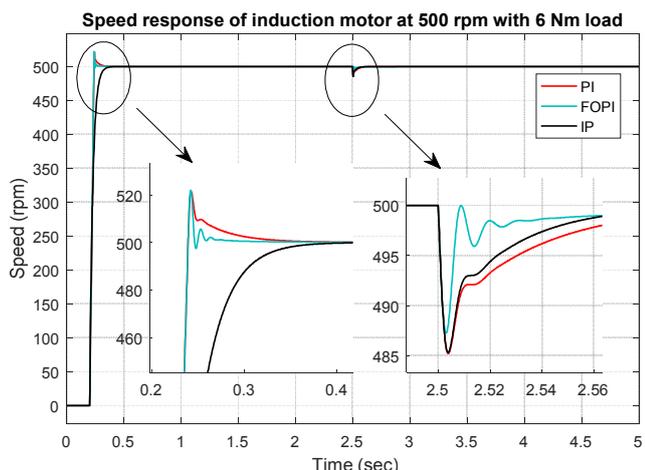


Figure 11 Simulation results of PI, FOPI and IP at 500 rpm with 6 Nm

Afterwards, these tests have been repeated on the experimental rig. Figure 12 shows the experimental results of IM at 1500rpm with a 6 Nm load applied at 14.55 seconds with PI, IP and FOPI controllers. Figure 12 shows that similar to the simulation result shown in Figure 9, the FOPI controller gives a quicker 2 percent speed tracking and a better load rejection insofar it is able to recover the speed steady state quicker.

Figure 13 shows the experimental results of IM at 1000 rpm with a 6 Nm load applied at 13.28 seconds with PI, IP and FOPI controllers. It can be observed that similar to the simulation result shown in Figure 9, the FOPI controller gives a quicker 2 percent speed tracking and a slightly better load rejection. This is also confirmed in Figure 14, which shows the experimental results of IM at 500 rpm with a 6 Nm load applied at 7.92 seconds, together with PI, IP and FOPI controllers. In this last figure, after the load disturbance of 6 Nm, the FOPI controller drives the system quicker inside the 2% band of steady-state than the IP or PI.

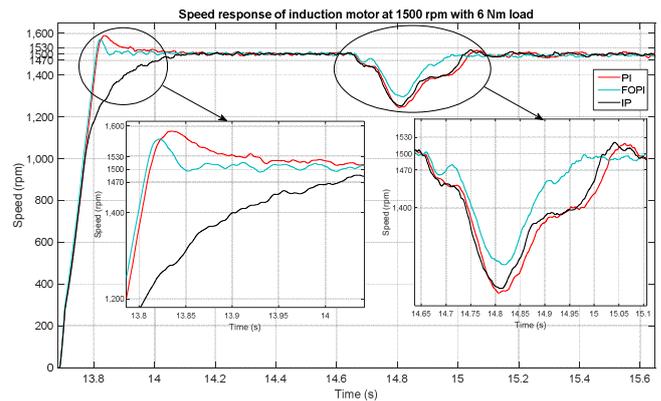


Figure 12 Experimental results of PI, FOPI and IP at 1500 rpm with 6 Nm

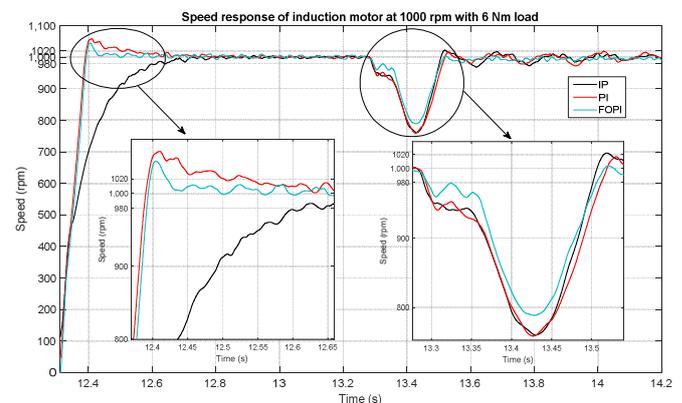


Figure 13 Experimental results of PI, FOPI and IP at 1000 rpm with 6 Nm

As a whole it is apparent that the FOPI has less overshoot and faster settling time than both the PI and IP controllers, confirming the validity of the use of the FOPI for the speed response.

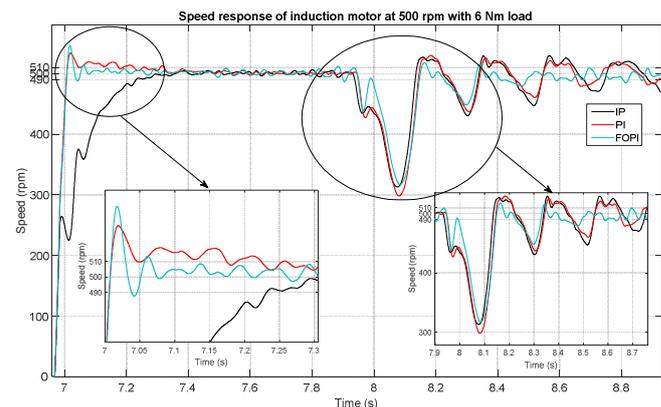


Figure 14 Experimental results of PI, FOPI and IP at 500 rpm with 6 Nm

## VII. CONCLUSION

This paper presents the application of Fractional PI (FOPI) to a classical field oriented control of an induction motor. After presenting a way to design them in the frequency domain, comparisons are made with respect to classical PI, IP control and the advantages of FOPI as for the quicker dynamical response and decrease of the overshoot in terms of speed tracking and load rejection. Simulation and experimental results are in accordance and show the goodness of the method.

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### List of Symbols:

$P_N$  = Rated Power  
 $U_N$  = Rated Voltage  
 $f_N$  = Rated Frequency  
 $p$  = Number of Pole Pairs  
 $R_s$  = Stator Resistance  
 $R_r$  = Rotor Resistance  
 $L_s$  = Stator Inductance  
 $L_r$  = Rotor Inductance  
 $L_m$  = Magnetizing Inductance  
 $J$  = Motor Inertia