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Full Length Article

Two degree of freedom fractional PI scheme for automatic voltage regulation

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ARTICLE INFO

Article history:

Received 21 March 2021

Revised 23 July 2021

Accepted 9 August 2021

Available online xxxx

Keywords:

Fractional controller

FOPI

AVR

Robustness

WOA

ABSTRACT

The effectiveness of the inferential control scheme based on robust fractional-order proportional integral (FOPI) controller is presented for automatic voltage regulation (AVR) applications. The method uses two degree of freedom (2DOF) in FOPI scheme, which is tuned with the whale optimization algorithm (WOA). Actually, any AVR needs to keep the reactive power of synchronous generator at demand level, stable voltage and frequency of the electrical power supplies. In this study, the 2DOF FOPI controller is proposed to deviate away from the standard integer order, to show the superiority of extra degree of freedom in both structure and controller. To improve the AVR performance, a new performance measure is proposed for the parameter tuning. The method acquires the significant robustness in parameter perturbation and disturbance interruptions. It is observed in the step response quality that the overshoot and settling time can be reduced to approximately by half than the recently published scheme. The various analyses are shown to accept the dominance of the proposed controller in terms of robustness.

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1. Introduction

The electrical supply industry has always faced a major challenge with perturbation, sustained power system and their effective amelioration. This is especially the case in the context of the rapidly growing integration of distributed generation and loads into power transmission and distribution networks [1]. The power transfers from the power plants to end users through distribution grids. In this process, it is necessary to maintain the specific rated voltage for each power equipment in overall power systems [2]. In the real world, power systems are subject to a wide range of small or large load disturbances during its operations. The disturbances cause voltage variations from the desired levels. Severe voltage fluctuations lead to equipment shut-down, system islanding, and transmission line tripping. Therefore, a voltage regulator is essential to achieve the satisfactory performance of a power system. Deviation to this voltage, causes instability in the power system, and reduces the life of such equipment [3]. Therefore, it has been the most vital control challenge, to perpetuate consistency and stability of this rated voltage level. In theory, the proffering of stability to the reactive power maintains the balance in the power system.

Due to its high value, reactive power has been brought to an equilibrium via numerous tools which includes stabilizers, shunt capacitors, power electronics devices such as flexible alternating current transmission system (FACTS), shunt and series reactors, static VAR compensator (SVC) and AVR. Currently, two major voltage regulation methods are utilized [4], of which AVR system has proven to be the optimum method to achieve this balance. The AVR system works by controlling the DC exciter voltage in the synchronous generator in the closed-loop form, which in turn creates balance of the reactive power. According to NERC (North American Electric Reliability Corporation), the voltage tolerance for the steady-state output of the generator is $\pm 2\%$ [4]. Therefore, a robust controller against disturbances has to be designed for the AVR system, so as to keep the steady-state output of the generator at the nominal level within the $\pm 2\%$ limits. Since the AVR needs to be controlled, researchers have employed various control strategies for this purpose. Upon reviewing recently published literature, it has been noted that researchers focus on the control strategies which are linear, robust and optimal. The robustness is one of the key aspects in control engineering. The PID controller has been the most commonly used control technique due to its simple structure and implementation. However, the PID tuning is crucial and not always optimally robust. Since the last decade, a fractional-order PID (FOPID) controller is attracting much importance. Some research have been done on robustness with fractional-order integral and derivative for uncertainty issues [5].

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<https://doi.org/10.1016/j.jestch.2021.08.003>

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In recent years, the rise of meta-heuristic optimization algorithms have spiked for optimal tuning purposes. Since these are problem-independent methods, it can be applied to a broad range of applications. Though the optimization has been developed aging back to the 1950s, over the last decade, a number of nature inspired optimizations have been adopted for AVR controllers. The comparative study with various PID structures, tuning methodology and systems are reviewed in Table 1.

2. Literature review

A summary of existing studies on achieving appropriate control for AVR systems is shown in Table 1. It shows the most recent studies up to a decade ago. Various methods of optimizing parameters, type of controllers and application of areas are outlined. It is found that the dynamic response for a closed-loop system is mainly characterized by rise time, the settling time and the overshoot. Also, the use of various meta-heuristic algorithms in application of AVR is comprehensively given in the table. In addition Fig. 1 illustrates that the PSO is the commonly considered application. Recently, Gozde [2] has considered the PSO for tuning better dynamic response, making it simple for the practical implementation. From the literature reviewed, it is distinguished that most of the works have been done with one degree of freedom (1DOF) PID for single area network (1A). This paves way for other areas of control structure to be explored, with more than one area network to be applied. Although, a very important step for designing

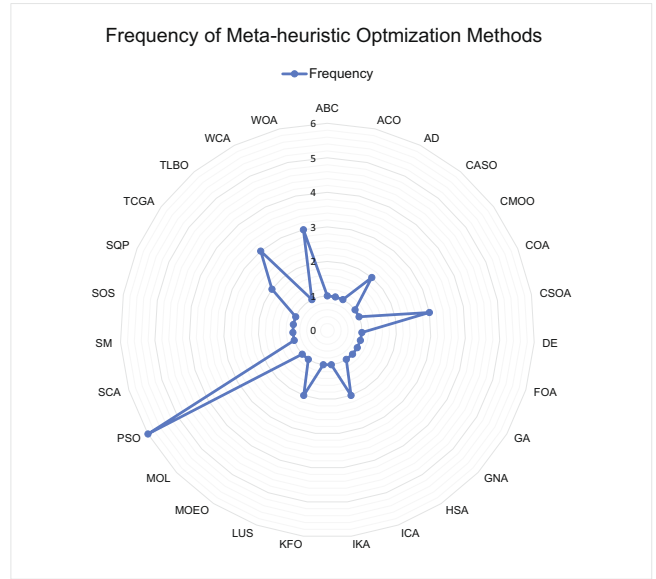


Fig. 1. Frequency of Meta-heuristic Algorithm Usage.

meta-heuristic algorithm, is the choice of objective function. It is found that PID works well with Integral Time Square Error (ITSE) criterion for optimization. While the research field into getting the best tuned parameters was done, the application of fractional

Table 1 Review of the meta-heuristic optimized controllers for AVR.

No.	Reference	Year	Tuning algorithm	Type of controller	Application
1	[6]	2021	COA	PI ² DND ² N ²	1A
2	[7]	2021	WCA	2DOF-PI	1A
3	[2]	2020	PSO	2DOF-PI	1A
4	[8]	2020	ICA	Cascade FOID + filter	2A
5	[9]	2020	SCA	FOPID	1A
6	[10]	2020	SM-MRF	PID	1A
7	[11]	2019	ACO-NM	PIDF (filter on derivative)	1A
8	[12]	2019	IKA	PID	1A
9	[13]	2019	CSOA	Fuzzy PI ² -DF	1A
10	[14]	2019	DE	2DOF-PID	2A
11	[15]	2019	ICA	Fuzzy- FOPI- FOPD	2A,3A
12	[16]	2019	WOA	PID and PID-A	1A
13	[17]	2018	TCGA	PID	1A
14	[18]	2018	KFO	PID	1A
15	[19]	2018	SOS	PID	1A
16	[20]	2018	HSA, LUS, TLBO	PID and PID-A	1A
17	[21]	2018	CSOA	PID	1A
18	[22]	2018	CSOA	FOPID	1A
19	[23]	2017	PSO and GNA	PID	1A
20	[24]	2017	WOA	Fuzzy FOPD ^μ +I	1A
21	[25]	2017	WOA	Fuzzy PD + I	1A, MA
22	[26]	2016	PSO	2DOF-PID	1A
23	[27]	2016	TLBO	PID	1A
24	[28]	2016	SQP, PSO, GA	FOPID	1A
25	[29]	2016	PSO	PID	1A
26	[30]	2015	TLBO	Fuzzy-PID	2A
27	[31]	2015	MOEO	PID	1A
28	[32]	2015	PSO	PIDD ²	1A
29	[33]	2014	N/A	2DOF-PID	1A
30	[34]	2014	FOA	2DOF-PID	1A
31	[35]	2014	LUS	PID	1A
32	[36]	2013	TCGA	PID	1A
33	[37]	2013	CMOO	FOPID	1A
34	[38]	2012	AD	2DOF-PI, 2DOF-PID	1A
35	[39]	2012	MOL	PID	1A
36	[40]	2012	CASO	FOPID	1A
37	[41]	2011	ABC	PID	1A
38	[42]	2009	CASO	PID	1A

1A: single area, 2A: two area, 3A: three area, MA: Multi area.

calculus (FC) has been emerging in a notable fast pace since the last decade for the best tuned parameters. On this theory, the FOPID has been developed and has given remarkable results in the area of control engineering. The FC theory has not only been employed in PID, but also in the area of fuzzy logic as shown in the literature outlined in Table 1.

It can be deduced that the most commonly used meta-heuristic algorithm in the past decade is PSO, WOA is second most utilised. Moreover, integer order and 1DOF schemes are most commonly researched using PID. However, in [2,6,14,26,33,32,38], 2DOF control structure has emerged as a prominent approach for a robust control strategy in particular for AVR systems. This is due to an extra degree of freedom which caters for the load side control. The 2DOF is considered to be simple structured and easily implemented strategy when compared to other complicated solutions. On the other hand, inclusion of fractional-order in actions has shown more promising results in [6,8,9,13,15,22,24,28,37] than classical PID.

Therefore, in this work the fractional-order PI is designed, namely 2DOF FOPI, controller which has a new form with change in system feedback. The new controller is derived for AVR system whose properties have been investigated in number of ways. The paper shows promising results than the recent works on 2DOF PI for the same plant. Briefly the work can be summarized as below.

- A focused review is tabulated to know the number of meta-heuristic algorithms that are adopted for AVR systems.
- Design a new form of controller namely 2DOF FOPI and its evaluation with 2DOF PI [38] and 2DOF PI with state feedback [2]. To note that it eliminates the proof to see if it performs better than 1DOF PI or classical PID which is already shown in [2].
- Tuned various forms of 2DOF controllers with a new performance index for optimization using PSO and WOA.
- Validated the performances with robustness and various time parameters.

In general, to illustrate the superiority of 2DOF FOPI controller, results from PSO and WOA tuned 2DOF PI, WOA tuned 2DOF FOPI controller and recently published paper [2] are compared and discussed. Also, for the considered application the objective is to achieve multiple requirements without increasing the solution complexity.

The article is organized as follows. After comprehensive review of AVR control methods, a brief information on AVR system model is given in Section 3. Followed by usefulness of 2DOF to motivate the design scheme in Section 4 and the proposed scheme of 2DOF FOPI in Section 5. The detailed analysis with various results are explained to prevail this technique in Section 6. Finally, Section 7 includes suggestions for further studies. To note, the integer order has been replaced in this work with fractional order. The definition and implementation used for fractional operator in this work, can be found in Appendix A to start with the approach.

3. AVR system model and its parameters

An AVR system is the DC excitation controller in a Synchronous Generator (SG). This system keeps the terminal voltage of the SG managed which in turn keeps the reactive power at an equilibrium and contributes to system stability and reliability. In general, the AVR system to be controlled is a single area network, which consists of an amplifier, exciter, generator, voltage sensor and comparator as represented in Fig. 2. Moreover, the synchronous generator's operating parameters have been chosen from Fiji's Energy Fiji Limited [43] website, where there is a total of 14 generators having a total power output of 112 MW. Therefore one gen-

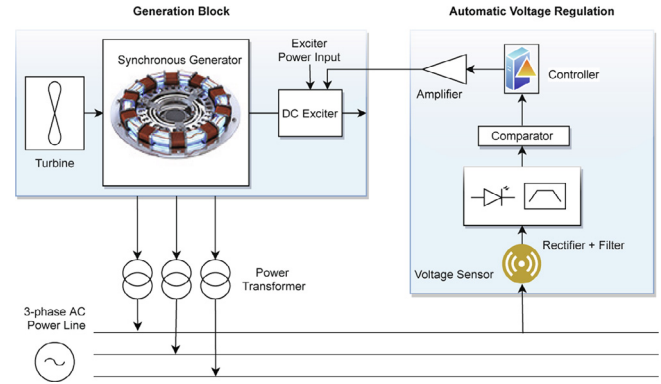


Fig. 2. A controlled AVR system for SG.

erator has power output of 8 MW, voltage of 240 V and frequency of 50 Hz. In this study, the linearized model is considered as shown in Fig. 3, where the terminal voltage V_t is received by the voltage sensor. This signal is rectified, filtered and then compared to set voltage denoted as V_{ref} . Now the controller after comparison, an error signal is generated. This helps to make decision to alter action to the DC exciter. As seen in Fig. 3, an amplifier is in between the DC exciter and the controller. The alteration to DC exciter is done via the controller governing the field windings of the generator over the exciter circuit. The values of linearized model parameters are given in Table 2 [44,2]. Each part of the AVR system has been utilized from the first degree time constant transfer function. Table 2 shows the transfer function model for each block, parameter limits and its values used in this study.

4. 2DOF controlling approach

Due to the fact that the operating conditions of SG is variable with load and system parameters over time, it has been made necessary for the controller to be robust and adaptive. For this reason, a system needs a suitable type of controller and structure. The feedback provides a difference between the input and the output which is usually used for setpoint tracking and disturbance rejection. As it is shown in theory that the degrees of freedom in control engineering simply means the number of closed-loop transfer functions that can be regulated independently [45]. Generally, one degree of freedom (1DOF) is mainly suggested in study. However, it is proven from the study that 1DOF only supports for the setpoint side of the system. While in the two degree of freedom (2DOF) structure, the load side is also fully covered in terms of control. In [45], 2DOF approach from 1DOF has been well verified and it is conceptually illustrated by Fig. 4.

In Fig. 4, say A is the disturbance optimal, B is the setpoint optimal, C is the realization area by 2DOF controller and A to B is the pareto optimal. The above mentioned hurdle has left control engineers with two options. Either to choose one of the pareto optimal points or to use the disturbance optimal parameters and impose limitation on the change of the setpoint variable (i.e., to use a rate limiter for V_{ref}). Under the process engineering situation of early

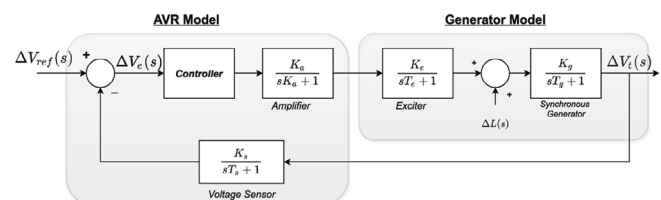


Fig. 3. Linear model of AVR system.

Table 2
Transfer functions and parameter values of considered AVR and Generator models.

Block	Transfer function	Parameter limits	Parameter value used
Amplifier	$K_a/(1 + sT_a)$	$10 \leq K_a \leq 40, 0.02 \leq T_a \leq 0.1$	$K_a = 10.0, T_a = 0.1$
DC Exciter	$K_e/(1 + sT_e)$	$1 \leq K_e \leq 0.4, 0.4 \leq T_e \leq 1$	$K_e = 1.0, T_e = 0.4$
Synchronous Generator	$K_g/(1 + sT_g)$	$0.7 \leq K_g \leq 1.0$ (depends on load), $1.0 \leq T_g \leq 2.0$	$K_g = 1.0, T_g = 1.0$
Voltage Sensor	$K_s/(1 + sT_s)$	$0.001 \leq T_s \leq 0.06$	$K_s = 1.0, T_s = 0.01$

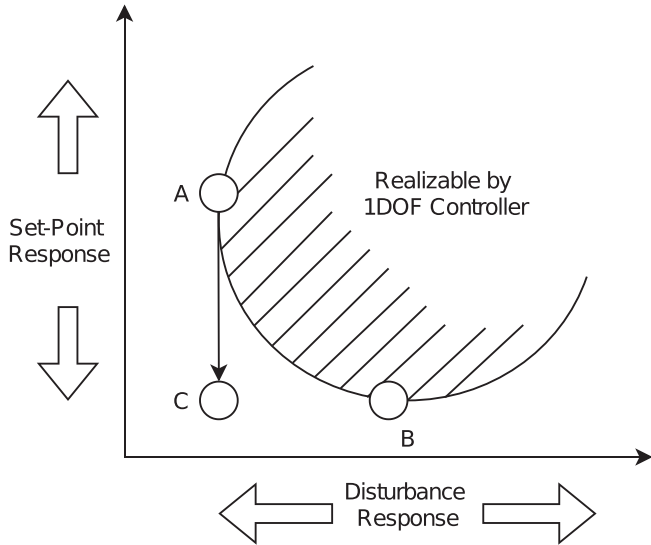


Fig. 4. Conceptual illustration of the effect of the 2DOF structure.

days, when the setpoint variable was not changed very often, the second alternative was satisfactory enough. Therefore, many of the optimal tuning methods presented only with ‘disturbance optimal’ parameters. However, the situation has been changed in the last few decades and the process control systems are required to change the setpoint variable frequently. In this case, the 2DOF PID offers a powerful means to manage with dual requirement. This concludes that the 2DOF structure enables control engineers to make both the setpoint response and the disturbance response practically optimal at once within the linear framework. This has also been proved mathematically in literature such as, Feedforward type (FF type), Feedback type (FB type), setpoint filter type (Filter type), Filter and preceded-derivative type and component-separated type expressions of the 2DOF PID [45].

5. Proposed scheme with FOPI

In recent years, several researchers have established the FOPID controller for most of the applications with fractional integrator and derivative. But, it is evaluated that 2DOF PI is sufficient to deal with steady state error [38] for the AVR system. Also, the fractional derivative from FOPID is very sensitive to noise, so can be difficult to approximate the equivalent transfer function. When the high frequency noise deals with high sample rates, the complexity can be increased with steep slopes. In addition, we have to obtain multiple objectives from the designed controller without adding the solution complexity. With this motive the FOPI is adopted for 2DOF with simple structure. In fact, the 2DOF FOPI assists in the speed of the response by utilizing the proper performance index.

The classical 2DOF PI is presented for the AVR system recently [2]. This scheme is shown in Fig. 5 to understand the designed system. In this system, there is a controller at load side and a controller at setpoint side. It ensures the proper regulation and appropriate tracking ability. In this figure, $C_r(s)$ represents setpoint

controller, which is used for setpoint tracking and $C_y(s)$ represents the feedback controller or load side controller which is used for disturbance rejection. For clarity, given are same transfer functions (1) and (2) and control signal (3), $u(s)$ as below [2].

$$C_r(s) = K_c \left(\beta + \frac{1}{sT_i} \right) \tag{1}$$

$$C_y(s) = K_c \left(1 + \frac{1}{sT_i} \right) \tag{2}$$

where K_c is the controller gain, T_i is the integral time constant, which must be positive and has units of time. As T_i gets smaller, the integral value increases as T_i is in the denominator and β is the setpoint weighting factor, having range from 0 to 1 complying to small gain theory. This is because, the value of β becomes 1, $C_r(s)$ and $C_y(s)$ will be equal. Thus defeating the whole purpose of 2DOF structure and reducing it to classical 1DOF PI controller. Furthermore, if β is more than 1, this increases the value of K_c which leads to instability.

$$u(s) = C_r(s)\Delta V_{ref}(s) - C_y(s)\Delta V_s(s). \tag{3}$$

where $\Delta V_{ref}(s)$ or $V_{ref}(s)$ is the generated voltage which is used to compare to the terminal voltage, $\Delta V_t(s)$ or $V_t(s)$ to achieve a determined control. $\Delta V_s(s)$ or $V_s(s)$ is the sensor voltage in the form of feedback to ascertain suitable load side voltage drop tracking.

Aiming to obtain the optimal values of K_c, T_i and β , the method used is PSO based metaheuristic algorithm. The suggested values for 2DOF PI and 2DOF PI with amplifier feedback (AF) by Gozde [2] are given in Table 3. Indeed, the state feedback can make the closed-loop system more efficient with less sensitive to parameter variations and disturbance inputs [2]. It was claimed the PSO algorithm was preferred due to its well-known superiority and short program code. In fact this is validated from the comprehensive review of methods more for AVR systems using PSO in Section 2.

Now, the aim of the proposed work is to prepare a fractional 2DOF scheme which can further improve the performances in AVR systems. It is well-known state feedback can make the overall system more stable, robust, less sensitive to parameter changes and less steady state error [2]. We extend the fractional 2DOF scheme with state feedback, by taking one feedback from the amplifier out of the AVR circuit, which is the input to the exciter of the SG and another feedback from the terminal voltage $\Delta V_t(s)$. This additional feedback loop drastically improves the dynamic response of the whole system, this has been verified when the pro-

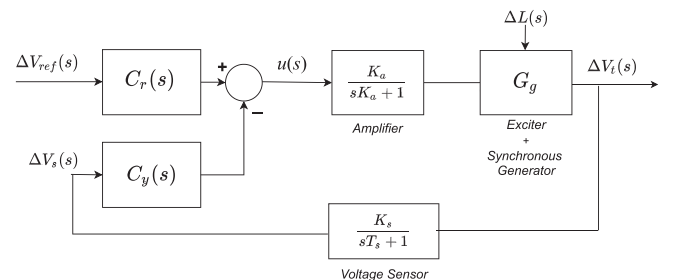


Fig. 5. AVR model with 2DOF PI (PSO tuned) presented in [2].

Table 3
Controller parameters from proposed and others.

Controller	K_c	T_i	β	λ
2DOF-PI with AF [2]	0.958	0.069	0.827	-
2DOF-PI [2]	0.494	0.871	0.332	-
2DOF-PI (proposed)	0.302	0.9525	0.3727	-
2DOF-FOPID with AF (proposed)	0.800	0.101	0.514	0.809

posed scheme was designed. Now aiming to convert 2DOF scheme with fractional order integrator, by adding a fractional order, (1) and (2) can now be represented as,

$$C_r(s) = K_c \left(\beta + \frac{1}{s^\lambda T_i} \right) \quad (4)$$

$$C_y(s) = K_c \left(1 + \frac{1}{s^\lambda T_i} \right) \quad (5)$$

Here, the new parameter λ is a fractional positive real value between 0 to 1. The value for λ cannot be negative, as this will invert the pole to zero. To have a more precise optimum value of λ , upper bound and lower bound for tuning has been set to 0 and 1 respectively. Thus, the values of K_c, T_i, β and λ are required to be calculated in order to satisfy better performance requirement. The modified scheme for AVR system with fractional-order derivatives, is shown in Fig. 6. Due to two state feedback used as shown in figure, the voltage sensor models have been employed. The input to the new voltage sensor has been taken from the output of the amplifier, due to its physical reachability. Without making any change on overall system model in [2], the gain of the voltage sensors has been halved in the scheme. The outputs of the voltage sensors are connected to the load side controller ($C_y(s)$). To note before tuning approach, a good 2DOF design is still maintained to achieve better setpoint and disturbance responses simultaneously.

Though the algorithm PSO is widely used by researchers, adopted is the swarm based technique namely WOA for the required optimization problem. The brief pseudo code of WOA [46] in given in Appendix B and its modification to generate the best search agent after satisfying the performance criterion is explained in the following sub-section.

5.1. Robust tuning performance index

After constituting fractional 2DOF structure for AVR system, it is important to determine the suitable values of controller parameters. In general, it is desired to receive the quick response with almost no or little overshoot and less settling time. On the other side, the response must be protected with load disturbances. Even though 2DOF offers both load side and setpoint side improved performances, as it is presented in [2], there is a scope of improvement with respect to parameter variations and minimum controller efforts. It is well known that perturbations in the system param-

eters may occur in the real field environment. Same way, the optimal controller is only be claimed if it achieves the desired outputs in a less input signal variation. Practically it is defined for less maintenance cost from controlling actions. Applied is the dual performance measures while the WOA algorithm obtains the controller parameters. The controller is designed using the minimization of Integral Squared Time Error (ISTE) criterion usually resulted satisfactory results. This index is defined below for a good step response.

$$J_{ISTE}(\eta) = \int_0^\infty (te(\eta, t))^2 dt \quad (6)$$

where η denotes the parameter to tune for minimization. Secondly, the optimal tuning in real sense is called when the controller output maintains its limit and variations. It is obvious that if the AVR is propelled beyond it's limits the excitation field destroy. It may take the huge wear and repairing cost. In this contest, less attention was paid so far in the literature studied on AVR systems. Analytically, the controller performance measures can be indexed as follows.

$$J_u = \min_{\eta} \int_0^\infty \left| \frac{du}{dt} \right| dt \quad (7)$$

The quality of controller is also defined by its smoothness, meaning less input usage (energy) for reduce maintenance cost of the overall system. Now, as per the AVR system in this work, the controller will be designed by satisfying both criteria stated in (6) and (7). The new performance measures for obtaining the fractional 2DOF PI parameters as follows.

$$J_{ISTE} = \min_{\eta^*} \int_0^\infty (t\Delta V_e(t))^2 dt \quad (8)$$

subject to $J_u = \min_{\eta^*} \int_0^\infty \left| \frac{du}{dt} \right| dt$

where $\Delta V_e(t) = (\Delta V_{ref}(t) - \Delta V_s(t))$ and η^* is the best controller parameters obtained from the WOA. Upon creation of 2DOF FOPID controller the related parameters were tuned with WOA optimization technique as stated. The computed parameters of all controllers identified in this study are summarized in Table 3.

5.2. Summarized implementation flow

The controller parameters, with the addition of the fractional-order integrator have been tuned using the WOA and on the basis of the dual performance indices. To consider the calculated values, the conformity analyses must be conducted to find if the approach is observable and controllable. Therefore, we have analyzed the closed loop system with the observability and controllability analyses. Systematically first, the absolute stability of the controllers are proved by using pole-zero mapping. Similarly, the time domain and frequency domain analyses are performed in order to evaluate their dynamic responses and relative stability. Finally, the robustness analyses are conducted and the proposed scheme is documented as the best method. To ease in implementation flow, the presented design method is given through simple block diagram in Appendix C. It is to be noted that a real time implementation of FOPID controller is possible using a microcontroller unit [47].

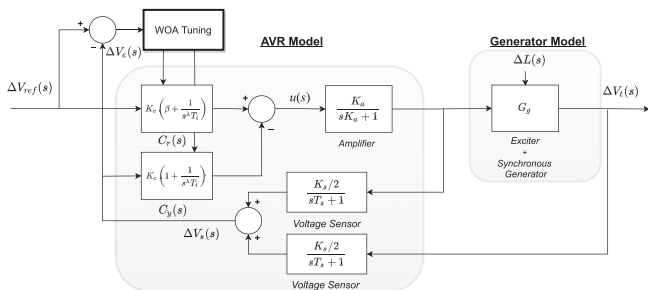


Fig. 6. Modified scheme for AVR system with fractional-order integrator (WOA tuned).

Moreover, in the following section, the results obtained from the new controller structure and improved performance index in (8), are presented in details. The result is compared with the latest strategy in [2] and proved the efficacy mainly with parameter perturbations and disturbance rejections.

6. Validations

It is important to verify the performance with various situations, measurement parameters and analysis. The results are obtained in MATLAB/Simulink environment at the computer including core i5 at 2.90 GHz CPU and 4 GB RAM. The parameters from both techniques are given in Table 3. Note that first two rows in the table show the values given in [2] for the same AVR system models. To establish the concrete acceptance of the proposed method, various analyses have been conducted such as stability analysis, step response analysis, bode analysis, disturbance analysis and parameter perturbation analysis.

6.1. Stability analysis

There are a number of techniques to test the stability of a control system. Out of the many techniques, investigation of localizations of the system poles in pole-zero map is used due to its simplicity. The logic of pole-zero map is to determine if the system is stable or not and to locate that all poles to the left side of the plane. The pole-zero maps can be computed manually or computed via the use of MATLAB Simulink, linear analysis feature. Since the system operation is designed to be functioning in conjunction with self-optimizing structure via a meta-heuristic optimization algorithm, we can consider this analysis as sufficient for normal operating conditions for these kinds of operations. The main comparison is to the recently published paper [2]. This paper has used PSO for tuning. To keep the comparison in fair playing field and prove the superiority of the WOA over PSO, PSO is used in the integer 2DOF PI system as well which has been labeled as 2DOF PI. The pole-zero maps of the designed systems are represented separately in Fig. 7.

According to the logic of pole-zero plots, all the systems are stable. However, if examined in detail, when the conjugate poles, called dominant poles are investigated, it can be seen that the poles of 2DOF PI-controlled system depicted in Fig. 7(b) and (d) is close to the origin and imaginary axis. This means that these systems has a lower response and higher rise time. On the other hand, Fig. 7(a) and (c), are showing very clear differences from the real axis scale. It can be said that the proposed 2DOF fractional PI is more stable than 2DOF PI with AF [2]. Also, second one makes a system more faster and has more oscillation possible in disturbances. But, the additional dominant pole (fractional) can make the overall system more stable. It will be observed later in disturbance and parameter perturbation analysis. Thus, the proposed controller is better than the one in [2].

6.2. Step response analysis

The time response behaviour is commonly used for verification. From a practical standpoint, knowing how the system responds to a sudden input is important because large and possibly fast deviations from the long term steady state may have extreme effects on the synchronous generator and on other portions of the overall system dependent on this component. In addition, the overall system cannot act until the component's output settles down to some vicinity of its final state, delaying the overall system response. Formally, knowing the step response of a dynamical system gives information on the stability of such a system, and on its ability

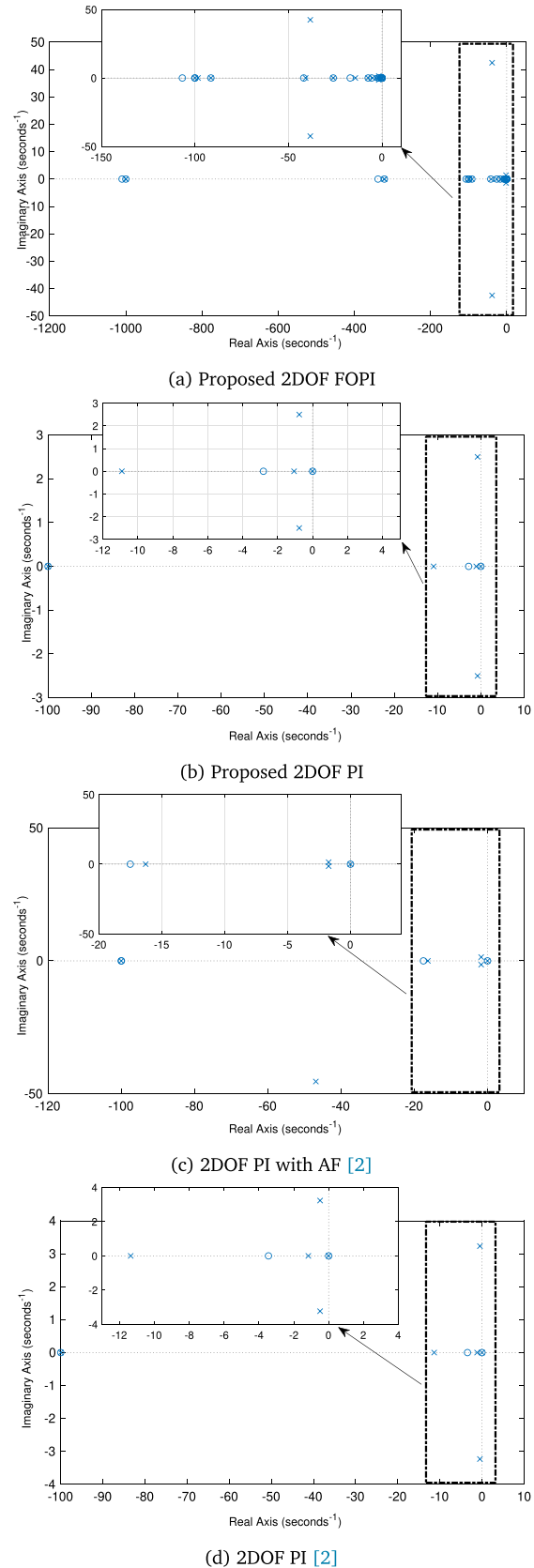


Fig. 7. Pole-Zero plots of the designed systems.

to reach one stationary state when starting from another. From the obtained best parameter values, the step responses are plotted in Fig. 8. As it can be seen easily that the proposed fractional 2DOF

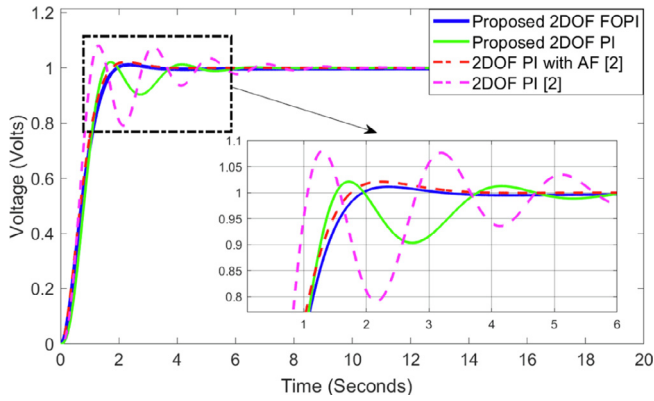


Fig. 8. Step Response Plot with zoom in overshoot area.

PI controller has better response when compared to the 2DOF PI with AF in [2].

The technique gives the better step response with less or almost no overshoot and faster settling time. When comparing the results in a wide picture, the proposed 2DOF FOPI performs better in many ways. From the measured time parameters, the output is about 86% less overshoot, 76% faster settling time, 54% lesser control signal variations and no oscillations. The maximum overshoot in percentage, peak value, peak time, settling time and rise time values are given in Table 4. Interestingly, the better step response is achieved with less control input signal variations. It shows the fractional 2DOF works well with less control efforts.

6.3. Bode analysis

Frequency response or bode plot analysis is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system. It is a measure of magnitude and phase of the output as a function of frequency, in comparison to the input. Therefore, the common parameters to check for the AVR system are the gain and phase margins. For that, the bode plot is simple to plot and to check the stability of the AVR system with controller. All four designed techniques' bode plots has been plotted. To note, for any stable plant the positive phase margin (PM) and gain margin (GM) are required and greater the margins, the greater the stability.

The PM and GM values for studied controllers are tabulated in Table 5. They indicate that 2DOF FOPI and 2DOF PI with AF are most stable than 2DOF PI. Thus, the frequency-domain analysis also agrees with the performances. For the reference, Fig. 9 illustrates the bode plots of the AVR system controlled by the four controllers examined. The key indicator is that AVR system controlled by 2DOF PIs have lower values of gain as shown in Fig. 9(b) and (d). It makes them the least stable plant when compared to others.

6.4. Disturbance analysis

The disturbance input analysis is key verification for any controller design. This analysis is based to assess how the closed-

Table 5
Bode plot values (GM and PM)

Controller	Gain Margin (dB)	Phase Margin (deg)
Proposed 2DOF FOPI	34.6 @ 1660 rad/s	67.8 @ 59.8 rad/s
Proposed 2DOF PI	20.3 @ 1.68 rad/s	67.5 @ 0.33 rad/s
2DOF PI with AF [2]	33.6 @ 2150 rad/s	59.9 @ 45.9 rad/s
2DOF PI [2]	20 @ 2.41 rad/s	51.1 @ 0.62 rad/s

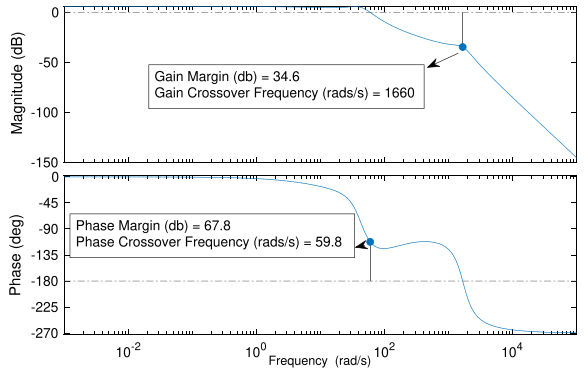
loop system behaves while unknown disturbance is observed. In AVR system it is widely possible for grid-based failures or manipulation such as load change or demand to occur. It is an essential robustness verification. In order to complete this task, a disturbance signal is chosen, denoted by $\Delta L(s)$. Let us assume the positive pulse of amplitude +0.5 pu and pulse width of 5% of the period as shown in Fig. 10. Similarly, considering another negative pulse of amplitude -0.5 pu and pulse width of 5% of the period as shown in Fig. 11. These disturbance signals are injected in between the exciter and synchronous generator at 10 s, individually. The output responses in Figs. 10 and 11 clearly reveals that the proposed 2DOF FOPI method performs well than others. The numerical values of the results has been tabulated in Tables 6 and 7. The new fractional structure of control in the AVR system has the best disturbance rejection by recovering to the original state with the fastest speed, approximately 8% lesser for positive disturbance and 44% lesser for negative disturbance when compared to 2DOF PI with AF [2]. Considering the other results, the proposed 2DOF FOPI performs the best with minimal overshoot and undershoot. Also to note that the proposed 2DOF FOPI does not cause any oscillation when the disturbance interrupts the system.

6.5. Parameter perturbation analysis

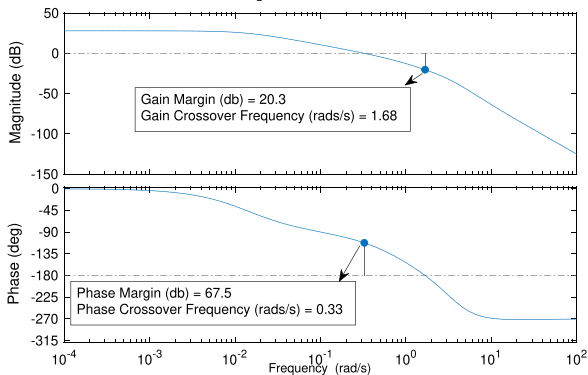
The parameter perturbation test closely relates to practical scenario, as perturbations may occur in the parameter(s) of the plant after initial setup. Even small changes in the parameter values can cause drastic power system failures. Taking that into consideration, the parameters of the amplifier, exciter, synchronous generator and voltage sensor are supposed changes in intervals of 25% from 0% to 100%. These changes are only done in the tolerable range of the nominal values of the parameters. Therefore, whereby the nominal value if is a maximum value from the range, the values taken is minus 25%. And if the nominal value is minimum, the values taken is plus 25%. Considered are four cases as shown in Table 8 for parameter variations. With the new values as per variations in the respective parameters, all four cases are simulated again by keeping the same controller settings. The overshoot and settling times are investigated mainly and compared case by case. Fig. 12 shows that the proposed controller displays the least overshoot compared to other controllers for all four cases. In cases, one, two, three and four the proposed 2DOF FOPI controlled AVR system has overshoot which is approximately 25%, 20%, 13% and 20% respectively lesser when compared to 2DOF PI with AF [2]. Thus, the overshoot value by 2DOF FOPI is almost four times lesser than the most overshoot value of other approach. Note that the PSO tuned 2DOF-PI with AF is performing well but the fractional scheme provides overall more robustness.

Table 4
The results of step response analysis

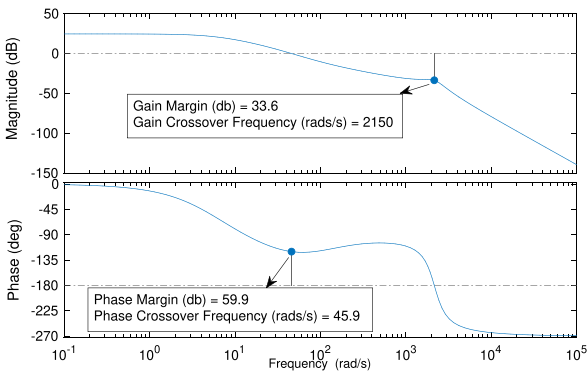
Methods	Max OS (%)	Peak Value (V)	Peak Time (s)	Settling Time (s)	Rise Time (s)	Variations in u
Proposed 2DOF FOPI	1.17	1.01	2.4	1.74	1.12	0.32
Proposed 2DOF PI	2.22	1.02	1.75	3.55	0.91	0.37
2DOF PI with AF [2]	8.15	1.08	2.23	7.32	0.64	0.91
2DOF PI [2]	2.22	1.02	3.30	3.44	0.69	0.69



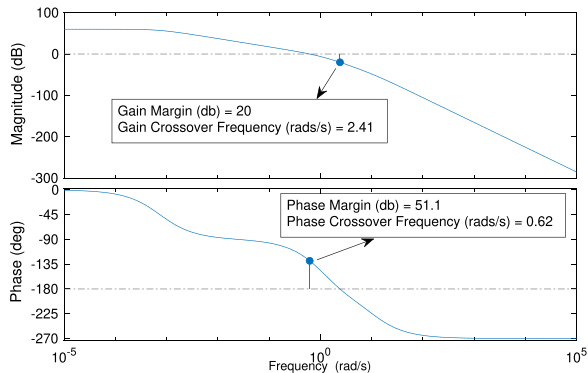
(a) Proposed 2DOF FOPI



(b) Proposed 2DOF PI



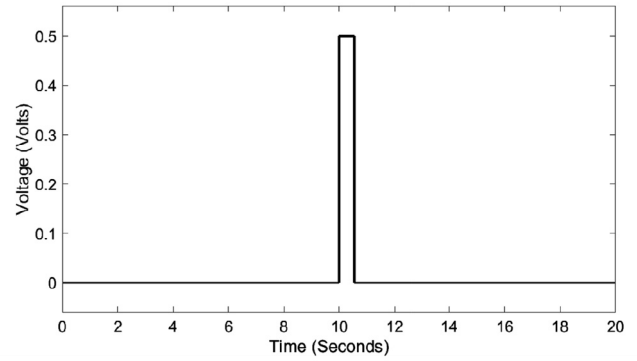
(c) 2DOF PI with AF[2]



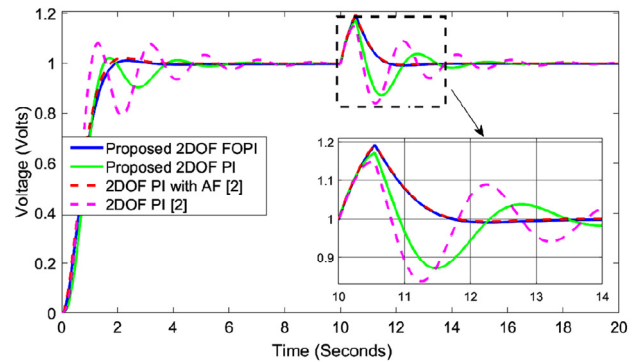
(d) 2DOF PI[2]

Fig. 9. Bode plots.

It can be learnt from Fig. 13 that the settling time from the proposed 2DOF FOPI is the least in all four perturbation cases. In perturbation cases, one, two, three and four the proposed 2DOF

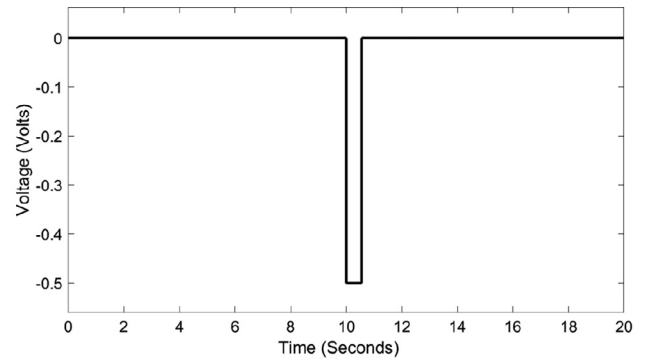


(a) Disturbance signal

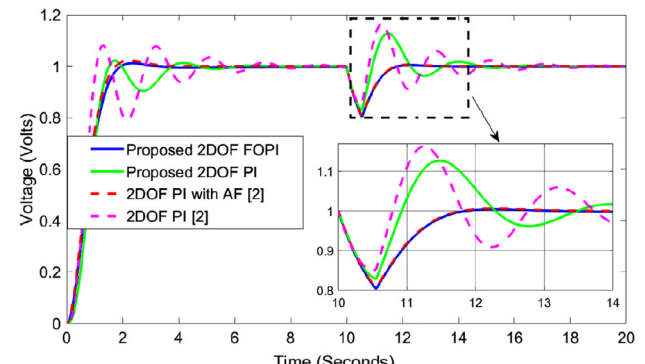


(b) After disturbance

Fig. 10. Positive disturbance analysis.



(a) Disturbance signal



(b) After disturbance

Fig. 11. Negative disturbance analysis.

Table 6
Results after positive disturbance

Methods	Peak Value (V)	Recovery Time(s)	Undershoot(V)
Proposed 2DOF FOPI	1.191	3.950	0.007
Proposed 2DOF PI	1.171	7.949	0.126
2DOF PI with AF [2]	1.193	4.294	0.006
2DOF PI [2]	1.148	9.456	0.609

Table 7
Results after negative disturbance.

Methods	Nadir Value (V)	Recovery Time(s)	Overshoot(V)
Proposed 2DOF FOPI	0.805	3.187	0.005
Proposed 2DOF PI	0.830	7.977	0.128
2DOF PI with AF [2]	0.807	5.725	0.006
2DOF PI [2]	0.852	9.456	0.163

Table 8
Various cases for parameter perturbations.

CaseNo.	T_a variance	T_e, T_g, T_s variances
Case 1	-25%	+25%
Case 2	-50%	+50%
Case 3	-75%	+75%
Case 4	Min	Max

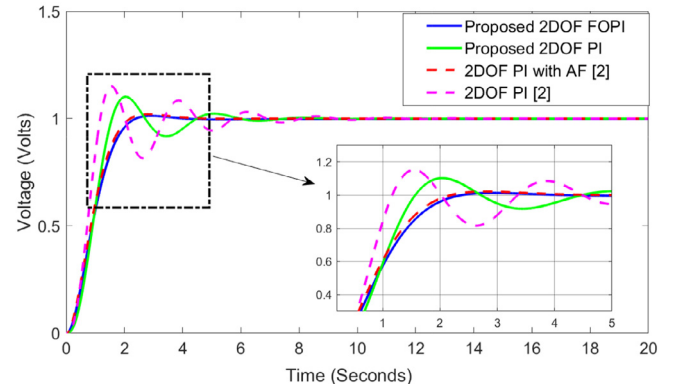


Fig. 14. Perturbed responses with Case 1.

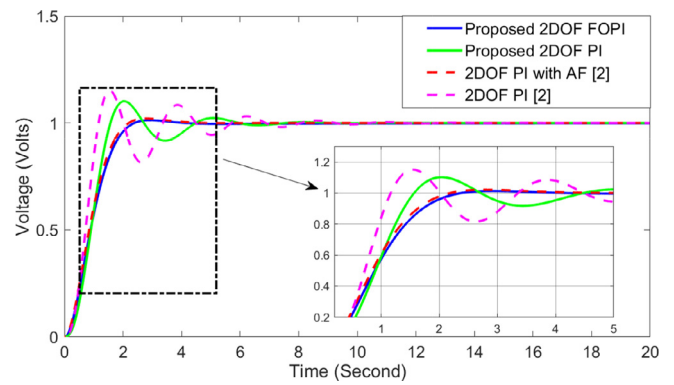


Fig. 15. Perturbed response with Case 2.

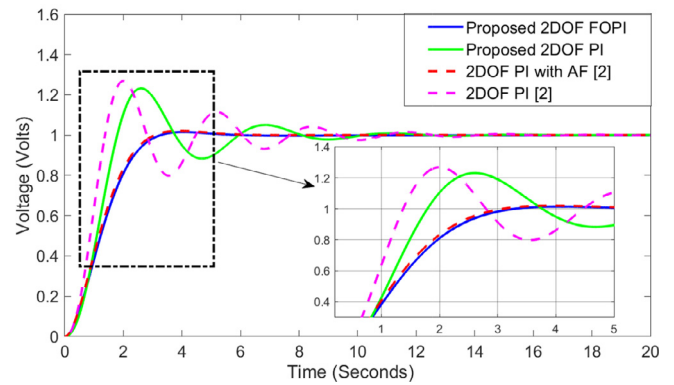


Fig. 16. Perturbed response with Case 3.



Fig. 12. Overshoot data in parameter perturbations.

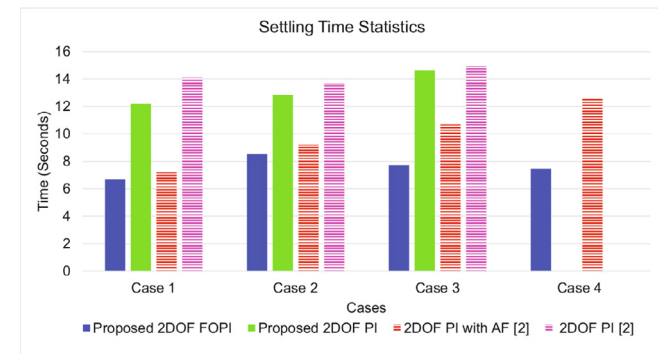


Fig. 13. Settling time data in parameter perturbations.

FOPI resulted the settling time which is approximately 7%, 7%, 27% and 40% respectively lesser when compared to 2DOF PI with AF [2]. Besides that the output performance with zoom in overshoot area can also be verified for various cases in

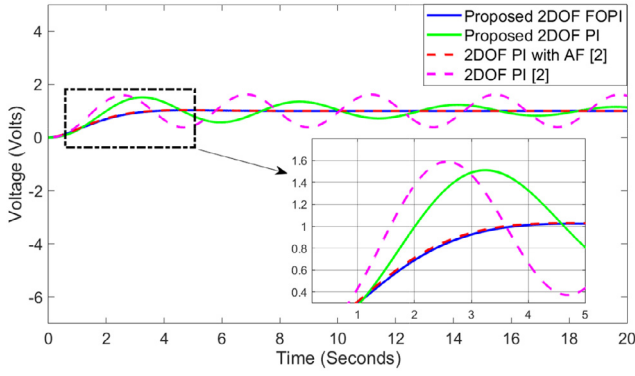


Fig. 17. Perturbed response with Case4.

see Figs. 14–17. They suggest that the new 2DOF FOPI controller has stronger robustness, as there is valid numerical data proof of better performances in the presence of maximum parameter variations.

7. Conclusions

The fractional 2DOF structure, namely 2DOF FOPI was designed for the AVR system. Since this is a very important component of synchronous generators, it is always required to have accurate and robust control in terms of power system quality and reliability. From the obtained results it was observed that the novel 2DOF FOPI was remarkably good towards load disturbances and parameter variations, to define its superiority in robustness. With the help of fractional-order theory, an extra degree of freedom in tuning is applied in the controller’s transfer function. The parameters of the controller are obtained using the dual performance index of integral of error and control signal variations. The WOA is adopted to check the performance of 2DOF FOPI by satisfying the performance criterion. With the analyses done, the proposed scheme proved better than the recent literature’s results. In particular, the proposed scheme resulted almost two times lesser overshoot and settling time. Moreover, its robustness tests showed that the technique performed better by 26% in average in disturbance rejection and in average of approximately 20% better with parameter of the system changed from its nominal values. Thus, the new technique has high immunity towards the disturbance and also during the uncertainty of system parameters. In future work, the approach can be verified for multi-area hybrid power systems where advanced fractional control schemes could be useful.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Definition of fractional-order operator

Fractional calculus is a generalization of integration and differentiation to the (fractional) noninteger order operator ${}_aD_t^\lambda$, where a and t denote the limits of the operation and λ ($\lambda \in R$) is the order of operation [48]. There exist numerous definitions to characterize fractional integration and differentiation in the literature. In our work, we use the R-L (Riemann–Liouville) definition which can be written as

$${}_aD_t^\lambda f(t) = \frac{1}{\Gamma(n - \lambda)} \left(\frac{d}{dt} \right)^n \int_a^t \frac{f(\tau)}{(t - \tau)^{\lambda+1-n}} d\tau \quad (9)$$

where $n - 1 < \lambda < n, n \in N$, and Γ denotes gamma function. Its operator is generally defined as ${}_aD_t^\lambda$. Here, when $\lambda > 0$ gives $\frac{d^\lambda}{dt^\lambda}$, $\lambda = 0$ gives 1 and $\lambda < 0$ gives $\int_a^t (d\tau)^{-\lambda}$. Generally, the FO derivative and integral are represented in the Laplace domain with zero initial as below.

$$L[{}_0D_t^\alpha f(t)] = s^\alpha F(s) \quad (10)$$

$$L[I_0^\alpha f(t)] = \frac{1}{s^\alpha} F(s) \quad (11)$$

where s^λ is a fractional Laplacian operator.

Appendix B. Whale optimization algorithm

The WOA was first suggested in 2016 [46] and is purely inspired from nature considering foraging behavior of humpback whales. The foraging behavior is basically the bubble netting technique of humpback whales. Due to the lack of speed of the whales, they cannot chase and devour schools of fish easily. This tempted them to innovate a technique to trap fish instead therefore coming up with the plan of creating bubbles to trap fish. This trapping technique works as one or many whales create bubbles in a spiral path around the school of fish. The spiral movement with continuous bubbles forces the school of fish towards the surface. The radius of the spiral decreases as they approach the surface. The whales eventually attack the school of fish when they are very close to the surface. This is key logic behind development of WOA. The core mechanisms simulated are encircling the prey, bubble net attack and searching for prey. The pseudo code of the algorithm is presented in Table 9.

Table 9
The pseudo code for WOA [46].

Initialize the whale’s population
Set algorithm parameters
Set performance index
Calculate the fitness of all search agent
While (termination criteria not satisfied)
Encircle prey
Bubble net hunt
Search the prey
Compute the fitness by index (8)
End While
Return the best result
End

Appendix C. Adapted design method

Illustration of the design method flow can be shown in Fig. 18.

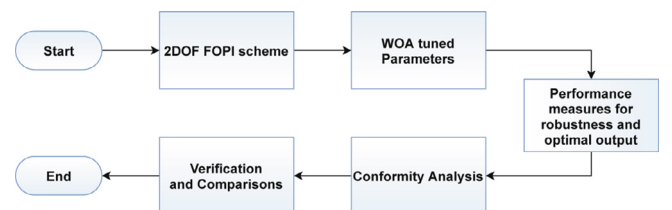


Fig. 18. Design Method Flow.

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