

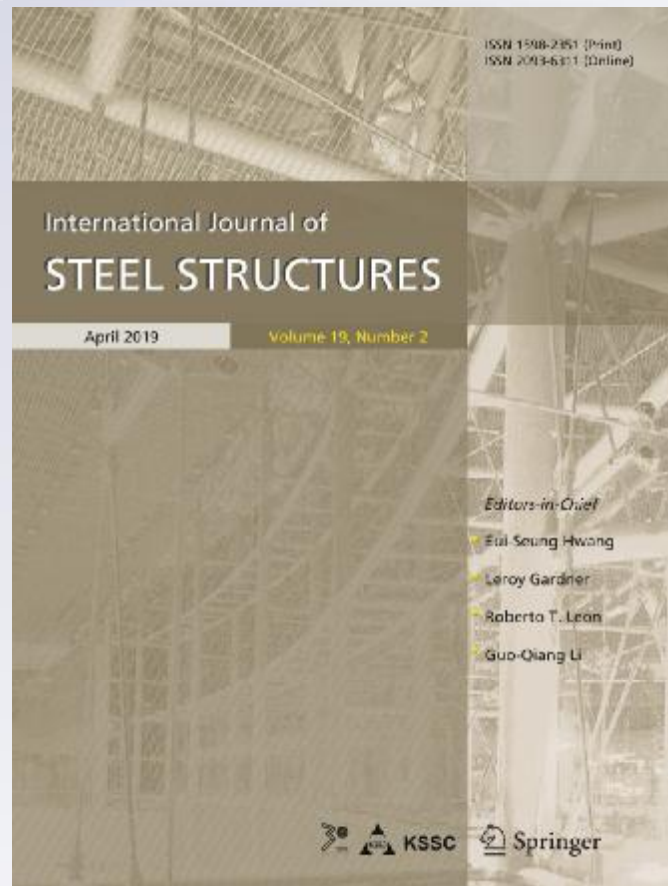
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**Farid Mahboubi Nasrekani & Hamidreza Eipakchi**

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# Axisymmetric Buckling of Cylindrical Shells with Nonuniform Thickness and Initial Imperfection

Farid Mahboubi Nasrekani<sup>1</sup> · Hamidreza Eipakchi<sup>1</sup>

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## Abstract

In this article, the axial buckling load of an axisymmetric cylindrical shell with nonuniform thickness is determined *analytically* with the initial imperfection by using the first order shear deformation theory. The imperfection is considered as an axisymmetric continuous radial displacement. The strain–displacement relations are defined using the nonlinear von-Karman formulas. The constitutive equations obey Hooke's law. The equilibrium equations are nonlinear ordinary differential equations with variable coefficients. The stability equations are determined from them. The stability equations are a system of coupled linear ordinary differential equations with variable coefficients. The results are compared with the finite element method and some other references.

**Keywords** Buckling · Nonuniform thickness · Initial imperfection · Finite element · First order shear deformation theory · Analytical method

## 1 Introduction

The thin cylindrical shells are highly useful structures and they have the vast applications in the field of civil, marine, petrochemical industries, aerospace and so on. The cylindrical shells may be prone to initial imperfections, due to their production processes. These imperfections affect the buckling load of these structures directly. The imperfections are grouped into three important categories: geometrical, structural and loading imperfections. In the case of determining the buckling load of structures, geometrical imperfections are more prevalent. Sobey (1964) determined the buckling load of a cylindrical shell with imperfections, numerically. Arbocz and Babcock Charles (1969) did an experimental and numerical study on the effect of geometrical imperfections on the axial buckling load of thin circular cylinders. Hui and Du (1987) investigated the torsional buckling load of imperfect anti-symmetric cross-ply closed cylindrical shells. The equations are extracted by applying the Koiter's theory and they are linearized by considering

the general solution as sinusoidal functions. Kim and Kim (2002) applied a numerical analysis to evaluate the buckling strength of circular cylinders subjected to axial load by considering the initial geometrical imperfections. Gristchak and Golovan (2003) presented an asymptotic method based on the perturbation technique in conjunction with the Boobnov–Galerkin method for investigating the effect of local thickness defects and imperfections on the buckling load of cylindrical shells. Papadopoulos and Papadrakakis (2005) studied the effect of thickness and material initial imperfections on the buckling load of isotropic shells using the finite element method (FEM). They used Monte Carlo method to determine the buckling load. The effect of initial geometrical imperfection on the buckling load subjected to different loadings was performed by Shahzad et al. (2007) using the finite element method. Eglitis et al. (2009) investigated the effects of initial imperfections on the axial buckling load of composite circular cylinders using numerical and experimental methods. Gotsulyak et al. (2009) offered a numerical solution for stability analysis of cylindrical shells with nonuniform thickness under surface pressure by considering initial geometrical imperfections. Prabu et al. (2010) studied the effect of local geometrical imperfections on the buckling load of circular cylindrical shells numerically using the finite element method. Dung and Hoa (2012) solved the nonlinear stability equations of imperfect functionally

✉ Farid Mahboubi Nasrekani  
farid.mn83@gmail.com

<sup>1</sup> Faculty of Mechanical and Mechatronics Engineering,  
Shahrood University of Technology, Shahrood,  
Islamic Republic of Iran