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MULTI-OBJECTIVE OPTIMIZATION USING SIMPLEX METHOD

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Abstract: The solution of multi-objective optimization problem always has much attention to the decision makers in management science as the objectives typically conflict each other. In this article, we devise a simplex technique approach to solve multi-objective linear programming problem (MOLP), in which all objectives are optimized simultaneously. Illustrations of computational details of the proposed technique is indicated via numerical methods. A real-life situation of a food processing problem is discussed and applied to the proposed technique to demonstrate the formulation of a MOLP. A comparative study is explicated by emphasizing on the comparison of the proposed approach with the preemptive goal programming approach. The computed results clearly show the usefulness, practicality and strength of the proposed technique in optimizing MOLP with reduced computational effort as compared to other goal programming technique. The results of the proposed technique are efficient, convenient and shows that the algorithm is applicable to almost all MOLP.

Index Terms: Multi-objective linear programming, Goal programming, Simplex algorithm, multi-criteria optimization.

1. Introduction

Most linear programming methods traditionally have dealt with problems under the presupposition of a single quantifiable objective, for instance, either to minimize loss (or cost) or maximize profit. Conversely, when considering any kind of scenario, whether considering daily or professional setting, multiple conflicting criteria typically exist that must be evaluated in making ultimate decisions. Therefore, presenting a single objective is not considered as of practical use. Multiple dependent criteria or objectives may be required to have problems solved and have realistic and increased realization of the reality that most real-life decision problems typically comprise of multiple objectives. These decision-making problems alongside multiple linear objectives are generally recognized and so-called as multi-objective linear optimization or multi-objective linear programming (MOLP) problems. Numerous attempts have accomplished the development of techniques and algorithms to solve multi-objective linear programming problems. Goal programming concept was initially created and established by Charnes, Cooper, and Ferguson [2] with an application and implementation to a single objective linear programming problem. Subsequently, it was developed by Lee [8] and Ignizio [6] and as a result goal programming became an invent for solving and decision-making, specifically, multi-objective problems that allowed decision makers to incorporate organizational managerial and environmental deliberations into the model via goal priorities and levels. Preemptive Goal Programming was developed by Ignizio [6]. This programming approach, highlighted that the decision maker is compelled to prioritize goals into unrelated and non-identical priority levels, each of which consist of one or more goals. Later, Flavell [4] introduced Chebyshev goal programming, whereby the maximum deviation in any goal from the target is minimized. The ε - constraint method was initially proposed by Haimes et al. [5] that was based on revising the multi-objective optimization problem and recommendation was to set up a

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procedure by keeping one of the objectives and restricting the rest of the objectives within user-specified values. Most publications in the past and latterly have been on the subject of fuzzy logic. A method to the solution of MOLP problems in fuzzy environments was proposed by De and Yadav [3] by means of which recognition is focused on the study of ideal compromise solutions for multi-objective fuzzy linear programming problems. Similarly, Yano and Sakawa [15] proposed an interactive fuzzy decision-making method for problems of multi-objective fuzzy random linear programming fractile criteria optimization to acquire an acceptable solution from amidst an extended Pareto optimal solution. Zangiabadi and Maleki [16] extended fuzzy goal programming application to the linear multi-objective transportation problem. The bi-level linear multi-objective problem was proposed by Pieume [9]. Similarly, an algorithm was developed that is incorporated in group of interactive methods to solve multi-objective problems by Sadrabadi and Sadjadi [10]. Implementation of MOLP to factual situations have been researched and applied by many authors. New algorithms and newly discovered techniques have been introduced and put to an application to factual situations, taking into account multiple-objective linear programming problems. A MOLP model on an injection reservoir recovery system was proposed by Xiao et al. [14]. Some researchers concentrated on generating every one of the efficient solutions of multi-objective integer linear programming (MOILP) similarly as Jahanshahloo [7], who instigated a method for creating all the efficient solutions of MLP 0-1 problem with a bounded feasible region. Tohidi and Razavyan [12] elongated the same method to detect efficient solutions, possibly all, of an MOILP problem which are categorized when the number of efficient solutions is limited. The concept and development of MOLP techniques and algorithms have been deliberated upon by several authors, who have developed and implemented several techniques that are available in the literatures. However, the commonly used technique that deals with multi-objective problems is the goal programming which seeks a compromise solution by positioning the relative significance of each objective referred to as a goal. Nevertheless, there are some circumstances, when no feasible solution is available in satisfying all the goals, it is necessary for the decision makers to search for a technique that optimizes several objectives simultaneously. In this article, a simplex like solution procedure is developed to obtain a compromise solution of the problem of multi-objective linear programming that optimizes all the objectives of the problem simultaneously. The proposed technique is illustrated with an application to a MOLP of an advertising and local food processing company. Finally, the results of the proposed technique are compared with other techniques and methods to demonstrate and substantiate the strength of the proposed method.

2. The Problem of Multi-objective Linear Programming (MOLP)

Consider that a Multi-objective Linear Programming problem (MOLPP) has p objective functions of n variables subjected to m constraints where the objective functions and constraints are linear. The standard form of the MOLPP may be written as:

$$\begin{aligned} \text{Maximize } & \left(z_1 = \boldsymbol{C}_1\boldsymbol{x},\ z_2 = \boldsymbol{C}_2\boldsymbol{x}, \ldots, z_p = \boldsymbol{C}_p\boldsymbol{x}\right) \\ \text{subject to } & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq 0 \end{aligned} \tag{1}$$

where $\boldsymbol{C_1} = \left(c_{11}, c_{12}, \cdots, c_{1n}\right)$, $\boldsymbol{C_2} = \left(c_{21}, c_{22}, \cdots, c_{2n}\right)$, …, $\boldsymbol{C_p} = \left(c_{p1}, c_{p2}, \cdots, c_{pn}\right)$ are $1 \times n$ coefficient matrices, $\boldsymbol{x} = \left(x_1, x_2, \cdots, x_n\right)^T$ is a $n \times 1$ matrix of decision variables, $\boldsymbol{A}$ is a $m \times n$ coefficient matrix of the m constraints:

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

and $\boldsymbol{b} = \left(b_1, b_2, \cdots, b_n\right)$ is a $m \times 1$ constants matrix of the constraints.

2.1 A Necessary and Sufficient Condition of Optimality

In order to solve the MOLPP (1) utilizing simplex method, it is important that the necessary and sufficient condition of the optimality is met as given in Theorem 2.1. (See Swarup et al. [11] and Bazaraa [1])

Theorem 2.1: If a necessary and sufficient condition for a basic feasible solution $\left(\boldsymbol{x_B}\right)$ of MOLPP (1) with a single objective function $z_k; (k=1,2,\ldots,p)$ to be an optimum (maximum) is $\Delta_{kj} = z_{kj} - c_{kj} \geq 0$ for all $j; (j=1,2,\ldots,n)$ for which the column vector $\boldsymbol{a_j} \in \boldsymbol{A}$ but $\boldsymbol{a_j} \notin \boldsymbol{B}$, then a necessary and sufficient condition for $\boldsymbol{x_B}$ to be an optimum to $z = \sum_{k=1}^{k} z_k$ is $\Delta_j = \sum_{k=1}^{p} \Delta_{kj} \geq 0$ for all j for which $\boldsymbol{a_j} \notin \boldsymbol{B}$.

Proof: Let the MOLPP (1) be to determine $\boldsymbol{x}$ and $\rho(\boldsymbol{A}) = m$ be the rank of $\boldsymbol{A}$ so that we can choose as $m \times m$ submatrix $\boldsymbol{B}$ of $\boldsymbol{A}$ as a basis matrix.

Assuming that a basic feasible solution $\boldsymbol{x_B}$ exists to the MOLPP (1). Let $\boldsymbol{c_{Bk}}$ be the cost vector confirming to the basic variables in z_k.

Then $$\boldsymbol{B}\boldsymbol{x_B} = \boldsymbol{b},\ \boldsymbol{x_B} \geq \boldsymbol{0} \text{ and } \boldsymbol{z}_0 = \sum_{k=1}^{p} \boldsymbol{c_{Bk}} \boldsymbol{x_B}$$

Now, for all those j for which $\boldsymbol{a_j} \notin \boldsymbol{B}$, we are given that $\Delta_j = \sum_{k=1}^{p} \Delta_{kj} \geq 0$.
Let $\boldsymbol{a_j} = \boldsymbol{b_j}$ for all such j for which $\boldsymbol{a_j} \in \boldsymbol{B}$. Then, for a column vector $\boldsymbol{y_j}$ in $\boldsymbol{A}$

$$\boldsymbol{y}_j = \boldsymbol{B}^{-1}\boldsymbol{b}_j = \boldsymbol{e}_j \text{ is a unit vector}$$

and $$\Delta_j = \sum_{k=1}^{p} \Delta_{kj} = \sum_{k=1}^{p} \left(z_{kj} - c_{kj}\right).$$

As $z_{kj} = \boldsymbol{c_{Bk}} \boldsymbol{y}_j$, Δ_j possibly be expressed as:

$$\Delta_j = \sum_{k=1}^{p} \left(\boldsymbol{c_{Bk}} \boldsymbol{y}_j - c_{kj}\right) = \sum_{k=1}^{p} \left(\boldsymbol{c_{Bkj}} - c_{kj}\right) = 0$$

Because $\boldsymbol{c_{Bkj}} - c_{kj} = 0 \quad \forall k$, $\boldsymbol{a_j} \in \boldsymbol{B}$ and $\boldsymbol{c_{Bkj}} = c_{kj}$.

Thus, $\Delta_j \geq 0$ for all j for which $\boldsymbol{a_j} \in \boldsymbol{A}$.

Now, let $\boldsymbol{x}$ be a feasible solution. Then

$$\sum_{j=1}^{n} \Delta_j x_j \geq 0$$

or $$\sum_{j=1}^{n} \left(\sum_{k=1}^{p} \left(z_{kj} - c_{kj}\right) \right) x_j \geq 0$$

or $$\sum_{j=1}^{n} \sum_{k=1}^{p} z_{kj} x_j \geq \sum_{j=1}^{n} \sum_{k=1}^{p} c_{kj} x_j$$

or $$\sum_{j=1}^{n} \sum_{k=1}^{p} \boldsymbol{c_{Bk}} \boldsymbol{y}_j x_j \geq \sum_{j=1}^{n} \sum_{k=1}^{p} c_{kj} x_j \tag{2}$$

Also note that $\boldsymbol{c}_{\boldsymbol{B}k}\boldsymbol{y}_j = \sum_{i=1}^{m} \boldsymbol{c}_{\boldsymbol{B}ik} y_{ij}$. Thus, (2) becomes

$$\sum_{i=1}^{m}\sum_{k=1}^{p} \boldsymbol{c}_{\boldsymbol{B}ik} \sum_{j=1}^{n} y_{ij} x_j \geq \sum_{k=1}^{p}\sum_{j=1}^{n} c_{kj} x_j \tag{3}$$

for all j for which $\boldsymbol{a}_j \notin \boldsymbol{B}$.

Note that $\boldsymbol{x}_{\boldsymbol{B}} = \boldsymbol{B}^{-1}(\boldsymbol{A}\boldsymbol{x}) = (\boldsymbol{B}^{-1}\boldsymbol{A})\boldsymbol{x} = \boldsymbol{Y}\boldsymbol{x}$

Thus, $x_{Bi} = \sum_{j=1}^{n} y_{ij} x_j; \quad i = 1, 2, \ldots m$.

Therefore, the equation (3) can be altered as

$$\sum_{i=1}^{m}\sum_{k=1}^{p} \boldsymbol{c}_{\boldsymbol{B}ik} x_{Bi} \geq \sum_{k=1}^{p}\sum_{j=1}^{n} c_{kj} x_j \tag{4}$$

or $$\sum_{k=1}^{p} \boldsymbol{c}_{\boldsymbol{B}k} x_B \geq \sum_{k=1}^{p} z_k^*$$

or $$\boldsymbol{z}_0 \geq \boldsymbol{z}^*$$

where z_k^* is the value of the kth objective function and $z^* = \sum_{k=1}^{p} z_k^*$.

2.2 Simplex Algorithm for Solving MOLP Problems

To solve an MOLPP using a simplex algorithm, express the problem as a standard form as in (1) by introducing artificial, slack and surplus variables. However, inclusion of artificial variable causes a violation of the corresponding constraints. Thus, allocate a very large penalty $-M$ for maximization and $+M$ for minimization to the objective functions in order to eradicate these variables from the final solution. Then, the following simplex like algorithmic steps explained below may be used to solve an MOLP:

Step 1: Examine whether the objective functions are to be maximized or minimized considering any objective, say z_k , is to be minimized, then convert it into a maximization as:

$$\text{Minimize } z_k = \text{Maximize } (-z_k)$$

Step 2: Check whether all b_i 's are positive. If any of the b_i 's is negative, multiply both sides of that constraint by –1 so as to make its right hand side positive.

Step 3: By introducing artificial, slack and surplus variables, convert the inequality constraints into equations and express the given MOLPP into its standard form as in (1).

Step 4: Set the simplex like table with the following changes:

(i) Insert p rows ($C_{1j}, C_{2j}, \ldots, C_{pj}$) for the coefficients of the variables of the p objective functions.

(ii) Insert p columns ($C_{B1}, C_{B2}, \ldots, C_{Bp}$) for the coefficients of the basic variables in the p objective functions.

(iii) Split the net-evaluations row into $p+1$ rows as:

$$\Delta_{1j} = z_{1j} - c_{1j} = \text{net-evaluations for the } z_1$$
$$\Delta_{2j} = z_{2j} - c_{2j} = \text{net-evaluations for the } z_2$$
$$\vdots \qquad\qquad \vdots$$
$$\Delta_{pj} = z_{pj} - c_{pj} = \text{net-evaluations for the } z_p$$

and $\Delta_j = \sum_{k=1}^{p} \Delta_{kj}$ = net-evaluations of variable j for the $z = \sum_{k=1}^{p} z_k$.

Where, z_{kj} ($k = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, n$) are calculated as:

$$z_{kj} = \boldsymbol{c}_{\boldsymbol{B}k}\boldsymbol{B}^{-1}\boldsymbol{a}_j = \boldsymbol{c}_{\boldsymbol{B}k}\boldsymbol{B}^{-1}\boldsymbol{y}_j .$$

Step 5: Test the optimality of the solution by using the rules given below:

Rule 1: The solution under the test will be optimal, provided if all $\Delta_j \geq 0,$. Alternate optimal solution will exist if any non-basic Δ_j is also zero.

Rule 2: The solution is not optimal if at least one Δ_j is negative, and further proceeds to amend the solution in the next step.

Rule 3: All elements of the column x_j are negative or zero, if corresponding to any negative Δ_j, then the solution under test will be unbounded.

Step 6: The variable entering the basis matrix and the variable leaving the basis matrix are determined as follows in order to improve the solution:

- ***Entering variable:*** The entering variable x_r (for some $j = r$), is selected every time when corresponding to the most negative value of Δ_j.
- ***Leaving variable*:** The method of leaving variable is the same as in simplex method for linear programming problem (LPP). Thus, the leaving variable (say, x_l) in MOLPP is selected corresponding to the basic variable that has least positive ratio value, that is, the ratio $\frac{X_{B_l}}{a_{lr}}$ is minimum.

Step 7: At the intersection of the leaving and entering variable, the pivot element is marked. The method of determining new values corresponding to new basic feasible solution is the same as in simplex method for LPP.

Step 8: Repeat the steps 5 through 7 till an optimal solution is acquired.

3. Numerical Example

The example on advertising presented in Section 3.1 as reported in Swarup et al. [12] and Winston and Venkataramanan [14] illustrates the computational details of the proposed solution procedure. The example is modified to formulate the problem as an MOLPP.

3.1 Example

The Leon Burnit Advertising Agency is attempting to decide on a TV advertising schedule for Priceler Auto Company; Swarup et al. [11]. The agency can purchase two types of ads and these are those shown during football games and those shown during soap operas. On the maximum, \$600,000 can be spent on ads and at least 35 million high-income women (HIW) should view the ads and these are Leon Burnit is expected to determine how many football ads and soap opera ads to purchase for Priceler that has 2 goals:

Goal 1: Its ads should be seen by at least 40 million high-income men (HIM)
Goal 2: Its ads should be seen by at least 60 million low-income people (LIP) (5)

The specific ads along with viewers, advertising costs and potential audiences of a one-minute ad of each type are shown in Table I.

Table I: Cost and number of viewers of ads

Ad	Viewers (in million)			Cost/minute
	HIM	LIP	HIW	
Football	7	10	5	100,000
Soap opera	3	5	4	60,000

Let x_1 and x_2 be the required number of minutes of ads that is expected to be shown during the football games and soap operas, respectively. Assuming that Priceler has two objectives that are to maximize the number of HIM viewers and to maximize the number of LIP viewers, as opposed to satisfying the goals provided in (5) on the expected number of viewers. The problem can then be formulated as MOLP problem with the following two objectives:

$$\text{Maximize } (z_1) = 7x_1 + 3x_2 \qquad \text{(HIM viewers)}$$
$$\text{Maximize } (z_2) = 10x_1 + 5x_2 \qquad \text{(LIP viewers)}$$
$$\text{subject to} \quad 5x_1 + 4x_2 \geq 35 \qquad \text{(HIW viewers)}$$
$$100x_1 + 60x_2 \leq 600 \qquad \text{(Budget constraint)}$$
$$x_1, x_2 \geq 0 \qquad (6)$$

Since the first constraint in (6) is '$\geq$' type, the initial basic feasible solution can be obtained using a two-phase simplex or Big-M method.

To use a Big-M method, introducing $s_1 \geq 0$ as slack variable, $s_2 \geq 0$ as surplus variable and $A_1 \geq 0$ as artificial variable, the standard form of the MOLPP is written as:

$$\text{Maximize } (z_1) = 7x_1 + 3x_2 + 0s_1 + 0s_2 - MA_1$$
$$\text{Maximize } (z_2) = 10x_1 + 5x_2 + 0s_1 + 0s_2 - MA_1$$
$$\text{subject to} \quad 5x_1 + 4x_2 + 0s_1 - s_2 + A_1 = 35$$
$$100x_1 + 60x_2 + s_1 + 0s_2 + 0A_1 = 600$$
$$x_1, x_2, s_1, s_2, A_1 \geq 0$$

Then, using the multi-objective simplex algorithm discussed in Section 2.2, the initial basic feasible solution and subsequently the iterative solutions are obtained and shown in Tables II and III. Note: X_B column values are in millions.

Table II: Is the initial iteration where the slack variable s_1, surplus variable s_2 and artificial variable A_1 form the initial basis. In iteration 1, entering variable is determined for Table II and the value of Δ_j which is $-10M-17$ a value lesser of all negative is chosen. This fifth column is the pivot column. The entering variable is x_1 with the pivot value of 100. For the leaving variable, the minimum ratio is calculated for the fourth and fifth rows. The minimum ratio is 6 (X_B / x_1 = 600/100) corresponding to the fifth row. Thus, entering variable is x_1 for Table III and the leaving variable is s_1 corresponding to 5th row.

Table II

			c_{1j}	7	3	0	0	-M	
			c_{2j}	10	5	0	0	-M	
C_{B1}	C_{B2}	*Basis*	X_B	x_1	x_2	s_1	s_2	A_1	Ratio
$-M$ 0	$-M$ 0	A_1 s_1	35 600	5 (100)	4 0	0 1	-1 0	1 0	7 6*
$\Delta_{1j} = z_{1j} - c_{1j}$			$-35M$	$-5M-7$	$-4M-3$	0	M	0	
$\Delta_{2j} = z_{2j} - c_{2j}$			$-35M$	$-5M-10$	$-4M-5$	0	0	0	
$\Delta_j = z_j - c_j$			$-70M$	$-10M-17$	$-8M-8$	0	M	0	

Table III: In iteration 2, the value of Δ_j in Table III is $-2M+11/5$, a value lesser of all negative is chosen and is the pivot column. The entering variable is x_2 with the pivot value of 1. For the leaving variable, the minimum ratio is calculated for the fourth and fifth rows. The minimum ratio is 5 (X_B / x_2 = 5/1) corresponding to the fourth row. Entering variable is x_2 and the leaving variable is A_1 corresponding to fifth row.

Table III

			c_{1j}	7	3	0	0	$-M$	
			c_{2j}	10	5	0	0	$-M$	
C_{B1}	C_{B2}	*Basis*	X_B	x_1	x_2	s_1	s_2	A_1	Ratio
$-M$	$-M$	A_1	5	0	(1)	-1/20	-1	1	5*
7	10	x_1	6	1	3/5	1/100	0	0	10
$\Delta_{1j} = z_{1j} - c_{1j}$			$-5M + 42$	0	$-M + 6/5$	$1/20M + 7/100$	M	0	
$\Delta_{2j} = z_{2j} - c_{2j}$			$-5M + 60$	0	$-M + 1$	$1/20M + 10/100$	M	0	
$\Delta_j = z_j - c_j$			$-10M + 102$	0	$-2M + 11/5$	$1/10M + 17/100$	M	0	

Table IV: The final iteration 3 indicates that all the coefficients in last row are positive, so the stop condition is fulfilled and the optimum solution to MOLPP (6) is obtained as indicated in equation (7).

Table IV

			c_{1j}	7	3	0	0
			c_{2j}	10	5	0	0
C_{B1}	C_{B2}	*Basis*	X_B	x_1	x_2	s_1	s_2
3	5	x_2	5	0	1	-1/20	-1
7	10	x_1	3	1	0	1/25	3/5
$\Delta_{1j} = z_{1j} - c_{1j}$			36	0	0	13/100	6/5
$\Delta_{2j} = z_{2j} - c_{2j}$			55	0	0	3/20	1
$\Delta_j = z_j - c_j$			91	0	0	7/25	11/5

Thus, the optimum solution to MOLPP (6) is:

$x_1 = 3$ and $x_2 = 5$ with maximum $Z_1 = 36$ million and maximum $Z_2 = 55$ million. (7)

Although the solution in (7), indicates that Priceler falls 4 million exposures short of meeting the Goal 1 and similarly 5 million exposures short of meeting Goal 2, however. Such results implies that the goal sets in (5) cannot be attained.

4. Application of MOLP Problem

In Section 4, the MOLP is applied to solve a local food processing company known as VitiFoods Limited (Fiji) that specializes in the production of tinned fish sold locally and abroad. Specifically, five different tinned products are produced as indicated in Table V.

Table V: VitiFoods Limited (Fiji) – Tinned Product details

Products	**Name & specification of product**
Product #1	Skipper Blue 425g (flakes)
Product #2	Skipper Blue 170g (flakes)
Product #3	Angel Gold 425g (chunks)
Product #4	Angel Yellow 425g (chunks)
Product #5	Sea King 425g (chunks)

VitiFoods Limited (Fiji) desires to determine the monthly optimum production of tinned products to achieve maximum profit with minimum labour hours. In order to produce a unit of each product, the company provides data based on selling price; cost incurred due to specific labour hours assigned to each product as shown in Table VI. The data, on which the calculations and formulations are made, is based on the month of June 2013.

Table VI: Selling price and Cost incurred per unit of each product

Products	Selling Price	Cost	Labor hours
Skipper Blue 425g (flakes)	\$2.50	\$0.94	0.040
Skipper Blue 170g (flakes)	\$1.10	\$0.85	0.026
Angel Gold 425g (chunks)	\$1.90	\$1.14	0.043
Angel Yellow 425g (chunks)	\$2.20	\$1.33	0.043
Sea King 425g (chunks)	\$1.75	\$0.98	0.043

Let $x_j; (j = 1, 2, ..., 5)$ be the number of units of *j*th product to be produced. Then, the two objectives of the VitiFoods Limited (Fiji) can be written as:

4.1 Objective 1: Maximize Profit

$$\text{Maximize}\,(z_1) = (2.50x_1 - 0.94x_1) + (1.10x_2 - 0.85x_2) + (1.90x_3 - 1.14x_3) + (2.20x_4 - 1.33x_4) + (1.75x_5 - 0.98x_5)$$

$$\text{Maximize}\,(z_1) = 1.56x_1 + 0.25x_2 + 0.76x_3 + 0.87x_4 + 0.77x_5 \quad (8)$$

4.2 Objective 2: Minimize labour hours

$$\text{Minimize}\,(z_2) = 0.040x_1 + 0.026x_2 + 0.043x_3 + 0.043x_4 + 0.043x_5 \quad (9)$$

Considering market demand as one of the various limitations, the company prefers to restrict the availability of raw materials, etc. on the production as depicted in the set of constraints provided below:

4.3 Constraints

i) $x_1 \le 48000$: The initial constraint specifies that the production of Skipper Blue 425g (flakes) that can be limited to 48000 units.

ii) $x_2 \ge 90000$: The second constraint forbids restriction in the production of Skipper Blue 170g (flakes) and would be at least 90,000 units to meet market demand.

iii) $x_3 \ge 39000$: This constraint forbids restriction on the production of Angel Gold 425g (chunk), that can be at least 39000 units.

iv) $x_4 \le 40000$: This implies that at most 40000 units of Angel Yellow 425g (chunk) should be produced.

v) $x_5 \ge 120000$: 120,000 or more units of Sea King 425g (chunk) can be produced to meet market demand.

vi) $x_1 + x_2 \ge 144000$: The constraint indicates that the total production of Skipper Blue 425g and 170g should be summed to 144, 000 units.

vii) $x_3 + x_4 + x_5 \le 220000$: The constraint indicates that the sum of the production of Angel Gold, Angel Yellow and Sea King should be at most 220,000 units.

Thus, from the objectives in (8) and (9) and considering the constraints above, the VitiFoods Limited (Fiji) problem can be formulated as an MOLP as:

$$\text{Maximize } (z_1) = 1.56x_1 + 0.25x_2 + 0.76x_3 + 0.87x_4 + 0.77x_5 \quad \text{and}$$
$$\text{Minimize } (z_2) = 0.040x_1 + 0.026x_2 + 0.043x_3 + 0.043x_4 + 0.043x_5$$
$$\text{subject to} \quad x_1 \le 48000$$
$$x_3 + x_4 + x_5 \le 220000$$
$$x_4 \le 40000$$
$$x_1 + x_2 = 144000$$
$$x_2 \ge 90000$$
$$x_3 \ge 39000$$
$$x_5 \ge 120000$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0 \quad (10)$$

Converting, the objective 2 from minimization to maximization and introducing $s_1 \ge 0$, $s_2 \ge 0$, $s_3 \ge 0$ as slack variables, $s_4 \ge 0$, $s_5 \ge 0$, $s_6 \ge 0$ as surplus variables and $A_1 \ge 0$, $A_2 \ge 0$, $A_3 \ge 0$, $A_4 \ge 0$ as artificial variables, the standard form of the MOLP (10) can be expressed as:

$$\text{Maximize } (z_1) = 1.56x_1 + 0.25x_2 + 0.76x_3 + 0.87x_4 + 0.77x_5 - MA_1 - MA_2 - MA_3 - MA_4$$
$$\text{Maximize } (-z_2) = -0.040x_1 - 0.026x_2 - 0.043x_3 - 0.043x_4 - 0.043x_5 - MA_1 - MA_2 - MA_3 - MA_4$$
$$\text{subject to} \quad x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + 0A_1 + 0A_2 + 0A_3 + 0A_4 = 48000$$
$$0x_1 + 0x_2 + x_3 + x_4 + x_5 + 0s_1 + s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + 0A_1 + 0A_2 + 0A_3 + 0A_4 = 220000$$
$$0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 + 0s_1 + 0s_2 + s_3 + 0s_4 + 0s_5 + 0s_6 + 0A_1 + 0A_2 + 0A_3 + 0A_4 = 40000$$
$$x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + A_1 + 0A_2 + 0A_3 + 0A_4 = 144000$$
$$0x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 + 0s_1 + 0s_2 + 0s_3 - s_4 + 0s_5 + 0s_6 + 0A_1 + A_2 + 0A_3 + 0A_4 = 90000$$
$$0x_1 + 0x_2 + x_3 + 0x_4 + 0x_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 - s_5 + 0s_6 + 0A_1 + 0A_2 + A_3 + 0A_4 = 39000$$
$$0x_1 + 0x_2 + 0x_3 + 0x_4 + x_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 - s_6 + 0A_1 + 0A_2 + 0A_3 + A_4 = 120000$$
$$x_1, x_2, x_3, x_4, x_5, s_1, s_2, s_3, s_4, s_5, s_6, A_1, A_2, A_3, A_4 \ge 0$$

Solving the problem (10) using the proposed simplex technique, the final iterative table obtained is shown in Table VII. The results of Table VII is obtained by iterating the tabulated data of equation (10).

Table VII: In **initial iteration**, the slack variables; $s_1, s_2, s_3,$ surplus variable; $s_4, s_5, s_6,$ and artificial variables; $A_1, A_2, A_3, A_4,$ form the initial basis for first tabulation. In **iteration 1**, value of Δ_j is $-2M - 0.224$ and minimum ratio is calculated to be 90000 (X_B / x_2 = 90000/1) corresponding to the eighth row. The leaving variable is A_2 for first tabulation and entering variable is x_2 for second tabulation. In **iteration 2,** Δ_j is $-M - 1.52$ and the entering variable is x_1 with the pivot value of 1 in fifth column. Minimum ratio is calculated to be 48000 (X_B / x_1 = 48000/1). Leaving variable is A_1 for second tabulation and the entering variable is x_1 for third tabulation. In **iteration 3,** Δ_j is $-M - 0.727$. This nineth column is called the pivot column with value of 1. Minimum ratio is calculated to be 120000 (X_B / x_5 = 120000/1) corresponding to the eleventh row. Therefore, the leaving variable is A_4 for third tabulation and the entering variable is x_5 for fourth tabulation. The value of Δ_j in **iteration 4** is $-M - 0.717$ and the entering variable is x_3 with the pivot value of 1. Minimum ratio is calculated to be 39000 (X_B / x_3 = 39000/1) corresponding to the nineth row. The leaving variable is A_3 for fourth tabulation and the entering variable is x_3 for fifth tabulation. In **iteration 5**, the value of Δ_j is $-M - 0.25.$ Minimum ratio is 6000. The leaving variable is A_1 for fifth tabulation and the entering variable is s_4 for sixth tabulation. In **iteration 6,** Δ_j is 0, minimum ratio is calculated to be 40000. The leaving variable is s_2 for sixth tabulation and the entering variable is x_4 for seventh tabulation. At **iteration 7**, the value of Δ_j is -0.727, minimum ratio is calculated to be 21000. The leaving variable is s_3 for seventh tabulation and the entering variable is s_6 for eighth tabulation. In the eighth tabulation, the coefficients are positive and this is indicated in Table VII, so the stop condition is fulfilled and the optimum solution to MOLPP is obtained for equation (10).

Table VII

			c_{1j}	1.56	0.25	0.76	0.87	0.77	0	0	0	0	0	0
			c_{2j}	-0.040	-0.026	-0.043	-0.043	-0.043	0	0	0	0	0	0
C_{B2}	C_{B2}	*Basis*	X_B	x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	s_6
1.56	-0.040	x_1	48000	1	0	0	0	0	1	0	0	0	0	0
0.87	-0.043	x_4	40000	0	0	0	1	0	0	1	0	0	0	0
0	0	s_6	21000	0	0	1	1	1	0	0	1	0	0	0
0	0	s_4	6000	1	1	0	0	0	0	0	0	0	0	0
0.25	-0.026	x_2	96000	0	1	0	0	0	0	0	0	-1	0	1
0.76	-0.043	x_3	141000	0	0	1	0	0	0	0	0	0	-1	0
0.77	-0.043	x_5	120000	0	0	0	0	1	0	0	0	0	0	-1
$\Delta_{1j} = z_{1j} - c_{1j}$			271890	0	0	0	0	0	1.31	0.1	0.77	0	0.01	0
$\Delta_{2j} = z_{2j} - c_{2j}$			-13876	0	0	0	0	0	- 0.014	0.1	0.77	0	0.01	0
$\Delta_j = z_j - c_j$			258014	0	0	0	0	0	1.296	0.1	0.727	0	0.01	0

Thus, the optimum solution is $x_1 = 48000,\ x_2 = 96000,\ x_3 = 141000,\ x_4 = 40000, x_5 = 120000$ with maximum $z_1 = 271890$ and minimum $z_2 = 13876$.

5. A Comparative Study and Discussion

An investigation is accomplished in this section to compare the efficiency of the proposed multi-objective simplex method to that of a preemptive goal programming approach - a well-known multi-objective optimization method (see Section 4.16, [13]). For this purpose, we use the numerical example discussed in Section 3 and its results presented in equation (7) obtained by the proposed method.

5.1 Preemptive Goal Programming Approach:

The Preemptive Goal Programming (PGP) approach compels the decision maker to rank goals into different priority levels, specifically, 1, 2, 3 etc. Thus, to apply PGP, the decision maker must prioritize the goals according to its importance levels i.e., from most important (goal 1) to least important (goal *p*), that is:

$$G_1 = P_1 > G_2 = P_2 > \cdots > G_p = P_n$$

Clearly, the weight for goal 1 is larger than the weight for goal 2 and so forth. The definition of $P_1,\ P_2,\ \ldots P_p$ assumes that the decision maker first tries to satisfy the goal 1. Then, amidst all points that fulfills goal 1, the decision maker attempts to come as precise as possible to satisfying goal 2, and so forth. The coefficient of the objective function for the variable representing goal i will be P_i. Use of the preemptive goal programming approach to solve Example 3.1, the following deviation variables is defined corresponding to the two goals given in (5):

s_i^+ = amount by which we numerically exceed the ith goal

s_i^- = amount by which we are numerically under the ith goal

Then, the goals in (5) can be expressed as:

$$7x_1 + 3x_1 + s_1^- - s_1^+ = 40$$
$$10x_1 + 5x_2 + s_2^- - s_2^+ = 60 \qquad (11)$$

We assume that each s_i^+ and s_i^- is measured in millions of exposures. If Priceler determined that the HIM goal was more important that the LIP goal, the preemptive goal programming formulation for the problem (6) is obtained by replacing the objective functions by $P_1 s_1^- + P_2 s_2^-$. Thus, using the deviational variables, we can express the problem as the PGP problem as:

$$\begin{aligned} &\text{Minimize } (Z) = P_1 s_1^- + P_2 s_2^- \\ &\text{subject to} \quad 7x_1 + 3x_2 + s_1^- - s_1^+ = 40 \\ &\qquad 10x_1 + 5x_2 + s_2^- - s_2^+ = 60 \\ &\qquad 5x_1 + 4x_2 \geq 35 \\ &\qquad 100x_1 + 60x_2 \leq 600 \\ &\qquad x_1, x_2 \geq 0 \end{aligned} \qquad (12)$$

In order to apply the preemptive goal programming technique in (12), the objective function must be separated into 2 components:

$$z_1 = P_1 s_1^-$$
$$\text{and} \quad z_2 = P_2 s_2^-$$

Then the PGP problem (12) can be solved by an extension of the simplex method, known as **goal programming simplex**, for which we compute 2 row 0's with the ith row 0 corresponding to goal i. Thus, we have:

$$\text{Row 0 (goal 1)}: z_1 - P_1 s_1^- = 0$$
$$\text{Row 0 (goal 2)}: z_2 - P_2 s_2^- = 0 \qquad (13)$$

If s_3 is the slack variable for the HIM constraint, s_4 and s_5 are the surplus and artificial variables respectively for the budget constraint, then $\{s_1^-, s_2^-, s_3, s_4, s_5\}$ is a starting basic feasible solution that could be used to solve (12) via the goal programming simplex algorithm. As with the regular simplex, we must first eliminate all variables in the starting basis from each row 0.

Adding P_1 (HIM constraint) to row 0 (goal 1) yields:

$$\text{Row 0 (goal 1)}: z_1 + 7P_1 x_1 + 3P_1 x_1 - P_1 s_1^+ = 40P_1 \qquad \text{(HIM)}$$

Adding P_2 (LIP constraint) to row 0 (goal 2) yields:

$$\text{Row 0 (goal 2)}: z_2 + 10P_2 x_1 + 5P_2 x_2 - P_2 s_2^+ = 60P_2 \qquad \text{(LIP)}$$

Then, using the goal programming simplex, the PGP (12) is solved (see Appendix) and the optimum solution is: x_1=3 and $x_2 = 5$ with the values of the goals that can be achieved are:

(i) Goal 1: $7x_1 + 3x_2 \Rightarrow 7(3) + 3(5) = 21 + 15 = 36$ million

(ii) Goal 2: $10x_1 + 5x_2 \Rightarrow 10(3) + 5(5) = 30 + 35 = 55$ million (14)

Thus, the results in (7) and (14) shows that the proposed method generates the identical results as those shown by the PGP approach.

Nonetheless, it can be noticed that the PGP technique consists of deviational variables for each individual goal. As the additional number of goals increases, so does the number of deviational variables. Such method requires additional columns in the simplex table. These extra columns are for positive and negative deviational variables. Furthermore, the method requires distinct objective function rows for every priority level. Thus, the introduction of additional columns for deviational variables and rows for priority levels increments immeasurably the computational time to solve MOLP problem using PGP, while in contrast, this is not the case in the proposed multi-objective simplex method as this new method keeps the number of constraints constant. In addition, no additional row and column is introduced regardless of the number of goals/objectives. Thus, MOLP problem can be solved with a reduced computational effort utilizing the proposed technique, in comparison with the preemptive goal programming technique.

6. Conclusion

Multi-objective optimization is a very significant topic in linear programming problems. This technique is broadly utilized in majority real-life situations as decision makers often light upon optimizing several functions simultaneously. There are several techniques mentioned by authors that deal with the MOLP problem. Some techniques are commonly used in MOLP and generally non-classical. These techniques belong to methods including genetic algorithm or class of evolutionary methods. The major difficulty in applying some of these methods is that if the number of objective functions is relatively big, then the computational exertion required to generate an efficient set of solutions is prohibitive. Moreover, there is no assurance in detecting an optimal solution within a finite amount of time. An additional drawback is that the population tends to converge to solutions which are higher ranked in one objective function, against to other objective functions.

In this article, a technique is developed for using the multi-objective simplex algorithm to solve MOLP problems. A new simplex tableau with additional rows and columns to cater for multiple objectives is created. Real life application is illustrated with computational details. In addition, a discussion on comparative study is also outlined. In the proposed simplex based multi-objective method, the number of constraints remains constant and no deviational variables are introduced. Thus, reduced computational effort is required to solve a MOLP problem as compared to other goal programming techniques, especially the PGP method. Comparative study along with discussion evaluate the efficiency of the proposed multi-objective simplex method.

7. References

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8. Appendix

The problem (12) can be solved by the goal programming simplex as discussed below:

Iteration 0: Initial tableau

The goal programming simplex requires *n* row 0's (one for each goal). Thus, the initial tableau is designed as follows:

The row with 0's is itemized in order of the goals priorities. The rows 2 and 3 consist of goals 1 and 2 respectively. The deviational variables $s_1^+, s_2^+, s_1^-, s_2^-$ and slack variables s_3, s_4, s_5 form the initial basis. The current basic feasible solution is $s_1^- = 40$, $s_2^- = 60$, $s_5 = 35$ and $s_3 = 600$. Because $z_1 = 40P_1$, goal 1 is not satisfied. To diminish the penalty associated with not meeting goal 1, we enter the variable with the most positive coefficient (x_1) in row 0 (HIM). Considering the x_1 column as pivotal column, the RHS of the constant is divided by the x_1 values. The minimum ratio is 40/7, which is corresponding to s_1^-. Thus, s_1^- is the leaving variable.

Table A1: Initial Tableau for PGP

	x_1	x_2	s_1^+	s_2^+	s_1^-	s_2^-	s_3	s_4	s_5	RHS		
Row 0 (goal 1)	$7P_1$	$3P_1$	$-P_1$	0	0	0	0	0	0	$z_1 = 40P_1$	X_B	Ratio
Row 0 (goal 2)	$10P_2$	$5P_2$	0	$-P_2$	0	0	0	0	0	$z_2 = 60P_2$		
HIM	7	3	−1	0	1	0	0	0	0	40	s_1^-	$40/7^*$
LIP	10	5	0	−1	0	1	0	0	0	60	s_2^-	6
HIW	5	4	0	0	0	0	0	−1	1	35	s_5	7
Budget	100	60	0	0	0	0	1	0	0	600	s_3	6

Iteration 1: First Tableau

After entering x_1 into the basis, we obtain Table A2. The current basic feasible solution is $x_1 = \frac{40}{7}$, $s_2^- = \frac{20}{7}$, $s_5 = \frac{45}{7}$, and $s_3 = \frac{200}{7}$. Since $s_1^- = 0$ and $z_1 = 0$, goal 1 is now satisfied. The variable with the most positive coefficient in row 0 is x_2. Now, to satisfy goal 2 while ascertaining that the higher priority goal is still satisfied. Entering x_2 into the basis will not increase *z*, because the coefficient of x_2 in row 0 (HIM is 0). Thus, after entering x_2 in the basis, goal 1 will still be satisfied. The ratio test is done considering column of x_2. The constraints of RHS is divided with the values of x_2. We arbitrarily choose to enter x_2 into the basis in the Budget constraint. Thus, the leaving variable will be s_3 and x_2 will be the entering variable.

Table A2: First Tableau for PGP

	x_1	x_2	s_1^+	s_2^+	s_1^-	s_2^-	s_3	s_4	s_5	RHS		
Row 0 (goal 1)	0	0	0	0	$-P_1$	0	0	0	0	$z_1 = 0$	X_B	Ratio
Row 0 (goal 2)	0	$5/7P_2$	$10/7P_2$	$-P_2$	$-10/7P_2$	0	0	0	0	$z_2 = 20/7P_2$		
HIM	1	$3/7$	$-1/7$	0	$1/7$	0	0	0	0	$40/7$	x_1	$40/7$
LIP	0	$5/7$	$10/7$	-1	$-10/7$	1	0	0	0	$20/7$	s_2^-	4
HIW	0	$13/7$	$5/7$	0	$-5/7$	0	0	-1	1	$45/7$	s_5	$45/13$
Budget	0	$120/7$	$100/7$	0	$-100/7$	0	1	0	0	$200/7$	s_3	$5/3^*$

Iteration 2: Second Tableau

After pivoting x_2 into the basis, we obtain Table A3. Since z=0, goal 1 is satisfied, however, goal 2 is still not satisfied. The current basic feasible solution is $x_1 = 5$, $s_2^- = \frac{5}{3}$, $s_5 = \frac{10}{3}$ and $x_2 = \frac{5}{3}$. Because s_2^- is the only variable with a positive coefficient in row 6 (HIW), the only way to come closer to meeting goal 2 is to enter s_1^- in the basis. The RHS is divided by the pivot values of the s_1 column, so s_5 will be the leaving variable.

Table A3: Second Tableau for PGP

	x_1	x_2	s_1^+	s_2^+	s_1^-	s_2^-	s_3	s_4	s_5	RHS		
Row 0 (goal 1)	0	0	0	0	$-P_1$	0	0	0	0	$z_1 = 0$	X_B	Ratio
Row 0 (goal 2)	0	0	$5/6P_2$	$-P_2$	$-5/6P_2$	0	$-1/24P_2$	0	0	$z_2 = 5/3P_2$		
HIM	1	0	$-1/2$	0	$1/2$	0	$-1/40$	0	0	5	x_1	$9/2$
LIP	0	0	$5/6$	-1	$-5/6$	1	$-1/24$	0	0	$5/3$	s_2^-	-
HIW	0	0	$-5/6$	0	$5/6$	0	$-13/120$	-1	1	$10/3$	s_5	4^*
Budget	0	1	$5/6$	0	$-5/6$	0	$7/120$	0	0	$5/3$	x_2	-

Iteration 3: Optimum Tableau for PGP

The pivot row and other row values are updated in Table A4 below.

Table A4: Optimum Tableau for PGP

	x_1	x_2	s_1^+	s_2^+	s_1^-	s_2^-	s_3	s_4	s_5	RHS	
Row 0 (goal 1)	0	0	$-P_1$	0	0	0	$-13/100P_1$	$-6/5P_1$	$6/5P_1$	$z_1 = 4P_1$	X_B
Row 0 (goal 2)	0	P_2	0	$-P_2$	0	0	$-3/20P_2$	$-P_2$	P_2	$z_2 = 5P_2$	
HIM	1	0	0	0	0	0	$73/200$	$3/5$	$-3/5$	3	x_1
LIP	0	0	0	-1	0	1	$-3/20$	-1	1	5	s_2^-
HIW	0	0	-1	0	1	0	$-13/100$	$-6/5$	$6/5$	4	s_1^-
Budget	0	1	0	0	0	0	$-1/20$	1	1	5	x_2

Since $z_1 \geq 0$ and $z_2 \geq 0$ the optimum solution is: x_1 =3 and $x_2 = 5$ with the values of the goals that can be achieved are:

(i) Goal 1: $7x_1 + 3x_2 \Rightarrow 7(3) + 3(5) = 21 + 15 = 36$ million

(ii) Goal 2: $10x_1 + 5x_2 \Rightarrow 10(3) + 5(5) = 30 + 35 = 55$ million