

Multimodal network routing problem using Multimodal Leg Outcome (MLO) table

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Abstract

Multimodal transportation is being increasingly adopted by a large number of freight companies. Multimodal transportation refers to using combinations of various transportation modes to move commodities from origin to destination. In this paper, a table is created to store the selected simulation results, which we shall refer to as a Multimodal Leg Outcome (MLO) table. This table is then utilized to find an optimal route in a multimodal network for given origin and destination. The model developed in this paper integrates total travel costs and total travel time of the routes subject to the availability of each mode for each leg, to generate the most preferred routes. Due to this model being multi-objective, where the desire is to minimise more than one objective function, we construct the MLO table using pareto optimal solutions.

In our approach, Monte Carlo simulations (MCS) are used to generate random feasible routes and the results are analysed to extract the pareto optimal solutions (pareto front) offering various routes with respective advantages in terms of duration and/or costs. These pareto optimal solutions are then used to develop a Multimodal Leg Outcome (MLO) table. The proposed approach is then applied to a simple situation and the results are presented. It is shown that the MLO table obtained can be used not only to find an optimal route in the respective multimodal network, given origin and destination, but also can be utilised after altering the availability of random vertices and/or modes of transportation in the original network.

1. Introduction

Multimodal transportation is being increasingly used to transport goods from origin to destination by local and global freight companies. It involves using combinations of various transportation modes (rail, road, water and air) to move commodities from origin to destination. Multimodal transportation potentially could improve efficiency, costs, safety and flexibility for the transportation industry as compared to the ordinary single mode transportation.

Identifying the best means of transporting goods from shippers to receivers is a common problem for freight forwarders. Trucks are commonly used in many regions since they have shorter travel times. However, other transport modes such as rail, air and sea can have a range of different benefits such as reduced operating costs, fuel consumption and emissions as well as greater levels of safety but they may have higher travel times and lower levels of reliability.

One of the operational problems that need to be solved in the multimodal setting is the freight routing problem. That is, to find and/or select the best route(s) in terms of time and/or

costs etc, while using the best available mode to move commodities from their origins to destinations through the transportation network. However due to the nature of this problem, a multi-objective approach needs to be taken and hence no one solution may exist that simultaneously optimises each objective function which in this case would be, at the very least, the cost function and the time function.

This paper tackles the multimodal freight routing problem by creating a table to store the selected simulation results, which we shall refer to as a Multimodal Leg Outcome (MLO) table. The Multimodal Leg Outcome (MLO) table is created from the pareto optimal solutions which are obtained by using Monte Carlo simulations (MCS) to generate random feasible routes.

The model developed in this paper integrates the availability of each mode for each leg to calculate total costs of transportation, including transfer costs from one mode of transportation to another, and the duration of trip, to generate an optimal route given the origin and destination.

The proposed approach is then applied to a simple situation and results presented. Also demonstrated is the ability to use the generated MLO table to generate an optimal route, even after altering the original network by changing the availability of random vertices and/or modes. This approach saves time and money by not having to regenerate routes all over again if the initially given modes and vertices are not available after simulation. We can still get the next best solution using the MLO table.

2. Literature review

Multimodal transport involves at least two modes being used to transport goods (Stadieseifi *et al.*, 2014). When considering alternatives to trucks, numerous options regarding the choice of terminals where other transport options are available must be considered. This requires routes to be designed in a way that involves an examination of a range of possible transport modes and terminal options. This can be challenging due to the number of alternatives available.

Typically, a single objective such as minimising travel time is considered when only one mode such as a truck is available. However, other objectives such as minimising financial costs are often considered when there are other modes available. Generally, different transport modes will have different operating costs as well as different journey times between terminals that will be conflicting (Sun and Lang, 2015).

Multi-objective optimisation approaches based on mathematical programming and network analysis can be applied to determine the best and most practical solutions for multimodal transport problems. Possible solutions consisting of routes comprising of paths using available transport modes that can be used to carry goods between terminals linking shippers and receivers need to be generated.

Multi-modal transport problems can be formulated by combining all objectives into one objective function, with each individual objective weighted according to its relative importance. However, it is challenging to determine the weighting of individual objectives and only a single solution is provided as an answer, where in practice a set of good options is generally desirable.

Multi-objective optimisation techniques can be employed to generate solutions that are better than all other feasible solutions considering multiple objectives without the need to specify the relative importance of each objective. Pareto optimal solutions are options that are not dominated by any other feasible solution where a better value for any objective is possible only with a poorer value of at least one other objective.

Genetic algorithms have been widely adopted as a solution method for generating pareto optimal solutions for Multi-modal transport problems, including the Vector Evaluated Genetic

Algorithm (Schaffer, 1985), the Non-dominated Sorting Genetic Algorithm (Srinivas and Deb, 1994), the NSGA-II (Deb *et al.*, 2002), the Normalized Normal Constraint Method (Messac, Ismail-Yahaya and Mattson, 2003) and the bi-level multi-objective Taguchi genetic algorithm (Xiong and Wang, 2014).

To the best of the authors' knowledge, there is no previously published work utilising the proposed approach of saving and applying the MCS simulation results of the extracted Pareto optimal solutions, for multimodal network routing problems.

3. Monte Carlo simulation (MCS)

Monte Carlo simulation involves building models of possible results by substituting all of the input values having inherent uncertainties, with probability distributions. It then calculates results repeatedly, each time using a different set of random values from the probability distributions. The results of Monte Carlo simulation are not single values but distributions of possible outcome values (Vose, 2008).

Generally, the following steps are involved in performing an MCS:

- Step 1: Create one (or more) parametric model(s),
 $F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})]$, where p is the number of objective functions and input $\mathbf{x} = (x_1, x_2, \dots, x_m)$, where m is the number of possible inputs.
- Step 2: Represent the inputs (x_1, x_2, \dots, x_m) using probability distributions.
- Step 3: Generate a set of random inputs $(x_{k1}, x_{k2}, \dots, x_{km})$ from the distributions for each iteration k , $k = 1$ to t .
- Step 4: Evaluate the model using the random inputs, $F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})]$ for each iteration, k .
- Step 5: Analyse the results of $F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})]$, obtained for all the iterations, $k = 1$ to t .

4. Pareto optimality

The origin of the term Pareto optimality goes back to the year 1906 applied in the area of economics and later on found its application in the field of Mathematics especially multi-objective optimisation (Marler and Arora, 2004). Multi-objective optimization problems (MOPs) are a branch of mathematical optimisation which involves having to optimize more than one objective function simultaneously. Usually, MOPs don't have a single optimal solution that optimises each objective function in which case Pareto optimal solutions may be used to represent the solution set. Pareto optimal solutions are solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives. The set of Pareto optimal outcomes is often called the Pareto front or Pareto boundary. Solutions which do not lay on the Pareto front are called Pareto dominated solutions. See Fig. 1.

Let $X \subset \mathbb{R}^n$ be a non-empty set of feasible solutions and $F = [F_1, F_2, \dots, F_p]: \mathbb{R}^n \rightarrow \mathbb{R}^p$ be a set of objective functions. Feasible solution, $\hat{\mathbf{x}} \in X$ is called a Pareto optimal solution of the

$$\text{MOP: } \min_{\mathbf{x} \in X} F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})],$$

if and only if there does not exist any $\mathbf{x} \in X$ such that $F(\mathbf{x}) \leq F(\hat{\mathbf{x}})$.

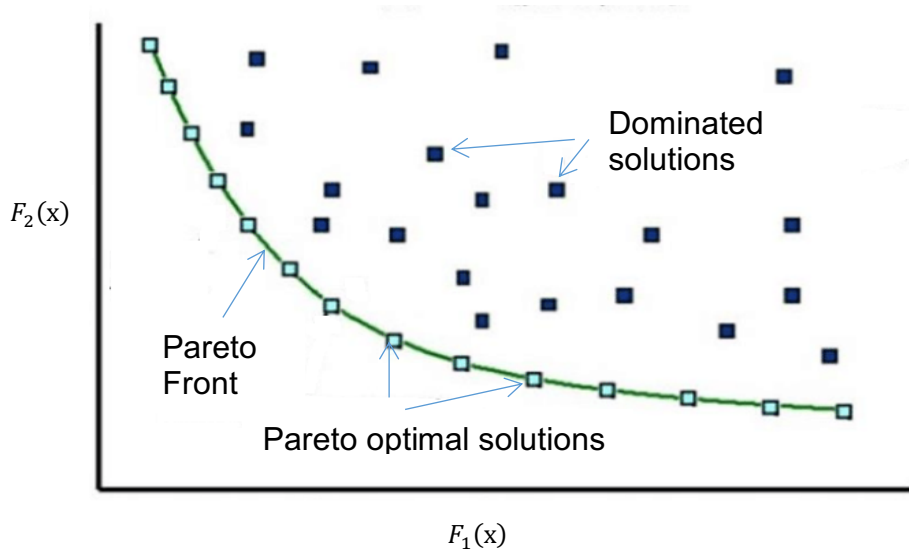


Fig 1: Pareto Front.

5. Multimodal freight route selection via Multimodal Leg Outcome (MLO) table

In this section, we present our approach to solving the multimodal freight route selection problem by creating and utilising Multimodal Leg Outcome (MLO) tables. We populate the MLO after analysing the results of the MCS and extracting the pareto optimal solutions. Note that the MLO table is based on the pareto optimal solutions and not on all the feasible solutions obtained.

The multimodal freight route selection problem involves choosing a combination of various routes from a selected origin to a particular destination, taking into consideration, the effects of implementing these combinations, including but not limited to travel costs and time taken, based on the availability of routes and modal options for each possible leg of transportation.

Step 1: Formulating the MOP:

Let $G = (V, E, M)$ denote the multimodal transportation network with the set of V vertices, a set of E edges and a set of M transportation modes. Let each edge, $e \in E$, connecting two vertices u and v , be denoted by (u, v) and the associated weight of the edge be denoted by $\delta(u, v)$.

Let $\phi(u^i, v^j)$ be the transfer cost from vertex u to vertex v from mode i to j , $i, j \in \{1: road, 2: rail, 3: water\}$, where i is the mode arriving at u and j is the mode arriving at v and hence the mode of transportation of (u, v) . Note if $i = j$, then $\phi(u^i, v^j) = 0$, i.e. there is no cost if no modal transfer occurred. Additionally, $\phi(u^i, v^j) = 0$, if u is the starting vertex.

Definitions

- u is adjacent to (or is a neighbour of) v , if $(u, v) \in E$.
- The set of all neighbours of u is the neighbourhood of u and is denoted $N(u)$.

- A path is defined as an ordered set of vertices $(v_1, \dots, v_t), t > 1$ where t is an integer, such that $(v_h, v_{h+1}) \in E$ for $h = 1, \dots, t - 1$, and the vertices are not repeated.

Step 2: Deciding on using some strategy to act and then act accordingly

We will use MCS, as described in Section 3, to randomly choose neighboring towns and modes to travel from origin until the destination town is reached. Note that a variant of this method called Monte Carlo Tree Search (MCTS) can also be used.

- I. Construct parametric model: $F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})]$, where:
 - $[F_1, F_2, \dots, F_p]$ are various applicable objective functions such as criteria, payoff functions, cost functions, time and value functions; and
 - $\mathbf{x} = (A_0, A_2^m, A_3^m, \dots, A_D^m)$ are the towns to be traversed to reach destination, A_D from the origin, A_0 ; $m \in \{1: road, 2: rail, 3: water, 4: air\}$ denotes the mode of transportation used to arrive at the vertex.
- II. For each iteration ($k = 1$ to t), starting with the town of origin, A_0 , randomly select the next town to be traversed from range of neighboring towns (A_e, \dots, A_f) represented using probability distribution. These input distributions can be derived from the adjacency matrix for the network. After the next town is selected, also randomly select an available mode from the list of available modes of transportation to this town from the preceding town. If this selected town is the desired destination town, A_D , then stop otherwise select the next town visited from respective range of neighboring towns represented by appropriate probability distribution until desired destination town, A_D is reached. If the next town does not exist, that is, the current town has no neighbor, then stop and restart the next iteration. The result of this step would generate, $\mathbf{x}^k = (A_0, A_2^m, \dots, A_D^m)^k$, for each iteration k .
- III. Evaluate the model, $F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})]$, using the random inputs of \mathbf{x} , generated in step 2, for each iteration, k

Step 3: Populate the Multimodal Leg Outcome (MLO) table

- IV. Analyse the results of $\mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_p(\mathbf{x})]$, obtained for all the iterations, $k = 1$ to t using the Pareto optimal approach. Section 5 describes Pareto optimality and the principle behind it.
- V. Extract the Pareto optimal solutions (i.e. Pareto front) of traversed towns denoted by $\mathbf{OF}(\dot{\mathbf{x}}) = [\dot{\mathbf{x}}_1, \dot{\mathbf{x}}_2, \dots, \dot{\mathbf{x}}_g, \dots, \dot{\mathbf{x}}_q]$, for $g = 1$ to q unique solutions and $\dot{\mathbf{x}}_g = (A_0, \dots, A_D)^g$.
- VI. For each unique extracted solution of $\mathbf{OF}(\dot{\mathbf{x}})$, regenerate random modes, $m \in \{1: road, 2: rail, 3: water, 4: air\}$ of traversal to obtain $\ddot{\mathbf{x}}^l = (A_0, A_2^m, \dots, A_D)^l$ for each iteration $l = 1$ to r . This step can be classed as a refinement step to ensure that the $\dot{\mathbf{x}}_g = (A_0, \dots, A_D)^g$ obtained in step V has sufficiently traversed the optimal combination of towns via the available modes thus improving our results.
- VII. Analyse the refined results, $\mathbf{F}(\ddot{\mathbf{x}}) = [F_1(\ddot{\mathbf{x}}), F_2(\ddot{\mathbf{x}}), \dots, F_p(\ddot{\mathbf{x}})]$, obtained for all the iterations, $l = 1$ to r , using the Pareto optimal front approach.

VIII. Create an empty MLO Table (3-Dimensional), [# of vertices by # of vertices by # of modes].

IX. Populate the MLO Table according to the Pareto optimal solutions obtained in step VII. The pseudocode for the MLO Table approach is provided as appendix 1.1:

6. Applied model

In this section, an example adapted from (Xiong and Wang, 2014) is presented to demonstrate MLO Table approach for best freight route selection in a multimodal network.

In this example there are 35 vertices and 136 edges as shown in Fig. 2 and the task is to go from vertex 1 to vertex 35. There are at most 3 possible modes of transportation which corresponds to A (road), B (rail) and C (water) between any vertices (Note, for the purposes of numerical table input we also refer to Road, Rail and Water as modes 1, 2 and 3 respectively). Appendix 1.2 provides the edge weights which are transportation distances between vertices for various modes if available. A dash (-) denotes that there is no connection between the associated vertices for the respective mode. Table 1 provides the speed and costs per unit load for each transportation mode while Tables 2 and 3 give the transfer costs per unit load and time per unit load for each change of mode.

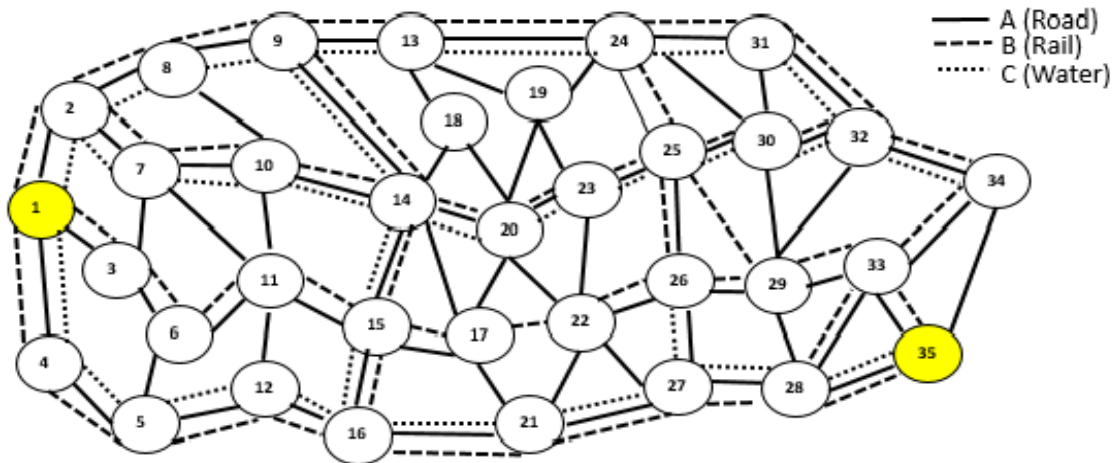


Fig.2 Example multimodal transport network (adapted from (Xiong and Wang, 2014)).

Table 1: Speed (unit distance per unit time) and Costs (\$ per unit load) with respect to transportation modes

	Road	Rail	Water
Speed	4.5	3	1
Costs	6	3	1

Table 2: Transfer cost rates (\$ per unit load) when changing transportation modes

	Road	Rail	Water
Road	0	4	7
Rail	4	0	7
Water	7	7	0

Table 3: Transfer time rate (unit time per unit load) when changing transportation modes

	Road	Rail	Water
Road	0	0.0067	0.0113
Rail	0.0067	0	0.0113
Water	0.0113	0.0113	0

The task at hand is to find the best route(s) to from town/vertex 1 to town/vertex 35 carrying $\mu = 20$ units, utilising available modes such that the objective functions $\mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x})]$, are minimized, where:

$$\text{Cost function: } F_1(\mathbf{x}) = \mu * \sum_{h=1}^{t-1} [\delta(A_h^i, A_{h+1}^j) * \alpha^j + \emptyset(A_h^i, A_{h+1}^j)];$$

and

$$\text{Time function: } F_2(\mathbf{x}) = \sum_{h=1}^{t-1} [\delta(A_h^i, A_{h+1}^j) / \vartheta^j + \mu * \theta(A_h^i, A_{h+1}^j)];$$

and:

- $\mathbf{x} = (A_0, A_2^m, \dots, A_D^m)$ are the towns to be traversed to reach destination, $A_D = 35$ from the origin, $A_0 = 1$; and $m \in \{1: \text{road}, 2: \text{rail}, 3: \text{water}\}$
- $t > 0$ is number of traversed town and $A_1 = 1$ and $A_t = 35$;
- $i, j = \{1: \text{road}, 2: \text{rail}, 3: \text{water}\}$ are the modes of transportation used arriving at respective vertices;
- $\delta(u^i, v^j)$ is the distance of two towns u and v traversed using mode j ;
- $\emptyset(u^i, v^j) = \begin{cases} 0; & i = j \\ 4; & i = 1 \text{ and } j = 2 \text{ or } i = 2 \text{ and } j = 1 \\ 7; & i = 1 \text{ and } j = 3 \text{ or } i = 2 \text{ and } j = 3 \end{cases}$ (Transfer rate of cost as provided in Table 2)
- $\theta(u^i, v^j) = \begin{cases} 0; & i = j \\ 0.0067; & i = 1 \text{ and } j = 2 \text{ or } i = 2 \text{ and } j = 1; \\ 0.0113; & i = 1 \text{ and } j = 3 \text{ or } i = 2 \text{ and } j = 3 \end{cases}$ (Transfer rate of time as provided in Table 3)
- $\vartheta^j = \begin{cases} 4.5; & j = 1 \\ 3; & j = 2 \\ 1; & j = 3 \end{cases}$ (as provided in Table 1).

- $\alpha^j = \begin{cases} 6; & j = 1 \\ 3; & j = 2 \\ 1; & j = 3 \end{cases}$ (as provided in Table 1).

7. Results and Discussion

The Pareto optimal solution (pareto front) routes obtained after step 3: parts IV-V, as described in section 5, are:

- 1-4-5-12-16-21-27-28-35
- 1-4-5-12-16-21-17-22-26-29-33-35
- 1-2-7-11-15-17-22-26-29-33-35
- 1-4-5-12-16-21-22-27-28-35

These routes were then used to run step 3: parts VI-VII, as described in section 5, where now only the modes are randomly chosen for these fixed routes. The Pareto optimal solutions of these results are provided in Appendix 1.3 and shown in Fig 3. The red straight lines in Fig. 3 contain the pareto front which represent all the pareto optimal solutions as described in Section 5.

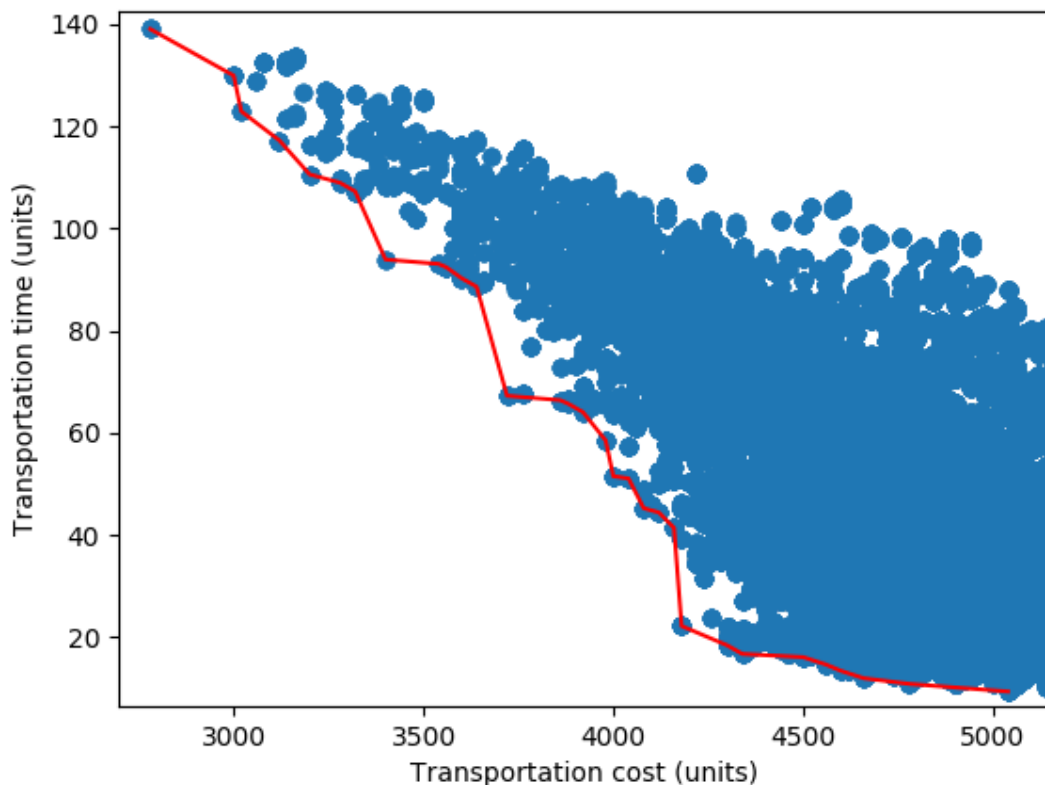


Fig.3 Pareto optimal solutions (pareto front).

After obtaining the pareto optimal results, the MLO table is created with dimensions of (35 x 35 x 3) representing the vertices and the available modes. This MLO table is then populated according to the results of all the solutions on the pareto front as shown below:

$$\text{New } Q(u, v, m) = Q(u, v, m) + R;$$

where: $Q(u, v, m)$ represents a path (u, v) traversed using mode $m = \{1: \text{road}, 2: \text{rail}, 3: \text{water}\}$ and R being the reward if it is traversed.

For instance, for solution #6 in Appendix 1.3, rewards would be awarded as follows assuming a standard reward, R of 1:

For

#	Route	Modes
6	1-4-5-12-16-21-27-28-35	B-B-A-A-C-C-C-C

$$\text{New } Q(1,4,2) = Q(1,4,2) + 1$$

$$\text{New } Q(4,5,2) = Q(4,5,2) + 1$$

$$\text{New } Q(5,12,1) = Q(5,12,1) + 1; \text{ and so on.}$$

Now that MLO table is created, the last step would be to use the MLO table to generate an optimal path from town 1 to town 35.

Hence starting from vertex (town) 1, using the MLO table to decide on the next town to traverse and via which mode. Firstly, the neighboring vertex which has the highest reward is chosen and then the mode that has the highest reward for that vertex is chosen as the transportation mode.

For instance, the best action from vertex 1, as provided by the MLO table is to go to vertex 4 via rail. This is because MLO table entry appearing for vertex 1 is:

$$Q(1,4,1) = 12 - [\text{implying from vertex 1 to 4 via mode 1 (road)}]$$

$$Q(1,4,2) = 15 - [\text{implying from vertex 1 to 4 via mode 1 (rail)}]$$

$$Q(1,4,3) = 4 - [\text{implying from vertex 1 to 4 via mode 1 (water)}].$$

Similarly, the route from vertex 1 is mapped using the MLO table until the destination vertex (#35) is reached. The resulting path provided is provided in Table 4.

Table 4: Solution from MLO table using the pareto optimal solutions

Route	Modes	Cost	Time
1-4-5-12-16-21-27-28-35	B-B-A-A-A-C-C-C	3600	90.25

Note that this solution is actually one of the pareto optimal solution (#11) provided in Appendix 1.3 hence the obtained solution cannot be improved for one objective without degrading the other. The path of the solution via MLO table is given in Fig 4. This solution is also presented while superimposed onto the pareto optimal solutions in Fig 5.

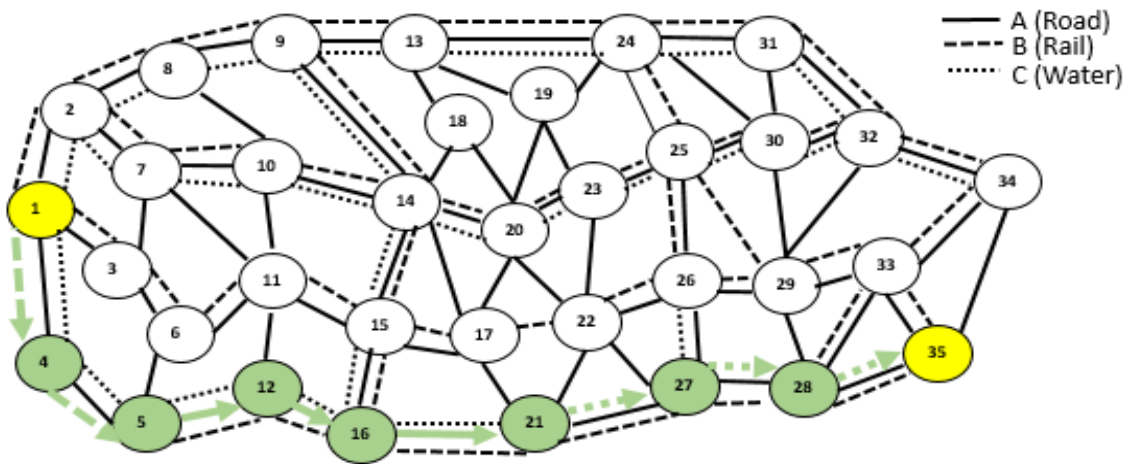


Fig.4 Generated path using MLO.

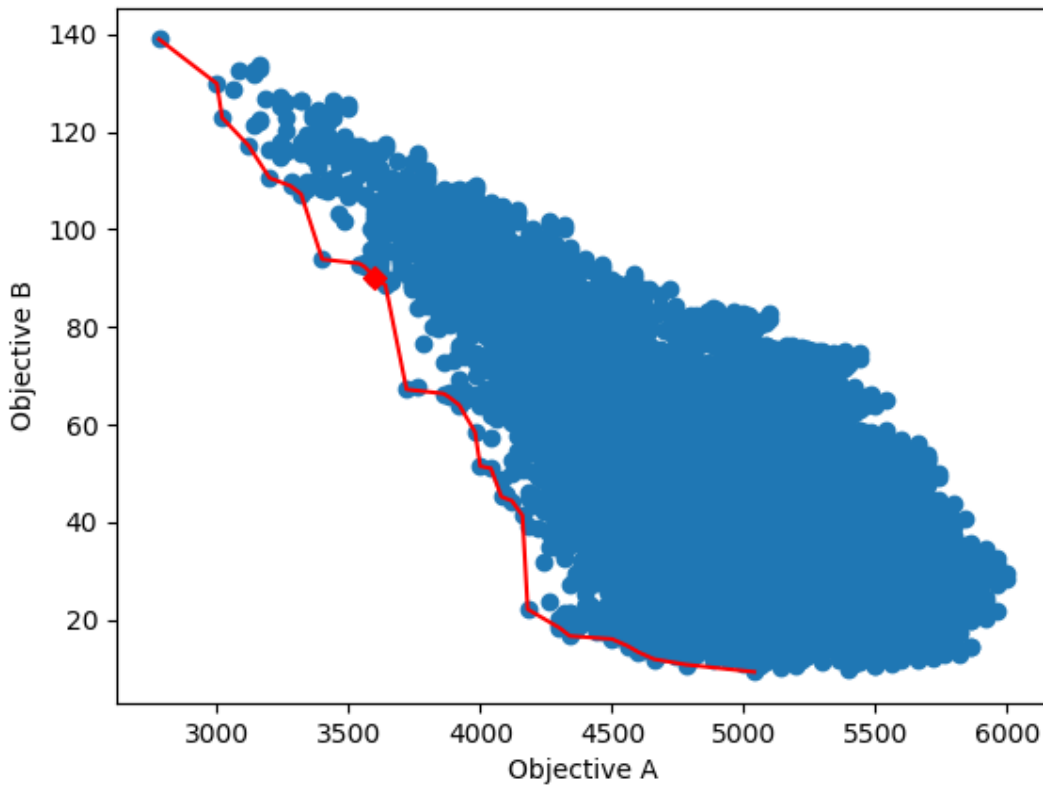


Fig 5. Solution from MLO table superimposed (shown as red diamond) on the extracted pareto optimal solution.

In reality, the transportation problem could be affected at any time because either the vertex is not available for traversing, or a particular mode is not available for transportation to/from that vertex. We carry out the test as follows to test the MLO table in the event that the original network is varied.

Assume that from original network (Fig 2), the water route from vertex 21 to 27 is not available. This implies that our generated solution in Table 4 and Fig 4 cannot be used since it traverses vertex/town 27. As such, we update our MLO table to $Q(21,27,3) = 0$, indicating that the reward from vertex 21 to 27 via water is zero. Restarting the process, to find the optimal route of transportation with this updated MLO table, provides us with an alternative route as provided in Table 5 and shown in Fig 6.

Table 5: Alternate solution from using the MLO table after varying the original network.

Route	Modes	Cost	Time
1-4-5-12-16-21-17-22-26-29-33-35	B-B-A-A-A-B-B-B-B- B	4300	18.38

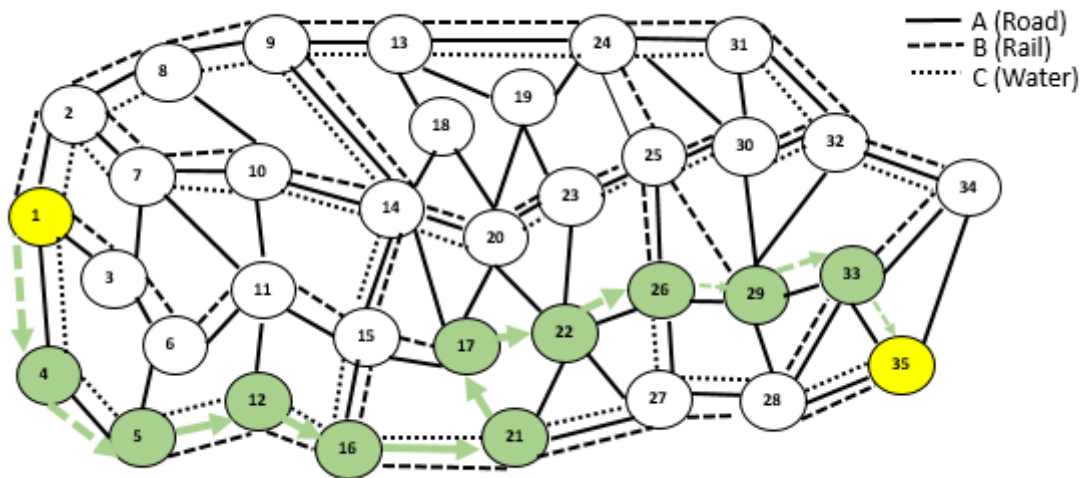


Fig.6 Generated alternate path using MLO.

The alternate path generated after MLO table modification is presented in Fig 6. As shown in Fig 7, this solution is also one of the extracted Pareto optimal solutions hence proving that the resulting solution after adjusting the MLO table due to changes in the original network is just as good as the original solution obtained in Table 4 since it is one of the Pareto optimal solutions.

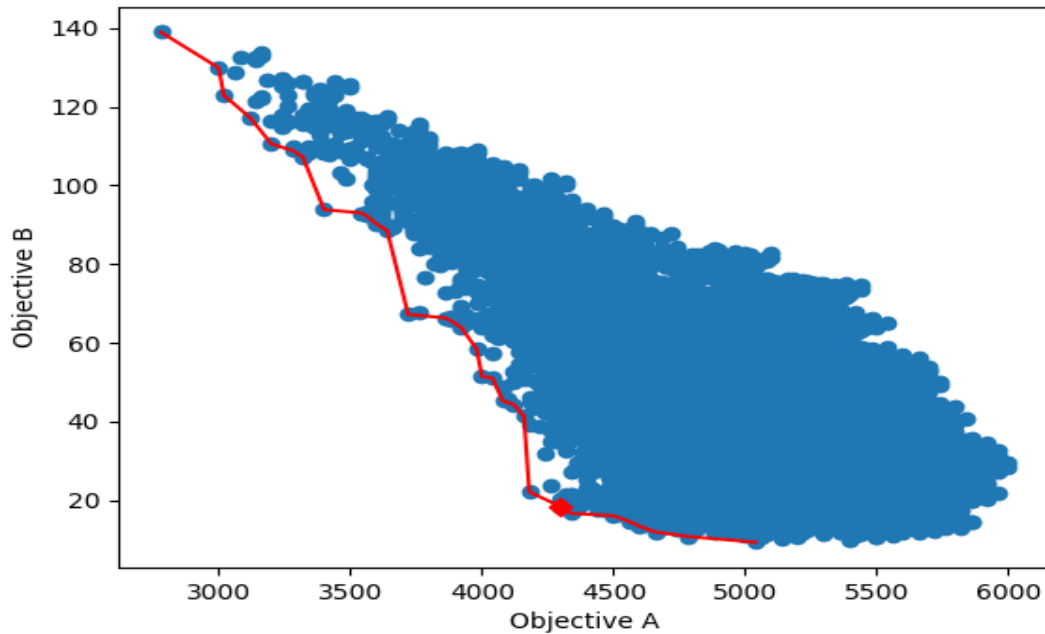


Fig 7. Alternate solution after adjusting the MLO table due to changes in the original network superimposed (shown as red diamond) on the extracted pareto optimal solutions.

8. Conclusion

In this paper, we solved the multimodal freight route selection problem by creating and utilising a Multimodal Leg Outcome (MLO) table. We used Monte Carlo simulation method to obtain many possible routes using the available modes of transportation and then extracted the pareto optimal solutions from these obtained routes. With the pareto optimal solutions in hand, a MLO table was constructed using these values which can be used to generate an optimal route for the given network. We applied our approach to an example problem and presented the results. We also demonstrated that the MLO table obtained can be used not only to find an optimal route in the respective multimodal network, given origin and destination, but is also applicable after altering the availability of random vertices and/or modes of transportation in the original network. This approach would enable users to use the created MLO table even if some of the nodes and/or modes are not available after the simulation, hence there is no need to redo the simulation.

9. References

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10. Appendix

Appendix 1.1: Pseudocode for MLO table approach

MCS phase

```

import network G
construct adjacency matrix, distance matrix and mode matrix
origin town_O
destination town_D
total_iterations_phase1
total_iterations_phase2
src = town_O
path = src
mode = []
MCS_phase1_cost = []
MCS_phase1_time = []
MCS_phase2_cost = []
MCS_phase2_time = []
for (counter =1 to total_iterations_phase1)
while (src is not town_D)
next_town = randomly choose a neighbour of src using adjacency matrix
while loop created
next_town = randomly choose another neighbour of src
if next_town = null
                break;
                next_mode = randomly choose a mode from SRC to next_town using mode matrix
path = array ( path + next_town)
                mode = array( mode + next_mode)
                src = next_town
MCS_phase1_cost = array (MCS_phase1_cost + F1(path, mode))
MCS_phase1_time = array (MCS_phase1_time + F2 (path, mode))
Pareto optimal solution extraction and rewarding
Pareto_optimal_front1 = extract Pareto optimal front solutions ( MCS_phase1_cost, MCS_phase1_time)
for each solution PO1 in Pareto_optimal_front1
                for (counter =1 to total_iterations_phase2)
                        mode = randomly select a mode for each connecting towns in PO1
MCS_phase2_cost = array (MCS_phase2_cost + F1(PO1, mode))
MCS_phase2_time = array (MCS_phase2_time + F2 (PO1, mode))
Pareto_optimal_front2 = extract Pareto optimal front solutions (MCS_phase2_cost, MCS_phase2_time)
Create a MLO table and use Pareto_optimal_front2 solutions to populate
Use the MLO table to generate an optimal route for the multimodal freight route selection problem.
Finish

```

Appendix 1.2: Vertex distances (units) with respect to available modes

Modes				Modes			
Edge	A	B	C	Edge	A	B	C
(1,2)	2	3	8	(18,20)	1	–	–
(1,3)	10	16	–	(19,20)	4	–	–
(1,4)	3	5	11	(19,23)	3	–	–
(2,7)	3	5	9	(19,24)	9	–	–
(2,8)	9	15	29	(20,22)	3	–	–
(3,6)	7	12	–	(20,23)	8	13	24
(3,7)	1	–	–	(21,22)	10	–	–
(4,5)	2	3	8	(21,27)	10	16	32
(5,6)	9	–	–	(22,23)	8	–	–
(5,12)	2	4	7	(22,26)	4	6	–
(6,11)	3	5	–	(22,27)	5	–	–
(7,10)	7	11	21	(23,25)	2	5	8
(7,11)	3	–	–	(24,25)	6	11	–
(8,9)	5	9	16	(24,30)	3	–	–
(8,10)	1	–	–	(24,31)	10	15	31
(9,13)	10	16	31	(25,26)	7	12	–
(9,14)	6	10	19	(25,29)	–	6	–
(10,11)	4	–	–	(25,30)	7	12	22
(10,14)	5	9	16	(26,27)	3	–	9
(11,12)	3	–	–	(26,29)	4	7	–
(11,15)	4	8	–	(27,28)	9	15	27
(12,16)	2	4	8	(28,29)	7	–	–
(13,18)	6	–	–	(28,33)	2	3	–
(13,19)	8	–	–	(28,35)	8	14	26
(13,24)	5	8	16	(29,30)	6	–	–
(14,15)	6	11	19	(29,32)	1	–	–
(14,17)	2	–	–	(29,33)	2	4	–
(14,18)	4	–	–	(30,31)	6	–	–
(14,20)	8	12	25	(30,32)	6	10	–
(15,16)	7	11	22	(31,32)	4	6	13
(15,17)	5	8	–	(32,34)	6	9	19
(16,21)	6	10	20	(33,34)	5	8	–
(17,20)	8	–	–	(33,35)	8	13	–
(17,21)	1	–	–	(34,35)	6	–	–
(17,22)	–	9	–				

Appendix 1.3: Pareto optimal front solutions

#	Route	Modes	Cost	Time
1	1-4-5-12-16-21-27-28-35	C-C-C-C-C-C-C-C	2780	139.00
2		B-C-C-C-C-C-C-C	3000	129.89
3		B-B-C-C-C-C-C-C	3020	122.89
4		B-B-B-C-C-C-C-C	3120	117.23
5		B-B-B-B-C-C-C-C	3200	110.56
6		B-B-A-A-C-C-C-C	3280	108.92
7		A-A-A-A-C-C-C-C	3320	107.23
8		B-B-B-B-B-C-C-C	3400	93.89
9		A-B-B-B-B-C-C-C	3540	93.03
10		B-B-A-A-B-C-C-C	3560	92.38
11		B-B-A-A-A-C-C-C	3600	90.25
12		A-A-A-A-A-C-C-C	3640	88.56
13		B-B-B-B-B-B-C-C	3720	67.23
14		A-B-B-B-B-B-C-C	3860	66.36
15		B-B-A-A-B-B-C-C	3880	65.72
16		A-A-A-A-B-B-C-C	3920	64.03
17		B-B-B-B-B-B-B-C	4080	45.23
18		A-A-A-A-A-A-A-A	5040	9.33
19	1-4-5-12-16-21-17-22-26-29-33-35	B-C-C-C-C-A-B-B-B-B-B	3980	58.47
20		B-B-C-C-C-A-B-B-B-B-B	4000	51.47
21		C-C-C-C-B-A-B-B-B-B-B	4040	51.05
22		C-C-C-B-B-A-B-B-B-B-B	4120	44.38
23		C-C-C-A-A-A-B-B-B-B-B	4160	41.36
24		B-B-B-B-B-A-B-B-B-B-B	4180	22.16
25		B-B-A-A-A-A-B-B-B-B-B	4300	18.38
26		A-A-A-A-A-A-B-B-B-B-B	4340	16.69
27		A-A-A-A-A-A-B-B-B-A-B	4500	16.07
28		A-A-A-A-A-A-B-B-A-A-B	4560	14.62
29		A-A-A-A-A-A-B-B-B-A-A	4600	13.38
30		A-A-A-A-A-A-B-B-A-A-A	4660	11.93
31		A-A-A-A-A-A-B-A-A-A-A	4780	10.82