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A comprehensive review of wireless power transfer systems: Focused study with fractional-order elements

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Abstract

Wireless power transfer (WPT) refers to promising technology that allows energy transfer between devices without wires or cables. Although wireless power transfer systems have come a long way, they still have a long way to go before they become truly viable. An approach that has been offered to address these issues is the implementation of fractional-order elements in WPT systems. Fractional-order wireless power transfer systems have emerged as a promising solution to address some challenges in classical approaches. This review aims to provide a valuable resource for researchers and engineers interested in designing and optimizing WPT systems using fractional methods by summarizing the current knowledge on fractional-based approaches in WPT. Furthermore, this article sheds light on the benefits, challenges, and prospects of integrating fractional elements into WPT strategies.

KEYWORDS

energy efficiency, fractional order elements, modelling, power, wireless power transfer

1 | INTRODUCTION

Nikola Tesla envisioned a future in which power (along with other signals) would be transmitted wirelessly using resonance phenomena. With the invention of the Tesla coil, undoubtedly his most renowned innovation, he came close to realizing this ambition, as it was the first system capable of delivering energy wirelessly. As the Tesla coil posed some hazards to human health and other electrical equipment, the project was forced to stop. As a result, the Tesla coil has few practical applications; however, its concept and principles have been incorporated into small gadgets such as cell phones, smartwatches, and electric toothbrushes to allow charging without physical connections between the source and the load. The concept of transmitting power from a source to a load without requiring a physical connection between the two is intriguing, especially given the enormous number of portable devices that require regular charging. Furthermore, this contactless charging solution is more reliable as it avoids the intrusion of dust and moisture and is a more hygienic solution for medical appliances.[1] Therefore, significant research is being conducted on WPT today.

WPT can be categorized into two subcategories: far-field transmission and near-field transmission. Far-field transmissions are also referred to as the radiative type and can achieve a transfer distance of up to a few meters but with low efficiency. Despite having limitations on transfer distance, the near-field transmission method, also referred to as the non-radiative type, has advanced significantly due to better efficiency.[2] A transformer, for example, functions using near-field WPT technology since it exploits the principle of magnetic induction to move energy from a primary coil to a secondary coil without requiring a direct electrical connection. These techniques have been applied in biomedical implants, consumer electronics, and electric vehicle (EV) charging.[3]

The operating concept of near-field WPT devices is primarily based on magnetic-field coupling. The electromagnetic induction technique (which is under magnetic field coupling) was frequently used in the early 1990s to charge the batteries of portable gadgets such as electric toothbrushes and cordless telephones.[4] Even today, the Qi standard for wireless charging implemented in smartphones uses the same electromagnetic induction concept, and these are commercially available wireless charging devices.[1] The second approach to magnetic coupling is the magnetic resonance method, which has recently gained popularity. The magnetic resonance method gained a lot of interest in 2007 after a team from MIT's Department of Physics, Department of Electrical Engineering and Computer Science, and ISN were able to demonstrate the transfer of 60 W of power over more than 2 m.[5]

The magnetic resonance coupling (MRC) method has been shown to resolve the issue of a drop in efficiency when the transfer distance is increased.[6] MRC has been shown to maintain transfer efficiency with gradual increases in transfer distance. The MRC method can be employed in the WPT system using four topologies: Series Series (S-S), Series Parallel (S-P), Parallel Series (P-S), and Parallel Parallel (P-P). The topologies define the connection of the capacitor in the primary and secondary circuits of the WPT system.

In MRC, even slight detuning of the resonance frequency of the receiving coil from that of the transmitting coil can significantly disrupt energy transfer efficiency.[7] The MRC technique achieves optimal transmission efficiency by carefully positioning the transmitting and receiving coils at an ideal distance from each other. Additionally, introducing a load on the receiving end alters the circuit's impedance, causing a misalignment with the transmitter circuit's resonant frequency.

This misalignment can be attributed to various factors, including frequency disparities, environmental influences, load-related effects, and manufacturing variations. When detuning occurs, it decreases the efficiency of energy transfer between coils, resulting in energy loss and the potential risk of overheating. Therefore, it is evident that special measures are needed to optimize and tune the receiving circuit to match the resonant frequency of the transmitting circuit.

The four topologies have a set of advantages and disadvantages that are associated with the MRC WPT. To begin with, the efficiency of the S-S topology decreases as load increases, indicating that this setup performs better with smaller loads.[8] The S-S topology can handle high-frequency changes, handle small load resistances, and improve efficiency with an increased coupling coefficient. The drawbacks of the S-S topology include frequency splitting when the transmission and reception coils are too close and loss of efficiency as load resistance increases. At the natural resonant frequency, the S-S and S-P topologies have very low input impedance, allowing a lot of input current to flow through the primary coil inductance, creating a maximum magnetic field around the coil and enough induced voltage and current in the secondary coil to recharge the battery. In P-S and P-P topologies, high input impedance at natural resonant frequency allows a very small current to pass through the main coil inductance, resulting in low induced voltage and current at the secondary, which are insufficient to recharge the battery.

In summary, WPT techniques, such as electromagnetic induction, magnetic resonance, and microwave transmission, face challenges in terms of efficiency, range, and safety. Electromagnetic induction involves transmitting energy through a magnetic field generated by a coil. However, its short range and low efficiency limit its application. Magnetic resonance employs resonant coils to improve efficiency, but it is still restricted in scope and can be influenced by neighboring metal objects. However, microwave transmission can transport power over more considerable distances, but it can be detrimental to human health and inefficient.

Recently, fractional-order wireless power transfer (fWPT) systems have emerged as a promising solution to address some of the challenges faced in WPT. Leveraging the power of fractional calculus for the precise design of components within WPT systems, this approach is critical to enhancing overall system performance.

This review critically assesses the fractional-order strategies utilized in WPT. Various engineering repositories such as IEEE Xplore, Elsevier, Springer, Wiley, CRC, and others were accessed for this research. The materials consulted included journal articles, book chapters, and conference proceedings. This review serves to provide a valuable resource for researchers and engineers interested in designing and optimizing wireless power transfer systems using fractional methods by summarizing the present knowledge on fractional-order approaches in WPT.

2 | FUNDAMENTALS OF FRACTIONAL ORDER SYSTEMS

Fractional calculus is a branch of mathematical analysis that expands the concepts of differentiation and integration to non-integer orders. This is distinct from traditional calculus, which only deals with integer powers of differentiation and integration and accomplishes it using fractional derivatives and fractional integrals. Traditional calculus theory requires repeated differentiation to obtain the n-th derivative of a given function $f(x)$ and is only valid for positive integers. A transformation that takes a function as input and outputs a function is necessary to compute fractional derivatives. The formulas mentioned in the following subsections have been accumulated from literature.[9–11]

2.1 | Cauchy's formula for repeated integration

The n-th order repeated integral of a function $f(x)$ is given by (1).

$$I^n f(x) = \frac{1}{(n-1)!}\int (x-t)^{n-1} f(t)\,dt. \tag{1}$$

Cauchy's formula for repeated integration is a powerful tool in fractional calculus, especially when dealing with fractional derivatives and integrals of functions. It allows computing the n-th order repeated integral of a function $f(x)$ by integrating the expression $(x-t)^{n-1}f(t)$ concerning t, n times, in (1), $(n-1)!$ can be replaced by the gamma function to expand the formula to accommodate any real positive numbers, not just positive integer values.

2.2 | Gamma function

The gamma function is a mathematical function that extends the factorial function to real and complex numbers. It is denoted by $\Gamma(n)$ and is defined as an improper integral, valid for all complex numbers n with a positive real part:

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt \tag{2}$$

where n is a complex number with a positive real part. The gamma function is a smooth curve that passes through the factorial points. Substituting (2) into (1) results in the *Riemann-Liouville fractional integral* (or R-L integral for short). This expression also allows the consideration of complex numbers; as long as the condition $Re(n) > 0$ is satisfied. The R-L integrals preserve the relation shown in (3).

$$D^\alpha I^\alpha f(x) = f(x) \tag{3}$$

(3) can be rewritten as follows:

$$D^\alpha f(x) = \frac{d^{\lceil \alpha \rceil}}{dx^{\lceil \alpha \rceil}}\left(I^{\lceil \alpha \rceil - \alpha} f(x)\right) \tag{4}$$

In (4) above, $\lceil\rceil$ denotes the ceiling function, which rounds any decimal to the nearest integer. Using (1) to further modify (4), the expression for the *Left R-L fractional derivative* can be obtained as shown in (5)

$$D^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_a^x (x-t)^{n-\alpha-1} f(t) dt \tag{5}$$

Using (5), fractional derivatives can be calculated to allow exploration between the lines of what traditional calculus theory offers. It is crucial to recognize that the R-L definitions for integration and differentiation are not the sole options. The field of fractional calculus encompasses a wide range of approaches and definitions of fractional derivatives and integrals. In addition to the Riemann-Liouville definition, the Caputo fractional derivative, the Grünwald-Letnikov fractional derivative, and others present alternative viewpoints and offer distinct advantages in specific applications. Each definition has distinct characteristics and mathematical properties, which makes it appropriate for various contexts.

Fractional calculus has been implemented in the development and analysis of WPT systems. This is done to comprehend the intricate nature of electromagnetic fields and the interplay between transmitting and receiving devices. As

WPT systems use electromagnetic waves, commonly in the form of magnetic fields, to facilitate the transfer of electrical energy, the accuracy of traditional methods that rely on classical calculus is limited when describing certain aspects.

When fractional derivatives and integrals are added to the equations that describe electromagnetic phenomena, memory effects, fractional orders of differentiation, and fractional orders of integration can be considered. Fractional calculus-based models can capture the fractional order behavior of components in WPT systems, including, but not limited to, coils, resonators, or antennas. The models can provide a more precise description of the characteristics of the distributed parameters, the impedance matching, and the efficiency of the WPT system.

2.3 | Fractional order elements (FOEs)

Fractional order elements (FOEs) are electrical components that exhibit unique fractional order behavior in their impedance or admittance characteristics. FOEs have fractional orders, represented by non-integer values, which introduce a level of complexity, versatility, and flexibilty in electrical circuits that has gained significant attention in recent years. To illustrate the practicality of FOEs, a study by Prasad et al[12] had investigated and recorded the presence of fractional characteristics in supercapacitors, showing that these elements can have variable fractional model structures. The experiment used three different supercapacitors of different values and manufacturers. The findings showed that the actual orders for supercapacitors varied according to the manufacturer and were close to, but not equal to 1. The fractional orders of the Kyocera AVX supercapacitors were 0.93 and 0.92, respectively, but the fractional orders of the other two brands ranged from 0.42 to 0.93 depending on capacitance values. This experiment proved the existence of supercapacitor fractional-order qualities and gave a flexible supercapacitor model for realistic and precise time-domain modelling. When fractional order properties are omitted, most capacitor orders are close to one; hence, it is commonly used to describe the basic capacitor model. As mentioned above, FOEs can be best understood by considering their impedance or admittance characteristics. Unlike typical electrical components such as resistors, capacitors, and inductors, FOEs exhibit fractional-order behavior, governed by parameters such as α or β. This fractional order indicates that the relationship between voltage and current in these components is not governed by integer exponents. FOEs have different properties from integer-order elements, such as having real and imaginary impedance parts, and can be used to construct various fractional-order circuits, which have more design flexibility and beneficial performance than integer order circuits.[13]

According to Boskovic et al,[14] there are two main types of FOEs: resistive-capacitive (RC-α) elements and resistive-inductive (RL-α) elements. The RC-α elements have a combination of resistance and capacitance behavior, while the RL-α elements have a combination of resistance and inductance behavior. RC-α and RL-α are also called fractional-order capacitors (FOC) and fractional-order inductors (FOI), respectively. The current–voltage relationships of an FOC and an FOI are given by (6) and (7), respectively.

$$i(t)_C = C_\alpha \frac{d^\alpha v(t)_C}{dt^\alpha} \tag{6}$$

$$v(t)_L = L_\beta \frac{d^\beta i(t)_L}{dt^\beta} \tag{7}$$

where $i(t)_C$ is the capacitor current, $v(t)_C$ is the capacitor voltage, C_α is the fractional capacitance, $v(t)_L$ is the inductor voltage, $i(t)_L$ is the inductor current, L_β is the fractional inductance, and $\frac{d^\alpha v(t)_C}{dt^\alpha}$ and $\frac{d^\beta i(t)_L}{dt^\beta}$ are the fractional derivative operators of orders α and β. The fractional capacitance is termed pseudo-capacitance,[15] and it can be obtained as

$$C_\alpha = \frac{\sin\left(\frac{\alpha\pi}{2}\right)}{\omega^{1+\alpha}(L_1)\sin\left(\frac{\pi}{2}\right)} \tag{8}$$

where ω is the angular frequency, α is the order in the range of 0 to 2, L is the inductance of the coil, and C_α is the pseudo-capacitance value in $F/s^{(1-\alpha)}$. The angular frequency influences the pseudo-capacitance value, as the relationship suggests. To note that the conventional capacitance (C) in Farads becomes:

$$C = \frac{C_\alpha}{\omega^{(1-\alpha)}} \tag{9}$$

Similarly, Zhang et al[16] state that the fractional inductance is termed pseudo-inductance and is expressed in $Hs^{(\alpha-1)}$. The order, denoted by α for the FOC and β for the FOI, represents the non-integer value that characterizes the behavior of these elements. FOEs are also sometimes referred to as constant phase elements (CPEs) because they exhibit a constant phase angle (CPA) over a wide range of frequencies.[17] The phase angle of a CPE is solely dependent on the fractional-order value and remains constant in the constant phase zone (CPZ). The phase response of the FOEs is characterized by[18]

$$\phi = \frac{x\pi}{2}, \tag{10}$$

where x can be either α or β.

To illustrate the diversity of FOEs, Figure 1 shows a classification scheme based on their order values. A fractional order in the range of $0 < \alpha, \beta < 1$ creates a passive FOE that consumes power, while $1 < \alpha, \beta < 2$ creates an active FOE, which has the characteristic of a negative resistor and supplies power.[3] FOEs can be constructed in one of two ways, as shown in Figure 2. The choice between single-component and multi-component realization of FOEs is influenced by several criteria, including the system's complexity, the level of accuracy required, and the available resources and expertise for implementation and analysis. However, it is essential to acknowledge that this section does not encompass all existing techniques for constructing FOEs. A more detailed review of the progress of FOE design techniques can be found in the literature.[19,20] In addition, the circuit symbols used to represent the FOEs differ from the generic inductor and capacitor symbols. The symbols shown in Figure 3 have been identified as common representations in the reviewed articles.

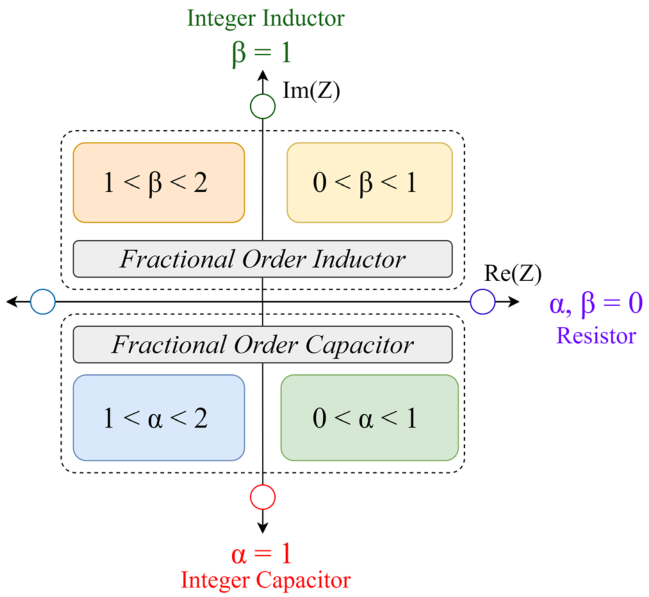

FIGURE 1 FOE classification. [Colour figure can be viewed at wileyonlinelibrary.com]

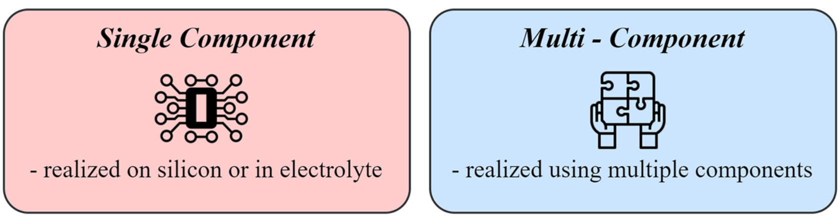

FIGURE 2 FOE realization techniques. [Colour figure can be viewed at wileyonlinelibrary.com]

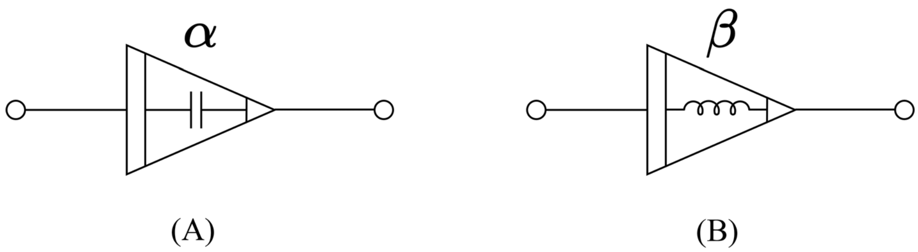

FIGURE 3 Symbols used to represent fractional elements: (A) FOC and (B) FOI.

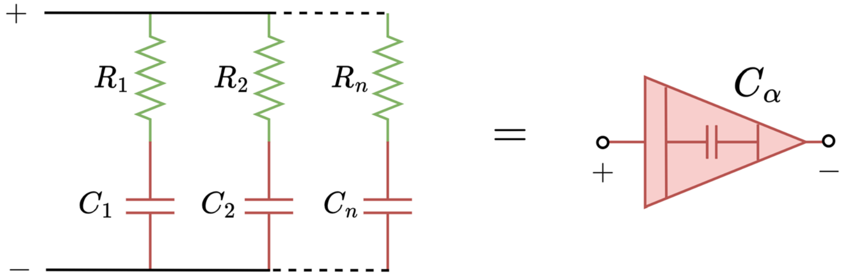

FIGURE 4 Foster II form representation of an FOC. [Colour figure can be viewed at wileyonlinelibrary.com]

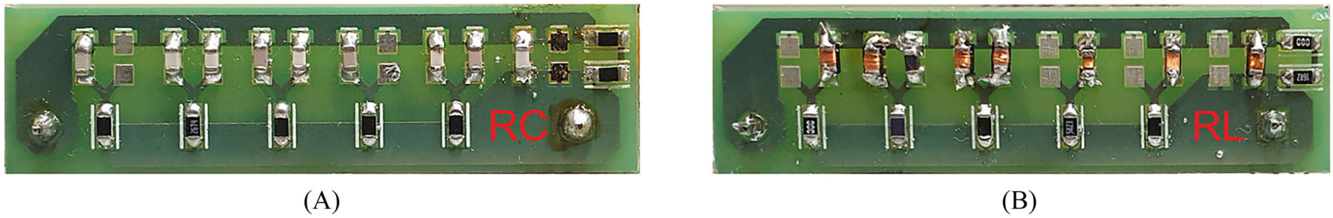

FIGURE 5 Multi-component FOEs fabricated by Mijat et al.[21]: (A) FOC; (B) FOI. [Colour figure can be viewed at wileyonlinelibrary.com]

2.4 | Multi-component realization

The multi-component realization approach employs network synthesis principles to implement fractional-order elements in a ladder structure using passive elements (active components can also be used). This method entails creating an equivalent electronic circuit capable of simulating the system's fractional dynamics and achieving a CPA for a CPZ and related impedance characteristics. The multi-component realization technique uses this methodology to create complex circuits that accurately capture the required fractional-order behavior and impedance characteristics. For example, an FOC can be realized in the Foster II RC ladder form, as shown in Figure 4.

A comprehensive procedure (focusing on the minimax approximation) for analog modelling of fractional components is provided in an article by Mijat et al.[21] Fundamental models of RC circuits proposed by Foster and Cauer are introduced for one-port elements of fractional-order systems. The study shows that a minimum of eight distinct canonical one-port configurations can be built for each constant phase value. These one-port arrangements create simple implementations of critical two-port fractional components, such as integrators and differentiators. The authors have made a MATLAB tool intended exclusively for producing constant-phase elements available on the Internet. This program was created as part of the study and is available online or directly from the creators (see literature[22]). Figure 5 shows the FOEs fabricated by Mijat et al.[21]

In addition to using passive elements, Kapoulea et al[23] propose an electronically tuneable fractional order impedance emulator that uses operational transconductance amplifiers (OTA) to change the emulated capacitance or inductance values and the bandwidth of operation for any order less than two. The proposed concept is independent of the integer-order method used for the approximation $s^{\pm\alpha}$, allowing several types of approximation. The post-layout simulation results confirm the correct operation of the proposed circuits, making it suitable for practical applications.

A study conducted by Jiang et al[24] presents the concepts of FOC realization, which encompass four distinct methods, each characterized by a different α value. The first three methods use $0<\alpha<1$, while the fourth method extends to $0<\alpha<2$. This study introduces a pioneering approach for achieving high-power FOC with $\alpha>1$. This innovative method involves employing a power converter to regulate the capacitor current, which requires precise processing of power levels and ensuring that the FOC operates at the desired frequency. A single port circuit is used to emulate the current–voltage relationship, as shown in Figure 6.

The performance of the high power FOC is rigorously tested, showing a maximum error between the set and the pseudo-capacitances achieved of 10. 9%. Furthermore, the study presents the practical application of high-power FOC in a resonance $RLC\alpha$ circuit, where the component successfully resonates with the inductor.

Varnshney et al[27] propose a novel approach for the construction of fractional-order capacitors based on active inductors. The circuit design uses a modified version of a previously proposed active inductor circuit,[28] which offers advantages in terms of mobility and the fabrication process. Two methods are presented to achieve higher-order transfer functions for approximating fractional-order capacitors: parallel connection of active inductors and impedance multiplication. The proposed circuit is validated through simulations using CMOS technology model parameters. Additionally, the functionality of the proposed circuit is demonstrated by implementing a fractional-order oscillator. Overall, this work provides a new perspective on the realization of fractional-order capacitors with minimal component count.

2.5 | Single component realizations

The single-component realization aims to fabricate FOEs to ensure their commercial viability and marketability. There are no commercially available FOEs, but significant research efforts are underway in the area. Mohapatra et al[29] have managed to realize an FOC using a combination of epoxy resin, conductive gel, and multi-walled carbon nanotube as the dielectric between two parallel copper plates (Figure 7A). Using only epoxy resin with the conductive gel resulted

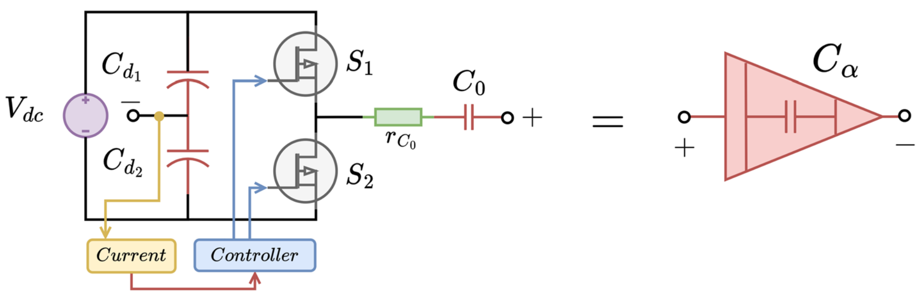

FIGURE 6 Proposed high power FOC network by literature.[24–26] [Colour figure can be viewed at wileyonlinelibrary.com]

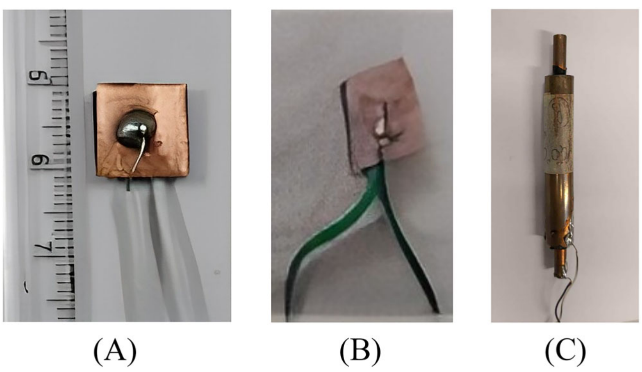

FIGURE 7 Single component realizations from literature,[29–31] respectively. [Colour figure can be viewed at wileyonlinelibrary.com]

in a CPZ from 10 mHz to 100 Hz, while adding the multi-walled carbon nanotube increased the CPZ from 10 mHz to 73 kHz. In both cases, fractional-order characteristics could be set in the $0.07 < \alpha < 0.3$ range simply by varying the percentages of chemicals used to prepare the dielectric mixture.

Similarly, Jha et al[30] fabricate an FOC with copper as electrodes, a dielectric mix of carbon, and nail enamel Araldite (a type of epoxy resin), arranged in layers between the two electrodes, with a porous film layer in the middle (see Figure 7B). The realization exhibits fractionally both resistive and capacitive behavior. For frequencies above 10 kHz, it works as a CPE with a CPA of 75° and an order of 0.8333. This realization technique is capable of replacing bulky RC ladder networks that are generated from network synthesis.

Another method developed by Buscarino et al[31] focuses on the use of nanostructured materials, particularly carbon black composites for the implementation of FOEs (Figure 7C). Similar to the previous case, the carbon-black composites make up the dielectric for the FOC. The fractional order obtained (α) was 0.81 with a pseudo-capacitance of 2.7 nF/$s^{(1-\alpha)}$. The authors also suggested that fractional-order characteristics can be controlled by altering the percentage of carbon black in the mixture, the curing time, and the curing temperature. Adhikary et al[32] presented an extensive comparison of alternative solid-state implementations of FOEs.

In the realm of WPT, FOEs have emerged as indispensable assets, primarily employed as coupling compensators. Their fractional nature allows for precise control over impedance matching and resonance tuning, optimizing energy transfer efficiency across varying load conditions. The versatility of FOEs extends beyond conventional capacitors and inductors, encompassing fractional-order transmission characteristics that exhibit superior signal integrity and reduced losses. Consequently, advances in FOE technology have revolutionized the design and performance of WPT systems and placed them at the forefront of cutting-edge wireless energy transmission research.

Furthermore, the application of fractional calculus plays a pivotal role in enabling a more comprehensive understanding of the dynamics and optimization of WPT systems. Specialized components and control strategies have been developed employing fractional calculus to enhance the efficiency, stability, and robustness of these systems.

3 | FRACTIONAL APPROACHES TO WIRELESS POWER TRANSFER

Fractional calculus allows for more accurate modelling of actual components such as capacitors and inductors, mainly when their fractional order properties are considered. A time-domain analysis of fractional-order circuits compared to classical integer components is presented in.[33] By replicating and comparing the behavior of a fractional-order derivative using ladder elements, it was proved that the classical integer derivative was highly inaccurate and could not capture the dynamic behavior of the process. The ladder network was discovered to accommodate rapid voltage changes during the initial charging condition, demonstrating a more true and dynamic system.

A recent study by Abdelhafiz et al[34] focuses on exploring the use of fractional-order capacitors in wireless power transfer systems. The work investigates a four-plate fractional capacitive power transfer system, where six orders of capacitors between the plates are varied along with the load resistance. The authors establish a mathematical model based on a 4×4 mutual fractional capacitance matrix for four equidistantly placed identical metal plates and vary the order of the mutual capacitances between 0.7 and 1. It was found that for most scenarios, the integer case outperformed the fractional case. However, when a fractional capacitor is used in the receiver, the load voltage increases by 29× compared to the integer case. The study concludes that fractional capacitive wireless power transfer produces more precise results than integer systems and has a different range of load values. An FOC with an order range of 0.7 to 1 for the receiving side is recommended for the best results.

In addition, Fathi et al[35] explore the concept of two-port network analysis in the context of fWPT circuits. The article investigates S-S and S-P topologies, employing fractional capacitors and inductors of fractional order 0.9. The simulations were performed using MATLAB Simulink, and the results were compared with the respective integer-order equivalent model. The study found that the integer circuit outperformed the fractional circuit in terms of efficiency. However, using fractional elements in the circuit adds more freedom to the system, providing more flexibility and stability for various applications, despite the lower efficiency of around 27% for each topology compared to the whole circuit.

Developing circuit models for FOC and FOI, Shu and Zhang[36] have analyzed the influences of these FOEs on a WPT system. Theoretical analysis revealed that different orders of FOC (α) and FOI (β) affect transmission efficiency differently. However, the effect of fractional orders on output power was found to be proportional, meaning that output power increased as the fractional orders of the elements increased. If the system is realized using physical components, such a theoretical analysis can be used to find the optimum orders for each FOE to obtain better characteristics.

Rong et al[37] conducted a theoretical analysis and experimental study of an fWPT system. The authors propose a fractional coupled model based on fractional calculus and coupled-mode theory to describe the dynamic characteristics of fractional systems. The results show that the fWPT system has better characteristics and design flexibility than integer-order systems. The fractional coupled model is validated through circuit simulation and experiment, demonstrating its accuracy and usefulness for further analysis of fWPT systems.

In a similar study,[15] the authors propose a series-parallel topology that uses resonance and includes an FOC and FOI on both the transmitting and receiving sides of the circuit (see Figure 8). This fWPT topology will be referred to as Type I hereafter.

Four cases were tested to evaluate the performance of the system: Case 1—classical WPT (where all orders are set to 1); Case 2—FOCs with integer order inductors (varying $\alpha_{1,2}$ while keeping $\beta = 1$); Case 3—all fractional components (varying α_1 between 0 and 2 while keeping α_2 constant at 1.2 and varying $\beta_{1,2}$); and Case 4—a special case that analyzes the effect of load resistance. The results highlight the optimal combination of fractional orders for efficient power transfer. A summary of the results indicates that a variation of Case 3 exhibits the highest efficiency of 97.22%, with Case 2 reaching a maximum of 96.23%—both outperforming the integer model, Case 1. The special case demonstrates that increasing the load resistance to a specific value contributes to better efficiency, but exceeding this value leads to decreased efficiency. The study concludes that selecting the appropriate fractional orders for a given load resistance results in optimal efficiency, surpassing the performance of the integer order model.

Zhang et al[3] proposed a S-S, FOE-based WPT system (see Figure 9) that can achieve higher power efficiency and lower resonant frequency compared to using conventional integer order components. This fWPT topology is denoted as Type II. By designing an FOC using the concepts of network synthesis and realizing it using the parameters obtained, the dynamic behavior of an FOC was tested before it was implemented in a WPT system.

Theoretically, the FOC should introduce a phase difference between the input voltage and the current, resulting in a leading phase angle of $-90°\alpha$ for the input current, where α represents the fractional order of the capacitor. During the realization process, it is discovered that the actual order of the implemented FOC is 0.54 instead of the intended 0.5, attributed to variations between the computed and actual RC values used for realization. Despite this discrepancy, the WPT system incorporating the FOC of order 0.54 achieves a unity power factor at a transfer distance of 0.3 m, indicating higher energy efficiency and successful MRC. However, it is important to note that the experimental analysis presented in the article does not provide specific information on transfer efficiency, a direct comparison with an equivalent integer-order WPT system, or an exploration of various fractional orders of FOC to determine the optimal α value.

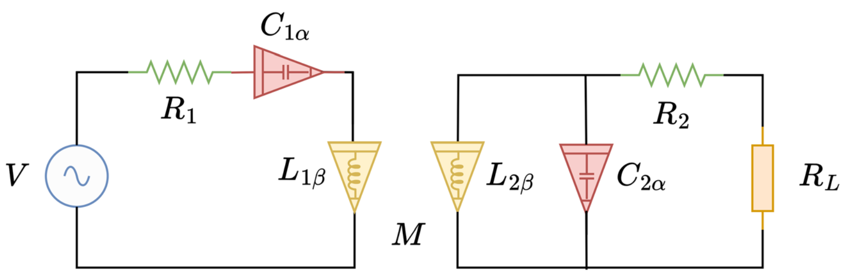

FIGURE 8 Type I—proposed S-P fWPT network by Prasad et al.[15] [Colour figure can be viewed at wileyonlinelibrary.com]

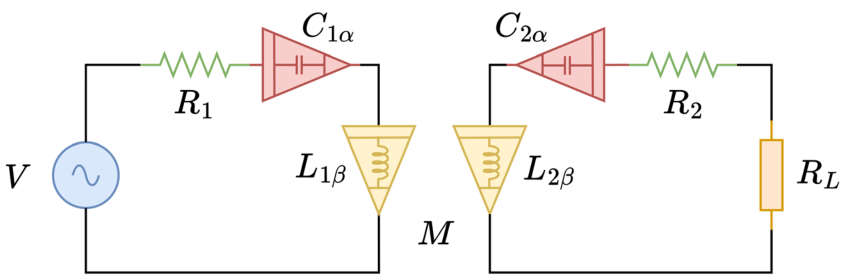

FIGURE 9 Type II—proposed S-S fWPT network by Zhang et al.[3] [Colour figure can be viewed at wileyonlinelibrary.com]

Jiang et al[26] proposed a similar MRC-based fWPT system, which incorporates the use of a high-power FOC. The proposed method uses the S-S topology with the FOC on the transmitter side with $\alpha = 1.013$, as shown in Figure 10. This fWPT topology will be referred to as Type III hereafter.

With an FOC of the order $\alpha > 1$, the system becomes insensitive to the resonant frequency. The theoretical results were consistent with the experimental results, where the input current increased as the load increased, but the output current remained unchanged. The proposed method did not require the communication to be established between the transmitter and the receiver. However, the drawback of the proposed system is that interference that affects the resistance of the coils can change the required order of the FOC.

In another study, Jiang et al[25] utilizing the same S-S compensation as shown in Figure 10 and the high-power FOC shown in Figure 6, the analysis revealed that the fWPT can produce a constant current output that is independent of the load if the suitable FOC parameters, namely, fractional order and pseudo-capacitance, are adopted. A prototype system was also developed with results identical to those of the theoretical analysis. The system has one flaw: If the inductance and internal resistance of the coil change as a result of interference from the surrounding environment, the required order of the FOC will change as well.

WPT technology for autonomous guided vehicles (AGVs) could become the go-to solution for charging AGV batteries. However, the main issue with AGV charging is the problem of misalignment, which results in power losses and prevents the output from achieving constant current and constant voltage modes. As a solution, Rong et al[38] propose a fWPT system that is 44.4% tolerant in the vertical direction and 22.2% tolerant in the horizontal direction. An experimental setup is used to charge a robotic lawnmower, with an output power range of 10.5 to 63 W. The efficiency of the system ranges from 69.2% to 82.4%. The novel work presented provides an interesting view of how fractional-order capacitors can benefit WPT systems. The proposed fWPT system shows promising results in terms of tolerance to misalignment and efficiency, making it a viable solution for charging AGV.

The discussion in Sections 2 and 3 emphasizes the progression of fractional calculus, fractional element design, and their integration into WPT systems over the years. Furthermore, a concise summary of the fractional-order methodologies applied in WPT, as detailed in this section, has been organized into Tables 1 and 2. These tables serve the purpose of presenting a succinct yet informative outline of the key details of the preceding research. These essential details encompass the characterization of the designed fractional order element, the range of orders employed, the configuration of the WPT system, utilization of resonance, the methodology employed, operation frequency, and the peak efficiency achieved.

Table 2 highlights a predominant preference for the S-S topology. However, it should be noted that contrary to this trend, the S-P topology was found to deliver the highest peak efficiency among fWPT systems compared in this review, as demonstrated in the study by Prasad et al.[15] The modification involved the use of the active FOC,[26] as depicted by Figure 6. This eliminated the need for a separate power supply. In a similar vein, Rong et al[38] introduced an active FOC structure in their design. However, they incorporated four MOSFETs instead of the original two, which also nullified the use of a separate power supply.

4 | PERFORMANCE ENHANCEMENT THROUGH FRACTIONAL-ORDER APPROACHES

The incorporation of FOEs into WPT systems presents a promising avenue for improving system performance. Specifically, the integration of fractional capacitors, with a particular focus on those positioned between distinct plates, has

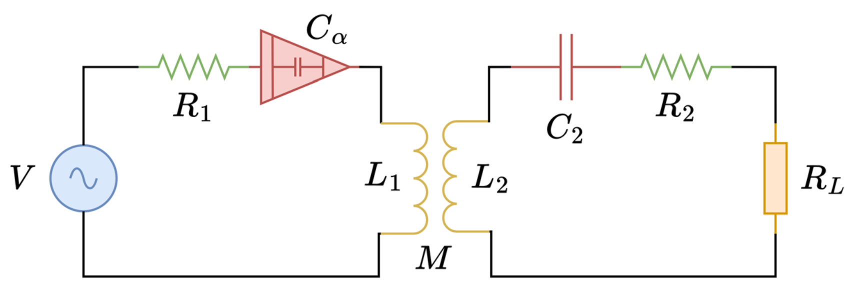

FIGURE 10 Type III—proposed S-S fWPT network by Jiang et al[25] and Jiang and Zhang.[26] [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Study characteristics of the discussed fWPT approaches.

Ref	Year	FOE	Order	Research approach
Shu and Zhang[36]	2018	FOC α_1 : Tx α_2 : Rx FOI β_1 : Tx β_2 : Rx	$0 < \alpha_{1,2} < 2$ $0 < \beta_{1,2} < 1$	Analytical Modelling
Zhang et al.[3]	2019	FOC α_1 : Tx α_2 : Rx	$0 < \alpha_{1,2} < 1$ $(\alpha_{1,2} = 0.54)$	Analytical Modelling, Simulation, and Experimental Validation
Jiang and Zhang[26]	2020	FOC α_1 : Tx	$1 < \alpha_1 < 2$	Analytical Modelling and Experimental Validation
Jiang et al.[25]	2020	FOC α_1 : Tx	$1 < \alpha_1 < 2$	Analytical Modelling and Experimental Validation
Rong et al.[37]	2020	FOC α_1 : Tx α_2 : Rx FOI β_1 : Tx β_1 : Rx	$0 < \alpha_1 \leq 1$ $\alpha_2, \beta_1, \beta_2 = 1$	Analytical Modelling and Experimental Validation
Prasad et al.[15]	2022	FOC α_1 : Tx α_2 : Rx FOI β_1 : Tx) β_2 : Rx	$0 < \alpha_{1,2} < 2$ $0 < \beta_{1,2} < 1$	Analytical Modelling
Abdelhafiz et al.[34]	2022	FOC β_1 : Tx β_2 :Rx	$0.7 < \alpha_n < 1$ $(0.7 \leq n \leq 1)$	Analytical Modelling
Rong et al.[38]	2022	FOC α_1 : Tx α_2 : Rx FOI β_1 : Tx β_2 : Rx	$\alpha_1 > 1$ $\alpha_2, \beta_1, \beta_2 = 1$	Analytical Modelling and Experimental Validation

yielded notable improvements in voltage precision. These enhancements result from a strategic reconfiguration of these capacitors.[34] As a consequence, voltage levels are effectively reduced, leading to significantly improved precision. Additionally, fractional capacitors have shown positive impacts on power transfer efficiency, especially when the order of capacitors is reduced within specific plate combinations. This integration of fractional capacitors has great potential to achieve increased precision and efficiency in capacitive wireless power transfer scenarios. The following enumeration outlines the enhancements that the incorporation of FOEs brings to WPT systems.

TABLE 2 Specifications of the discussed fWPT approaches.

Ref	Topology	Network structure	Resonance	Frequency range	Peak efficiency
Shu and Zhang[36]	S-S	Type II	Yes	kHz	≈ 95%
Zhang et al.[3]	S-S	Type II	Yes	kHz–MHz	-
Jiang and Zhang[26]	S-S	Modified Type III	Yes	kHz	>90%
Jiang et al.[25]	S-S	Type III	Yes	-	>90%
Rong et al.[37]	S-S	Type II	Yes	kHz	>35%
Prasad et al.[15]	S-P	Type I	Yes	kHz	97.22%
Rong et al.[38]	S-S	Modified Type III	Yes	kHz	92.7%

1. **Transmission Efficiency and Output Power:**
   The introduction of fractional-order components into WPT systems, as highlighted by Shu et al,[36] plays a crucial role in enhancing power transfer efficiency, ultimately resulting in increased output power. The strategic selection of fractional-order elements within the optimized operating range of the system realizes this attribute, which proves to be particularly advantageous in the world of WPT systems. This optimization enables the system to maximize the delivery of power from the source to the load.
2. **Accurate Modelling:**
   Fractional modelling based on fractional calculus provides a more precise and efficient description of fractional-order wireless power transfer (fWPT) systems.[37] This approach offers a clearer understanding of the dynamic characteristics of fractional systems, bypassing the intricacies associated with traditional circuit models. It also allows for more accurate predictions of system behavior under various conditions, which aids in the design and optimization process.
3. **Robustness:**
   fWPT systems, as demonstrated by Rong et al,[38] exhibit remarkable tolerance to misalignment. This attribute underscores the system's ability to maintain efficient power transfer, even in instances of less-than-optimal alignment between the coupling coils. Such robustness has significant implications for practical applications, including charging AGVs and electric vehicles, where operational flexibility and convenience are paramount. Robustness ensures that power transfer remains reliable and efficient despite real-world challenges.
4. **Insensitivity to Resonant Frequency Detuning:**
   The incorporation of FOEs imparts insensitivity to resonant frequency, enhancing the system's resilience.[26] This characteristic is especially crucial for practical applications, as the resonant frequency of a WPT system can unpredictably shift due to external environmental disturbances. FOEs help maintain efficient power transfer even when the system's resonance is disturbed, making it more reliable in dynamic operating environments.
5. **Design Freedom:**
   Incorporating FOEs offers greater flexibility in receiver design.[36] Compared to conventional WPT systems, fWPT systems provide a higher degree of design freedom, facilitating the development of compact and robust receivers. This flexibility is particularly advantageous in applications where size and weight constraints are of paramount importance. It enables engineers to design receivers that meet specific size, weight, and form factor requirements without compromising performance. In addition, it allows for the exploration of innovative receiver designs to optimize the overall system.

From the research mentioned, it is evident that the incorporation of FOCs with $\alpha > 1$ offers notable advantages to WPT systems. This results in higher power transfer efficiencies, even surpassing those achieved by traditional WPT methods. In contrast, when FOCs with $\alpha < 1$ are integrated with WPT, they do not match the maximum efficiencies achieved by classical WPT. This highlights that while the approximation and implementation of FOCs with $0 < \alpha < 1$ may be less complex compared to those with $1 < \alpha < 2$, it does not enhance the efficiency of the system. However, it can improve other factors, such as stability and bandwidth. Additionally, when FOCs with orders greater than 1 are implemented, they achieve higher efficiencies only when the order is close to, but greater than, 1.

5 | CHALLENGES AND MOTIVATION

The works discussed strongly emphasize the critical importance of advancing the development of FOEs and fWPT systems. When a thorough examination of various studies in the field is conducted, a clear and distinct pattern emerges, showcasing the multiple advantages that fWPT systems have in comparison to their WPT counterparts. However, a prominent and foundational challenge surfaces in this context, the absence of readily available commercial FOEs. This absence constitutes a substantial barrier to progress in the domains of both WPT and fWPT technology, hindering potential advancements. While addressing the unavailability of FOEs is a fundamental prerequisite, it represents just the tip of the iceberg in the spectrum of challenges facing the development of fWPT systems. The second group of challenges pertains to the intricate optimization of fWPT systems, each posing unique complexities. One of these challenges involves the delicate task of determining the optimal combination of fractional orders. This optimization seeks to maximize output power and transmission efficiency while skilfully navigating issues such as the avoidance of frequency splitting and parameter coupling, ensuring the seamless operation of the system. Furthermore, there exists a need to optimize the transmission characteristics of fWPT systems. This entails the suppression of magnetic fields and the strategic arrangement of the coils. These optimizations can be effectively executed using dedicated algorithms like particle swarm optimization, paving the way for enhanced performance and reliability.

Given that most studies have primarily concentrated on analytical modelling or experimental validation, there arises a compelling opportunity to conduct an in-depth examination of fWPT systems utilizing sophisticated software tools such as ANSYS Maxwell, Simplorer, or MATLAB Simulink. Utilizing ANSYS, for instance, would facilitate the visualization of magnetic-field distributions and strength, offering a comprehensive basis for comparative evaluations regarding the advantages introduced by FOEs in the context of a WPT system. Also, it would provide valuable insights into intricate phenomena, including but not limited to frequency splitting phenomena, thereby contributing significantly to the understanding of the underlying principles governing fWPT system behavior.

6 | CONCLUSION

In conclusion, this review has delved deeply into the realm of fWPT. Through a meticulous analysis of various fractional-order methods, their effectiveness, and their practicality, this study has illuminated the advantages and limitations of employing fractional calculus in the development of WPT components. The research findings underscore the potential for substantial improvements in efficiency, range, and safety by integrating fractional-order elements into wireless power transfer systems.

However, it is clear that further research is needed to improve the implementation efficiency of fractional-order methods. Designers of wireless power transfer systems must carefully balance efficiency, range, and safety considerations. This paper has shed light on the myriad benefits, challenges, and future prospects of integrating fractional-order elements into WPT strategies. Moving forward, a successful transition to fractional-order elements in WPTs holds great promise and invites continued exploration and development in this exciting field.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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