Eng. Sci., 2024, **31**, 1275

Engineered Science
DOI: https://dx.doi.org/10.30919/es1275

New Strategy Aiding the Motion Control of a Standard *n*-Trailer System in a Constrained Environment

Avinesh Prasad,[1] Sandeep A. Kumar[1,*] and Ravinesh Chand[1, 2]

Abstract

The motion planning and control of a tractor-trailer system is a complex and computationally intensive problem due to the coupling of the tractor robot and the trailer(s) which are all non-holonomic in nature. This paper proposes a new solution via the Lyapunov-based control scheme to control the motion of a standard *n*-trailer system in an obstacle-ridden workspace. The *n*-trailer system is comprised of a rear wheel driven car-like robot which is hitched to a series of multiple passive trailers. A constrained workspace cluttered with fixed elliptical and line obstacles of random sizes and positions is considered. For the avoidance of the obstacles, a new strategy of enclosing each body of the *n*-trailer system in rectangular regions is presented. The strategy helps maximize free-space to allow the robot to enter narrow passages or maneuver in between neighboring obstacles. The system singularities and bounds on velocities are considered as artificial obstacles for which the respective avoidance functions are constructed. Attractive and repulsive potential functions, which are part of a Lyapunov function, are designed and the control inputs are extracted using the Lyapunov-based control scheme. Stability analysis is carried out using the direct method of Lyapunov and the result is verified numerically via computer simulations.

Keywords: Standard *n*-trailer system; Lyapunov-based control scheme; Minimum distance technique; Stability.
Received: 11 March 2024; Revised: 18 September 2024; Accepted: 08 October 2024.
Article type: Research article.

1. Introduction

The development of advanced robotics technologies, particularly artificial intelligence and machine learning, has opened up new possibilities for the use of robots in new areas of applications. Many tasks which can be difficult or dangerous for humans or tasks that are repetitive or require high precision can easily be performed by robots without the need for human labor.[1-4] Robots have come a long way since their inception in the mid-twentieth century with the application of automated robots now expanding across various industries, and transportation is no exception. The advances in technology have resulted in mobile robots playing an increasingly important role in the transportation industry by improving efficiency, safety, and sustainability.[4] Mobile robots are revolutionizing the modern-day transportation system with their ability to navigate autonomously complete tasks with precision and thus, are successfully applied as autonomous vehicles,[5] delivery robots,[6] material handling robots,[7] and maintenance robots,[8] to mention some.

One such innovation under mobile robots is the trailer robotic system, a front wheel steerable car-like autonomous vehicle that is specifically designed for hauling and towing trailers in industrial settings. The demand for trailer robots has been on the rise in recent years, as they offer several advantages over traditional methods of trailer transport.[5] Apart from improving safety by reducing the risk of accidents and injuries associated with manual trailer movement, the ability to operate autonomously for increased productivity has led to increased use of trailer robots in modern industrial settings. Some of the recent and significant multi-factor applications of trailer systems that have become quite popular in many developed countries include transporting goods within warehouses and distribution centres, moving raw materials and finished products between production lines in the manufacturing sector,[6] and transporting crops and machinery around farms in the agriculture sector.[9]

Designing autonomous vehicles that can tow trailers without the need for a human driver is a vital component of the development of robotic trailer systems, which can be broadly categorized as active and passive systems. Passive trailer robotic systems, typically used for material handling,

[1] *School of Information Technology, Engineering, Mathematics & Physics, The University of the South Pacific, Suva 1168, Fiji.*
[2] *School of Mathematical & Computing Sciences, Fiji National University, Suva 3722, Fiji.*
**Email:* sandeep.kumar@usp.ac.fj (S. A. Kumar)

consist of a tow vehicle and a passive trailer that depend on sensors and motion control algorithms to detect the position and orientation of the trailer and to navigate it into the correct position for hitching or Unhitching.[9] However, it has always been a challenging task to design an autonomous passive trailer system because the maneuverability of the trailers depends on the movement of the tractor.[10] This can restrict the motion of the passive trailer system along sharp bends, narrow passages, narrow roads and even bridges. On the other hand, such problems may not be encountered by an active trailer system where the trailers are steerable. However, the overall implementation and operational cost of passive trailers is low when compared to active trailers, therefore transportation industries prefer passive trailer systems to save time and energy.[10]

One approach to developing trailer robotic systems is to use artificial intelligence (AI) and motion control algorithms to enable the vehicle to navigate constrained environments. This involves using sensors such as cameras, lidar, and radar to detect obstacles and track the position of the trailers. According to Dong *et al.*,[11] a key challenge faced by researchers in developing tractor-trailer systems is ensuring that the robot and its trailers comply with safety regulations and standards for operation in dynamic environments. Due to the amalgamation of the tractor robot and the respective trailers, the motion control problem of the tractor-trailer system is seen to be complex and computationally intensive.[11,12] The holonomic and nonholonomic constraints that are inherently present in the system make its kinematics complicated, nonlinear and highly underactuated.[10] Furthermore, the motion of the tractor-trailer robot is restricted because of the dynamic constraints associated with the system.[13] The bounds on the velocities, the limitations on steering and bending angles give rise to the dynamic constraints which can be difficult to incorporate into the control scheme. Moreover, the motion control problem is more challenging if the robot has to avoid collisions with static or dynamic obstacles along its path.[14-16]

Various techniques and strategies have been developed to solve the motion planning and control problem of trailer robotic systems, with the research focus mainly on addressing autonomous control of 1-trailer, 2-trailer, and 3-trailer systems. The development of trailer robots from a single trailer to multiple trailers has been driven by the need for greater flexibility and efficiency in large-scale material handling and transportation applications. Multi-trailer robots consist of multiple linked bodies arranged in a trailer-like configuration and provide greater flexibility in the types of tasks the robot can perform, as the trailer configuration allows for a wider range of motion and maneuverability. Despite the complexity of the problem and challenges faced in controlling the motion of multiple-trailer mobile robots when compared to multiple individual mobile robots, there have been many research achievements in the design and implementation of motion control methods of n-trailer mobile robotic systems. Some of the approaches to solving the find-path problem of multi-trailer systems given in literature include Virtual Link Tracking Method (VLTM),[17] H-infinity control approach,[18] Modified Transpose Jacobian (MTJ) control method,[19,20] fuzzy control approach,[21] and neural networks.[5] Some of the recent work to address the motion control on-trailer systems include Deng *et al.*'s distributed three node service scheduling algorithm,[22] Zhao *et al.*'s configuration estimation and trajectory planning algorithms,[23] Bertolani *et al.*'s adaptive inner-outer kinematic and dynamic controllers,[24] and Zhao *et al.*'s two-tier curvature-based path-tracking controller.[25] Moreover, while the control problem of a 1-trailer, 2-trailer and 3-trailer systems are solved using Lyapunov based control scheme (LbCS),[12,13,26,27] there is no general solution (using LbCS) proposed for a n-trailer system.

Due to its nature, simplicity, elegance and its ability to easily incorporate system singularities, limitations, inequalities, bounds and restrictions, this research utilizes LbCS to solve the find-path problem of an n-trailer system. The LbCS, which is a type of artificial potential field method, has recently become a popular method and has been utilized by many researchers to solve the motion control of various robotic systems ranging from a simple point mass to a complex 3-dimensional robot.[2,13,15,28-31]

In addition, the motion planning and control are integrated into one set of control inputs, thus LbCS eliminates the problems associated with path tracking which is seen in many other methods. Despite such advantages, one major drawback of LbCS is that the extracted control inputs can lead the system to a local minima trap, implying that the robot can be trapped behind an obstacle. However, this local minima issue can be avoided by choosing suitable initial conditions such that the robot's initial and goal positions and the obstacle positions are not collinear.[32] Given the significant advantages of LbCS which supersedes its drawback, this research utilizes the LbCS to develop a set of stabilizing nonlinear, time-invariant, acceleration-based, continuous control laws to navigate a standard n-trailer robot from an initial position to a goal position in a bounded 2D workspace, whilst avoiding fixed obstacles on its route and simultaneously obeying system constraints and singularities. The major contributions of this article are:

1. Design of acceleration-based controllers using LbCS to achieve a collision-free motion of the standard tractor-trailer robot which works for any arbitrary number of trailers. To the knowledge of the authors, while the motion planning and control problem of n-trailer systems has been addressed using several control algorithms,[5,17,18,20,21] LbCS as an approach has not been considered yet. In comparison to techniques applied to n-trailer systems such as H-infinity control,[18] which is computationally extensive and Virtual Link Tracking Method (VLTM),[17] which has limited applicability to complex robotic systems, LbCS is inherently robust and provides stability guarantees, making it more suitable for controlling the complex nonlinear n-trailer system considered in this research.

2. The use of elliptic and line obstacles that mimic real-life situations as most objects can be enclosed or represented by ellipses and lines. This strategy provides the *n*-trailer system with a more realistic representation of the obstacles (such as narrow passages, bridges, buildings, parking bays, and road lanes) in real-life situations for better navigation in complex environments and avoids collisions. In contrast, the fixed disk-shaped obstacles utilized in studies such as Refs. [13] and [27] are a very simple representation of obstacles as they fail to accurately capture the true shape of obstacles encountered by the robot in the environment.

3. A new strategy of enclosing each body of the n-trailer system in rectangular protective regions for the avoidance of obstacles. This strategy helps maximize free space to allow the robot to enter narrow passages or maneuver in between neighbouring obstacles. While the authors of Refs. [13] and [27] attempted to minimize the obstacle space by enclosing each body of the trailer system vehicle within separate protective circular regions to reduce the unnecessary growth of the obstacle space in their approaches, this research, for the first time, maximizes the free-space in the workspace by enclosing each component of the *n*-trailer system in rectangular regions. Furthermore, the minimum distance technique is utilized for the avoidance of obstacles for constructing repulsive potential functions for obstacle avoidance.

The remainder of the paper is organized as follows: A review of literature on tractor-trailer robots is provided in Section 2. Then, Section 3 presents the vehicle model where the dynamic equations that describes the motion of a standard *n*-trailer robot is given. Section 4 describes the motion planning and control of the robotic system and the respective attractive and repulsive functions are designed. Section 5 proposes a tentative Lyapunov function from which the control laws are extracted using the Lyapunov-based control scheme. Stability of the system is analysed in Section 6. In Section 7, the simulation results of interesting scenarios are presented. Finally, Sections 8 and 9 give a concluding remark and lists future work along this research area.

2. Literature review

Tractor-trailer robots are a subcategory of wheeled mobile robots with numerous applications in autonomous public transportation, material handling, and manufacturing tasks. Due to their vast range of benefits, such as optimal design and maneuverability in narrow passages, trailer systems have greater significance than single-unit mobile robots. This literature survey provides an overview of the different types of trailer systems that have been designed and implemented with their specific control algorithms, whereby an outline of developments from 1-trailer, 2-trailer, and 3-trailer to *n*-trailer systems is discussed.

2.1 Single-trailer system

In recent years, factors such as design simplicity and costs have prompted researchers to design single-trailer systems. Single trailer robots were the first generation of trailer robots and were primarily used for simple tasks such as moving materials from one location to another. These robots typically had limited maneuverability and could only transport a single load at a time. In 2013, Khalaji *et al*.[33] proposed feedback linearizing dynamic controller (FLDC) to control a nonholonomic tractor-trailer mobile robot using estimated upper bounds of uncertainties. Although the authors demonstrated the control of the 1-trailer system at a dynamic level in the presence of parameter uncertainties and external disturbances, the selection of inappropriate values for adaptive controller gains resulted in diminishing the robustness of the controller, which, in turn, violated the saturation limits of the actuators. Lashkari *et al*.[7] in 2016 designed a tractor-trailer omnidirectional type robot system with on-axle hitching for transporting heavy loads. The proposed sliding mode controller was developed from three meta-heuristic optimization techniques, namely Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Simulated Annealing (SA), while an optimal selection of control design parameters was required to attain desired results. Although the desired trajectories were obtained by the controller of the 1-trailer system, the proposed methodology could have been extended to include *n*-trailers for practical use in transportation.

2.2 Double-trailer system

The development of two-trailer robots allowed for greater flexibility and maneuverability, as the two trailers could be steered independently of each other. For instance, a 2-trailer robotic system comprising a mobile robot and two on-axle hitched passive trailers was presented in 2015,[26] where the desired motion of the system was achieved from a feedback control law directly derived from a designed Lyapunov function. Recently, in 2021, Bertolani *et al*.[9] presented a unicycle-like mobile platform subjected to pushing or pulling two passive trailers connected to it, mimicking an application in precision agriculture involving material transportation. To guide the movement of the robot in either pushing or pulling its trailers when required, a kinematic cascade controller, also known as an inner-outer loop control system was utilised. However, the kinematic controllers which were velocity-based led to producing undesirable errors which resulted in unfavourable oscillations during navigation. Nonetheless, the development of double-trailer robotic systems marked a significant advancement in robotic mobility, allowing for enhanced flexibility and maneuverability in various sectors, including transportation and agriculture.

2.3 Triple-trailer system

Three-trailer robots provided even greater flexibility and efficiency, as the additional trailer allowed for more materials to be transported in a single trip. These robots were particularly useful in applications where space was limited or where there was a need to transport large quantities of materials over long distances. Tanaka *et al*. in 2008,[21] presented the motion planning of an autonomous 3-trailer vehicle system based on sensor reduction using a linear-matrix-inequality (LMI) approach to design stable fuzzy controllers. The authors demonstrated that the LMI-based design achieved the backing-up control of the 3-trailer system while avoiding the jackknife phenomenon. Later in 2022, Leu *et al*.[5] attempted to address the challenges in planning and control of a standard 3-trailer system with a car-like tractor using an improved A-Search Guided Tree algorithm. The authors trained a neural network through reinforcement learning to model the maneuver costs of the trailers and used it as the heuristic value to better approximate the system's efficiency. While the desired results and trajectories were obtained in static environments, the approach needs to be implemented for reactive planning in dynamic environments. The development of triple-trailer robots further enhanced transportation efficiency, allowing for the transportation of larger loads, particularly in confined spaces or over long distances. The search for better motion control approaches indicates significant advancements, though further research is needed for application in dynamic environments.

2.4 Multi-trailer system

Technology advancements in automation and control have made it possible to develop multiple trailer robots. These robots are capable of coordinating the motion of multiple trailers and can be programmed to operate autonomously, allowing them to perform complex tasks with minimal human intervention. In Ref. [17], the motion control of a robot with n passive trailers using a backward Virtual Link Tracking Method (VLTM) motion controller was demonstrated. The challenging problem of backward motion control was addressed by providing a reference trajectory to the nth trailer which was subsequently followed by the rest of the attached trailers in the system. Although the motion of the passive multiple trailer system could be controlled in both directions, more precise and smoother motion control could have been achieved if acceleration-based controllers were used instead of velocity-based motion controllers. In 2022, Rigatos *et al*.[18] proposed a nonlinear optimal H-infinity control approach as an optimal solution to the control problem of a truck and n-trailer mobile robot. The control scheme was successfully implemented despite being computationally extensive as the approach involved approximating linearization around a temporary operating point, first-order Taylor series expansion, computation of the associated Jacobian matrices, and solving an algebraic Riccati equation at each time-step of the method. These studies indicate that advancements in automation and control have enabled the development of sophisticated multi-trailer robots capable of autonomously coordinating multiple trailers for complex tasks. The development of VLTM and H-infinity control approaches for complex multi-trailer operations can potentially enhance real-life applications such as autonomous logistics and transportation.

2.5 Control algorithms for trailer systems

Several algorithms have been proposed in the literature for the motion control of trailer systems. One of the simplest algorithms used to control the motion of tractor-trailer robotic systems is the Transposed Jacobian (TJ) control utilised in Ref. [19], which however, had drawbacks because the heuristic selection of control parameters and controller gains was done without any formal process. As a result, the TJ algorithm encountered problems in obtaining desired trajectories for task accomplishment. Inspired by the poor controller performance of the TJ algorithm, Khalaji *et al*.[20] in 2015 presented a Non-Model-Based (NMB) algorithm based on the Modified Transpose Jacobian (MTJ) control method to autonomously steer a multi-trailer wheeled robot asymptotically while following reference trajectories. Various algorithms have been explored for the motion control of trailer systems, including the TJ and NMB algorithms. These advancements enhance the autonomous navigation of multi-trailer robots by improving control precision and trajectory following in complex scenarios.

2.6 Motion planning and control challenges

The complex motion control requirements of trailer systems due to their nonholonomic constraints and the need to coordinate multiple trailers implies that effective motion planning and control algorithms must be designed to acquire successful navigation for task accomplishment. Due to easy implementation, Lyapunov techniques have been regarded as a powerful tool for analyzing and controlling the stability of trailer robotic systems. In 2004, Astolfi *et al*.[12] designed a Lyapunov-based controller to demonstrate the lateral stability and motion of a tractor-trailer vehicle along a desired geometric path. Although the only control variable used was the steering angle of the tractor's front wheels, the authors managed to account for the limitations arising from a saturation of the steering angle. Then, Raj *et al*.[27] in 2016 proposed a new solution to motion planning and control problem for a flock of 1-trailer systems via LbCS for the avoidance of swarm of boids and attraction to their designated targets. Later in 2019, Zhou *et al*.[34] presented a path following framework for a n-trailer systems for applications in Global Positioning System (GPS)-denied environments. While the authors proposed a new path following controller and used onboard sensors for navigation, the Lyapunov method was only utilised to prove the stability of the estimator and the controller. Moreover, the navigation of a system of 1-trailer robots in a dynamic environment cluttered with obstacles, including a swarm of boids, was studied just recently by Raj *et al*.[13] in 2021. In their research, LbCS was used to derive a set of continuous nonlinear controllers for avoiding collision with obstacles in a dynamic workspace.

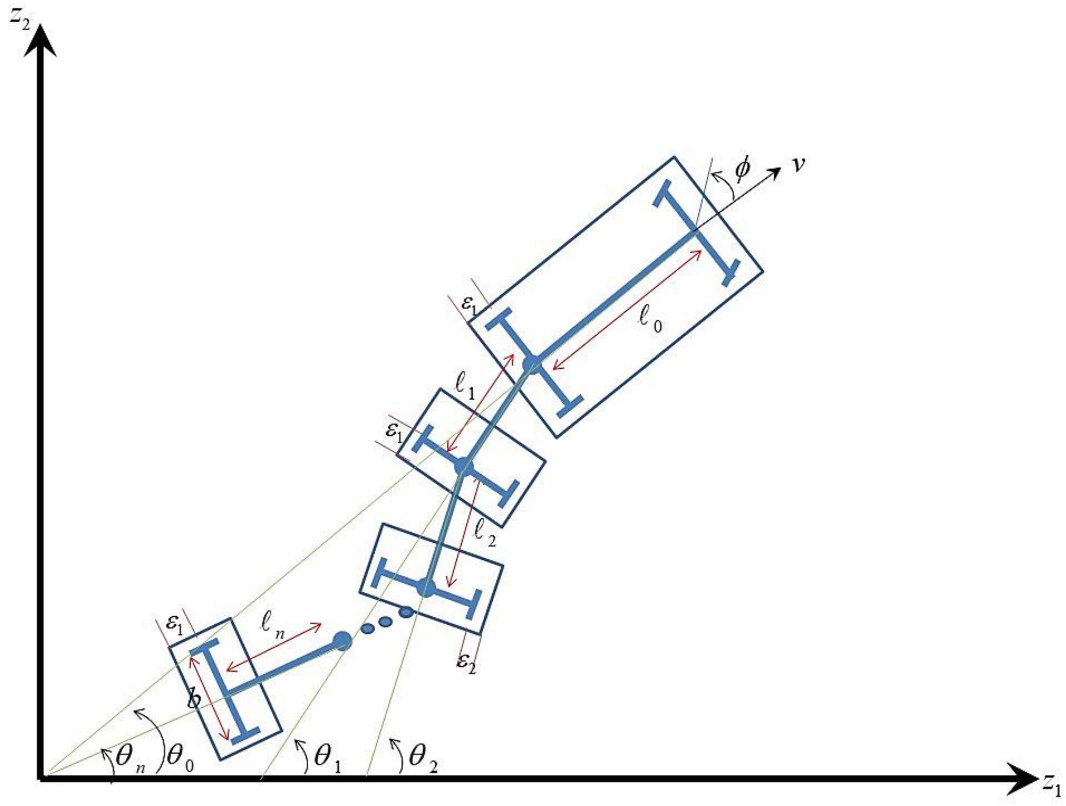

Fig. 1 The schematic representation of a standard *n*-trailer robot.

2.7 Addressing the research gap

This literature survey reveals that the motion planning and control problem of a 1-trailer, 2-trailer and 3-trailer systems are solved using various effective algorithms, including LbCS. Also, the motion control problem of the *n*-trailer system has been addressed using several control schemes as discussed above. However, due to the complexity of the motion control problem and the complicated kinematics structure of multiple trailers robotic systems, no general solution has been proposed for *n*-trailer systems using LbCS. In this research, the collision-free motion control problem of a standard *n*-trailer system, which is seen to be a challenging problem in literature, is addressed using LbCS in an obstacle-ridden bounded workspace.

3. The Standard *n*-trailer model

Consider a standard tractor-trailer robot which is comprised of a rear wheel driven car-like vehicle hitched to a series of multiple passive trailers as shown in Fig. 1.

Referring to Fig. 1, (x_0, y_0) represents the cartesian coordinates of the tractor robot, θ_0 gives its orientation with respect to the z_1-axis, while ϕ gives the steering angle with respect to its longitudinal axis. Similarly, (x_i, y_i) represents the cartesian coordinates of the *i*th passive trailer while θ_i gives its orientations with respect to the z_1-axis. Furthermore, ϵ_1 and ϵ_2 are the clearance parameters.

Letting ℓ_0 and ℓ_1 be the lengths of the mid-axle of the tractor and *i*th trailer, respectively, the dynamic model of a standard *n*-trailer system, adapted from Ref. [35], is given by

$$\left.\begin{array}{c} \dot{x}_0 = v\cos\theta_0 - \frac{\ell_0}{2} w \sin\theta_0\,, \\ \dot{y}_0 = v\sin\theta_0 + \frac{\ell_0}{2} w \cos\theta_0\,, \\ \dot{\theta}_0 = \frac{v}{\ell_0}\tan\phi := w, \\ \dot{\theta}_i = \frac{v}{\ell_i}\sin(\theta_{i-1}-\theta_i)\prod_{k=1}^{i-1}\cos(\theta_{k-1}-\theta_k)\,, \\ \dot{v} = \sigma_1, \\ \dot{w} = \sigma_2, \end{array}\right\} \tag{1}$$

where v and w are its translational and rotational velocities of the tractor robot while σ_1 and σ_2 are the translational and rotational accelerations. Note that since we are considering a passive trailer, there is no need to impose any additional controller for θ_i, hence the only controllers in system (1) are σ_1 and σ_2. However, we need to observe any mechanical singularities associated with the movement of the tractor-trailer body and each trailer must avoid collision with every obstacle in the path.

We shall use the vector notation $\mathbf{x} = (x_0, y_0, \theta_0, \theta_1, \ldots, \theta_n, v, w) \in \mathbb{R}^{5+n}$ to refer to the positions and velocities of tractor trailer system. Note that the position (x_i, y_i) of the center of the *i*th trailer can be expressed completely in terms of the state variables x_0, y_0, θ_0 and θ_i as follows:[36]

$$\begin{array}{l} x_i = x_1 - \frac{L_0}{2}\cos\theta_0 - \frac{L_i}{2}\cos\theta_i - \sum_{k=1}^{i-1} L_k\cos\theta_k\,, \\ y_i = y_1 - \frac{L_0}{2}\sin\theta_0 - \frac{L_i}{2}\sin\theta_i - \sum_{k=1}^{i-1} L_k\sin\theta_k\,. \end{array} \tag{2}$$

4. Motion planning and control

The main objective of this research is to use the Lyapunov-based control scheme (LbCS) to derive the acceleration-based controllers σ_1 and σ_2 so that the tractor-trailer system can move from an initial configuration to a goal position whilst avoiding collisions with obstacles along its path and simultaneously satisfying the mechanical singularities associated with the system. The LbCS, which was proposed by Sharma in Ref. [37] is a popular scheme that has been used by many researchers[2,13,15,29-31] in the recent years for robot navigation as this is one of the artificial potential fields methods that generate continuous time-invariant control laws. The governing principle behind this control scheme is to design an appropriate Lyapunov function which acts as an energy function of a system. The Lyapunov function (or the total potentials of the system) is comprised of attractive (for attraction to target) and repulsive potential functions (for avoidance of obstacles). The control laws are then designed such that the Lyapunov function is decreasing for all $t \geq 0$ and converges to zero as $t \rightarrow \infty$. This implies that any trajectory starting near an equilibrium point will converge to the equilibrium point or remain close to it.

Note that if control laws are designed using an artificial potential field method, the speed and the direction of a robot is determined by the gradient of the total potentials.[15] According to Khatib,[36] the direction of the robot's motion is via the notion of steepest descent. This implies that the path generated by the LbCS is guaranteed to be the shortest, safest and smoothest.[37]

4.1 Target attraction

The standard n-trailer system is required to move from an initial position $(x_0(0), y_0(0))$ to a goal position. The goal position or the target is a disk centered at (τ_1, τ_2) with radius r_τ is the set:

$$T = \{(z_1, z_2) \in \mathbb{R}^2 : \|(z_1, z_2) - (\tau_1, \tau_2)\|^2 \leq r_\tau^2\} \quad (3)$$

For the tractor-trailer system to be attracted to the target, we consider the following attractive potential functions:

$$V(\mathbf{x}) = \frac{1}{2}[(x_0 - \tau_1)^2 + (y_0 - \tau_2)^2 + v^2 + w^2] \quad (4)$$

which is a measure of the velocities and the Euclidean distance of the tractor robot's center of mass to the target point. When $V(\mathbf{x})$ is appropriately added to a Lyapunov function, the function will ensure that the robot moves from an initial position and converges to the goal position, the target.

4.2 Avoidance of obstacles

The function $V(\mathbf{x})$is capable of initiating the motion of the system, however, in practical sense, a robot's motion is restricted due to the system singularities and dynamic or static obstacles that it may encounter on its path. This research considers the following types of obstacles.

1. Static Obstacles - A priori known bounded workspace cluttered with stationary obstacles of various shapes and sizes is considered. While maneuvering to the target, the robots must avoid collisions with any static obstacles that lies along its route.

2. Artificial obstacles - The restrictions on steering (ϕ) and bending (θ_i) angles of tractor-trailer system and the bounds on velocities are treated as artificial obstacles. Practically, the steering and bending angles are limited due to the mechanical singularities of the system while the velocities must be bounded for safety reasons.[2,26,38]

3. Workspace Restriction - A rectangular constrained workspace is considered. The workspace needs to be bounded to ensure that the robot remains within the boundaries of the workspace at all time.

4.2.1 Fixed obstacles

This subsection considers elliptic and line obstacles as most objects can be enclosed or represented by ellipse and/or lines. The lth elliptic obstacle with center (o_{l1}, o_{l2}) and constants $a_l > 0$ and $b_l > 0$ on the $z_1 z_2$ plane is described as

$$EO_l = \left\{(z_1, z_2) \in \mathbb{R}^2 : \frac{(z_1 - o_{l1})^2}{a_l^2} + \frac{(z_2 - o_{l2})^2}{b_l^2} \leq 1\right\} \quad (5)$$

for $l = 1,2, \ldots, q$.

Normally seen in literature, for obstacle avoidances, the robots are enclosed in circular protective regions.[29] However, circular regions unnecessary take more space when the robot avoids obstacles. This may restrict the tractor-trailer robot from entering narrow passages or pass-through close neighboring obstacles.[15] Thus, in the research, we will deploy rectangular regions as a rectangle has a smaller area compared to the circle that inscribes the same rectangle. For the tractor-trailer robot to avoid collision with elliptic obstacles, each body of the articulated robot is enclosed in a separate rectangular region as shown in Fig. 2. This idea serves as an

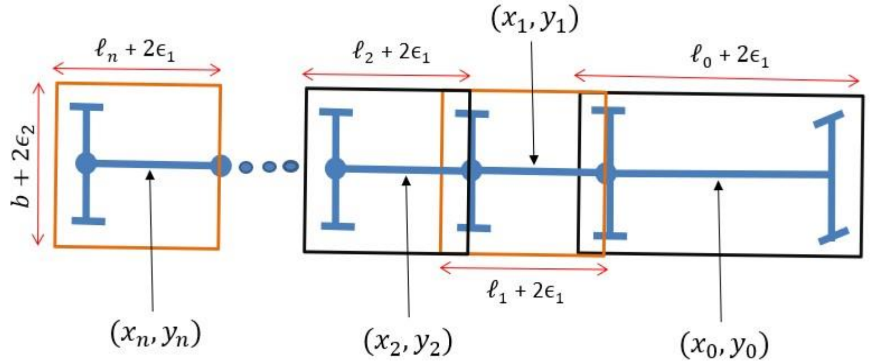

Fig. 2 Enclosing the standard n-trailer robot in rectangular protective regions.

advantage over enclosing the components in circular regions since the free-space is maximized.[15] This will also allow the robotic system to successfully pass through or maneuver over narrow passages such as a bridge or make right and left turns on roads.

For the entire system to avoid a fixed obstacle, it is important that every point on the boundary of the rectangular regions must avoid the obstacles. For this, we utilize the minimum distance technique (MDT) proposed by Sharma in Ref. [37]. The basic idea in the MDT is to find a point on the boundary of each region that is closest to an obstacle and at any time $t \geq 0$, the closest point (and hence the entire rectangular region) will avoid the obstacle.

For the avoidance of the elliptical obstacles by the mobile robot, we construct the following potential functions:

$$W_{ijl}(\mathbf{x}) = \frac{1}{2}\left[\frac{\left(x_i+\lambda_{il}(b+2\epsilon_2)\sin\theta_i-(-1)^j\frac{\ell_i+2\epsilon_1}{2}\cos\theta_i-o_{l1}\right)^2}{a_l^2} + \frac{\left(y_i-\lambda_{il}(b+2\epsilon_2)\cos\theta_i-(-1)^j\frac{\ell_i+2\epsilon_1}{2}\sin\theta_i-o_{l2}\right)^2}{b_l^2} - 1\right] \quad (6)$$

$$W_{ijl}^*(\mathbf{x}) = \frac{1}{2}\left[\frac{\left(x_i+(-1)^j\left(\frac{b+2\epsilon_2}{2}\right)\sin\theta_i+\lambda_{il}^*(\ell_i+2\epsilon_1)\cos\theta_i-o_{l1}\right)^2}{a_l^2} + \frac{\left(y_i-(-1)^j\left(\frac{b+2\epsilon_2}{2}\right)\cos\theta_i+\lambda_{il}^*(\ell_i+2\epsilon_1)\sin\theta_i-o_{l2}\right)^2}{b_l^2} - 1\right] \quad (7)$$

where

$$\lambda_{il} = min\left\{max\left\{-\frac{1}{2},\frac{(y_i-o_{l2})\cos\theta_i+(o_{l1}-x_i)\sin\theta_i}{b+2\epsilon_2}\right\},\frac{1}{2}\right\}$$

$$\lambda_{il} = min\left\{max\left\{-\frac{1}{2},\frac{(o_{l1}-x_i)\cos\theta_i+(o_{l2}-y_i)\sin\theta_i}{\ell_i+2\epsilon_1}\right\},\frac{1}{2}\right\}$$

for $l = 1, 2, \ldots, q$, $i = 0, 1, 2, \ldots, n$ and $j = 1, 2$. We shall see in Section 5 that when W_{ijl} and W_{ijl}^* are added as a ratio in the L yapunov function, these functions will ensure that the tractor-trailer robot will steer safely pass any elliptic obstacle in the workspace.

Next, let us consider line obstacles in the workspace. Suppose the workspace contains $m > 0$ line obstacles. The kth line segment in the z_1z_2 plane, from the point (a_{k1}, b_{k1}) to the point (a_{k2}, b_{k2}) is the set:

$$LO_k = \left\{(z_1,z_2)\in\mathbb{R}^2 : \left(z_1-a_{k1}-\beta_k(a_{k2}-a_{k1})\right)^2 + \left(z_2-b_{k1}-\beta_k(b_{k2}-b_{k1})\right)^2 = 0\right\} \quad (8)$$

where $\beta_k \in [0,1], k = 1,2,\ldots,m$. Again, utilizing the MDT, the following functions are constructed for the avoidance of the line obstacle:

$$R_{ijk}(\mathbf{x}) = \frac{1}{2}\left[\left(x_i+\gamma_{ijk}(b+2\epsilon_2)\sin\theta_i-(-1)^j\frac{\ell_i+2\epsilon_1}{2}\cos\theta_i-a_{k1}-\beta_{ijk}(a_{k2}-a_{k1})\right)^2+\left(y_i-\gamma_{ijk}(b+2\epsilon_2)\cos\theta_i-(-1)^j\frac{\ell_i+2\epsilon_1}{2}\sin\theta_i-b_{k1}-\beta_{ijk}(b_{k2}-b_{k1})\right)^2\right] \quad (9)$$

s:

$$R_{ijk}^*(\mathbf{x}) = \frac{1}{2}\left[\left(x_i+(-1)^j\left(\frac{b+2\epsilon_2}{2}\right)\sin\theta_i+\gamma_{ijk}^*(\ell_i+2\epsilon_1)\cos\theta_i-a_{k1}-\beta_{ijk}^*(a_{k2}-a_{k1})\right)^2+\left(y_i-(-1)^j\left(\frac{b+2\epsilon_2}{2}\right)\cos\theta_i+\gamma_{ijk}^*(\ell_i+2\epsilon_1)\sin\theta_i-b_{k1}-\beta_{ijk}^*(b_{k2}-b_{k1})\right)^2\right] \quad (10)$$

where,

$$\beta_{ijk} = min\left\{max\left\{0,\frac{(-1)^j(\ell_i/2+\epsilon_1)+(a_{k1}-x_i)\cos\theta_i+(b_{k1}-y_i)\sin\theta_i}{(a_{k1}-a_{k2})\cos\theta_i+(b_{k1}-b_{k2})\sin\theta_i}\right\},1\right\}$$

$$\beta_{ijk}^* = min\left\{max\left\{0,\frac{(-1)^j(b/2+\epsilon_1)+(b_{k1}-y_i)\cos\theta_i-(a_{k1}-x_i)\sin\theta_i}{(b_{k1}-b_{k2})\cos\theta_i+(a_{k2}-a_{k1})\sin\theta_i}\right\},1\right\}$$

$$\gamma_{ijk} = min\left\{max\left\{-\frac{1}{2},\frac{a_{k2}b_{k1}-a_{k1}b_{k2}+(b_{k2}-b_{k1})x_i+(a_{k1}-a_{k2})y_i}{b[(a_{k1}-a_{k2})\cos\theta_i+(b_{k1}-b_{k2})\sin\theta_i]}+\frac{(-1)^j(\ell_i+2\epsilon_1)[(b_{k1}-b_{k2})\cos\theta_i-(a_{k1}-a_{k2})\sin\theta_i]}{2b[(a_{k1}-a_{k2})\cos\theta_i+(b_{k1}-b_{k2})\sin\theta_i]}\right\},\frac{1}{2}\right\}$$

$$\gamma_{ijk}^* = min\left\{max\left\{-\frac{1}{2},\frac{a_{k2}b_{k1}-a_{k1}b_{k2}+(b_{k2}-b_{k1})x_i+(a_{k1}-a_{k2})y_i}{\ell_i[(b_{k1}-b_{k2})\cos\theta_i+(a_{k2}-a_{k2})\sin\theta_i]}+\frac{(-1)^j(b+2\epsilon_2)[(a_{k1}-a_{k2})\cos\theta_i+(b_{k1}-b_{k2})\sin\theta_i]}{2\ell_i[(b_{k1}-b_{k2})\cos\theta_i+(a_{k2}-a_{k2})\sin\theta_i]}\right\},\frac{1}{2}\right\}$$

for $k = 1, 2, \ldots, m$, $i = 0, 1, 2, \ldots, n$ and $j = 1, 2$.

4.2.2 Artificial obstacles

To observe the mechanical singularities of the system and the boundedness on velocities of the robot, the following restrictions, which are considered as dynamic constraints of the tractor-trailer system, are imposed:

1. $|v| \leq v_{max}$, where $v_{max} > 0$ is the maximum speed of the tractor robot. This will ensure that the robot steers safely pass an obstacle and converges smoothly to the target.
2. $|\phi| \leq \phi_{max}$, where $\phi_{max} < \pi/2$ is the maximum steering angle of the tractor robot. In reality, the steering angle of any nonholonomic robot is bounded to prevent the front wheel of the tractor robot from being jammed. Thus, this restriction ensures that the tractor robot's steering angle Φ is within a predefined range ($-\phi_{max}$,ϕ_{max}) at all time.
3. $|\theta_i - \theta_{i-1}| \leq \theta_{max}$, where $\theta_{max} < \pi/2$ is the maximum bending angle of the trailers. This condition is necessary so that any trailer does not collide with the preceding one during bending.

To include the above restrictions into the control laws, the following artificial obstacles are developed:

$$AO_1 = \{v \in \mathbb{R}:\ |v| > v_{max}\}$$
$$AO_2 = \left\{\omega \in \mathbb{R}:\ |\omega| > \frac{v_{max}\tan\phi_{max}}{\ell_0}\right\}$$
$$AO_{2+i} = \{\theta_i \in \mathbb{R}:\ |\theta_i - \theta_{i-1}| > \theta_{max}\},\quad i = 1.2\ldots,n \quad (11)$$

and for their avoidances, the functions:

$$S_1(\mathbf{x}) = \frac{1}{2}(v_{max}^2 - v^2)$$
$$S_2(\mathbf{x}) = \frac{1}{2}\left(\frac{v_{max}^2\tan^2\phi_{max}}{\ell_0^2} - \omega\right) \quad (12)$$
$$S_3(\mathbf{x}) = \frac{1}{2}(\theta_{max}^2 - (\theta_i - \theta_{i-1})^2),\quad i = 1.2\ldots,n$$

will be added as a ratio to a Lyapunov function of the system in Section 5.

4.2.3 Workspace restriction

The workspace is a fixed, closed and bounded rectangular region, defined, for some constants η_1 and η_2 as

$$WS = \{(z_1, z_2) \in \mathbb{R}^2 : -\eta_1 \leq z_1 \leq \eta_1, -\eta_2 \leq z_2 \leq \eta_2\}. \quad (13)$$

The tractor-trailer robot is required to stay within the workspace at all time $t \geq 0$. It suffices that the four vertices of the rectangular regions that encloses the tractor and the respective trailers must avoid collisions with the boundaries of the workspace. As such, we construct the potential functions:

$$U_{ij}(\mathbf{x}) = \frac{1}{2}\left[\eta_1^2 - \left(x_i - \frac{b+2\epsilon_2}{2}(-1)^j \sin\theta_i + \frac{\ell_i+2\epsilon_1}{2}(-1)^{\lfloor j/2 \rfloor}\cos\theta_i\right)^2\right]$$
$$U_{ij}^*(\mathbf{x}) = \frac{1}{2}\left[\eta_2^2 - \left(y_i - \frac{b+2\epsilon_2}{2}(-1)^j \cos\theta_i + \frac{\ell_i+2\epsilon_1}{2}(-1)^{\lfloor j/2 \rfloor}\sin\theta_i\right)^2\right] \quad (14)$$

for $i = 0, 1, 2, \ldots, n$ and $j = 1, 2, 3, 4$.

5. The Lyapunov function and the control laws

In this section, the Lyapunov-based control scheme is utilised to design the non-linear acceleration control laws σ_1 and σ_2. Firstly, a Lyapunov function for system (1) is defined by combining all the functions (15) - (17) as:

$$L(\mathbf{x}) = V(\mathbf{x}) + F(\mathbf{x})\left[\sum_{i=1}^{n+2}\frac{\alpha_i}{S_i(\mathbf{x})} + \sum_{i=0}^{n}\left(\sum_{j=1}^{2}\sum_{l=1}^{q}\left\{\frac{\xi_{ijl}}{W_{ijl}(\mathbf{x})} + \frac{\xi_{ijl}^*}{W_{ijl}^*(\mathbf{x})}\right\} + \sum_{j=1}^{2}\sum_{k=1}^{m}\left\{\frac{\varsigma_{ijk}}{R_{ijk}(\mathbf{x})} + \frac{\varsigma_{ijk}^*}{R_{ijk}^*(\mathbf{x})}\right\} + \sum_{j=1}^{4}\left\{\frac{\chi_{ij}}{U_{ij}(\mathbf{x})} + \frac{\chi_{ij}^*}{U_{ij}^*(\mathbf{x})}\right\}\right)\right] \quad (15)$$

where $\alpha_i > 0$, $\xi_{ijl} > 0$, $\xi_{ijl}^* > 0$, $\varsigma_{ijk} > 0$, $\varsigma_{ijk}^* > 0$, $\chi_{ij} > 0$, and $\chi_{ij}^* > 0$ for $i, j, k, l \in$ N are called the control parameters, while

$$F(\mathbf{x}) = \frac{1}{2}[(x_0 - \tau_1)^2 + (y_0 - \tau_2)^2] \quad (16)$$

is an auxiliary function required for the Lyapunov function to vanish at the target.

The Lyapunov function is positive, continuous and bounded over the domain:

$$D = \left\{\mathbf{x} \in \mathbb{R}^9 : S_i(\mathbf{x}) > 0, W_{ijl}(\mathbf{x}) > 0, W_{ijl}^*(\mathbf{x}) > 0, R_{ijk}(\mathbf{x}) > 0, R_{ijk}^*(\mathbf{x}) > 0, U_{ij}(\mathbf{x}) > 0, U_{ij}^*(\mathbf{x}) > 0\right\} \quad (17)$$

Using the Lyapunov-based control scheme, the following form of the non-linear acceleration-based time-invariant controllers is proposed:

$$\left.\begin{aligned} \sigma_1 &= -\frac{1}{1+\frac{\alpha_1 F}{S_1^2}}\left[\delta_1 v + \frac{\partial L}{\partial x_0}\cos\theta_0 + \frac{\partial L}{\partial y_0}\sin\theta_0 + \sum_{i=1}^{n}\frac{\partial L}{\partial \theta_i}\left(\frac{1}{\ell_i}\sin(\theta_{i-1} - \theta_i)\prod_{k=1}^{i-1}\cos(\theta_{k-1} - \theta_k)\right)\right] \\ \sigma_2 &= -\frac{1}{1+\frac{\alpha_2 F}{S_2^2}}\left[\delta_2 \omega - \frac{\partial L}{\partial x_0}\frac{\ell_0}{2}\sin\theta_0 + \frac{\partial L}{\partial y_0}\frac{\ell_0}{2}\cos\theta_0 + \frac{\partial L}{\partial \theta_0}\right] \end{aligned}\right\} \quad (18)$$

where $\delta_1 > 0$ and $\delta_2 > 0$ are called the convergence parameters.

6. Stability analysis

Let θ_i^*(for $i = 0, 1, 2, \ldots, n$) be the orientation of each body of the robot at the target. Then the point $\mathbf{e} = (\tau_1, \tau_1, \theta_{0,}^*, \theta_1^*, \ldots \theta_n^*, 0,0)$ is an equilibrium point of system (1). The stability issues pertaining to the equilibrium point e is given in Theorem 6.1.

Theorem 6.1. The equilibrium point $\mathbf{e} = (\tau_1, \tau_1, \theta_{0,}^*, \theta_1^*, \ldots \theta_n^*, 0,0)$ of system (1) is stable provided the controllers σ_1 and σ_2 are defined as in (18).

Proof: We use the Direct Method of Lyapunov to prove the **e** is a stable equilibrium point of system (1). The Lyapunov function $L(\mathbf{x})$ defined in (19) is positive, continuous, has continuous first partial derivatives and bounded over the domain D. Moreover, $L(\mathbf{e}) = 0$ and $L(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{e}$. Next, since $L(\mathbf{x})$ has continuous first partial derivatives on the domain D, the time derivative of $L(\mathbf{x})$ is

$$\dot{L}(\mathbf{x}) = \frac{\partial L}{\partial x_0}\dot{x}_0 + \frac{\partial L}{\partial y_0}\dot{y}_0 + \frac{\partial L}{\partial \theta_0}\dot{\theta}_0 + \sum_{i=1}^{n}\frac{\partial L}{\partial \theta_i}\dot{\theta}_i + \frac{\partial L}{\partial v}\dot{v} + \frac{\partial L}{\partial \omega}\dot{\omega} = \frac{\partial L}{\partial x_0}\left(v\cos\theta_0 - \frac{\ell_0}{2}\omega\sin\theta_0\right) + \frac{\partial L}{\partial y_0}\left(v\sin\theta_0 + \frac{\ell_0}{2}\omega\cos\theta_0\right) + \frac{\partial L}{\partial \theta_0}\omega + \sum_{i=1}^{n}\frac{\partial L}{\partial \theta_i}\left(\frac{v}{\ell_i}\sin(\theta_{i-1} - \theta_i)\prod_{k=1}^{i-1}\cos(\theta_{k-1} - \theta_k)\right) + \left(1 + \frac{\alpha_1 F}{S_1^2}\right)v\sigma_1 + \left(1 + \frac{\alpha_2 F}{S_2^2}\right)\omega\sigma_2 = \left[\frac{\partial L}{\partial x_0}\cos\theta_0 + \frac{\partial L}{\partial y_0}\sin\theta_0 + \sum_{i=1}^{n}\frac{\partial L}{\partial \theta_i}\left(\frac{1}{\ell_i}\sin(\theta_{i-1} - \theta_i)\prod_{k=1}^{i-1}\cos(\theta_{k-1} - \theta_k)\right) + \left(1 + \frac{\alpha_1 F}{S_1^2}\right)\sigma_1\right]v + \left[-\frac{\partial L}{\partial x_0}\frac{\ell_0}{2}\sin\theta_0 + \frac{\partial L}{\partial y_0}\frac{\ell_0}{2}\cos\theta_0 + \frac{\partial L}{\partial \theta_0} + \left(1 + \frac{\alpha_2 F}{S_2^2}\right)\sigma_2\right]\omega \quad (19)$$

With the non-linear acceleration control laws σ_1 and σ_2 defined as in (18), it follows that

$$\dot{L}(\mathbf{x}) = -\delta_1 v^2 - \delta_2 \omega^2 \leq 0. \quad (20)$$

It is clear that, in the domain D, $\dot{L}(\mathbf{x}) \leq 0$ and $\dot{L}(\mathbf{e}) = 0$. Hence, **e** is a stable equilibrium point of system (1).

7. Computer simulation

To numerically verify the effectiveness of the control scheme, the new strategy and the proposed control laws, interesting simulations were carried out using Matlab and Mathematica softwares. This is illustrated in the three scenarios below.

7.1 Scenario 1

A 4-trailer system is considered, as shown in Fig. 3, where the robot has to move from an initial position (-30, 50) to the target at (50, -50) avoiding collisions with elliptic and line obstacles (forming a square) in the workspace. Fig. 3 shows the path followed by the 4-trailer system and its convergence to a final configuration. The different parameters used in the simulation are given in Table 1.

The control and convergence parameters are selected using the brute-force method.[2,13,15,29-31] This is a limitation in the Lyapunov-based control scheme which can be considered as a future work. The graph of the velocities, v and ω are given in Fig. 4(a). One can clearly notice the asymptotic convergence of the velocities as $t \to \infty$. The graph also shows where the robots has speed up or speed down. Fig. 4(b) shows the behavior of the nonlinear acceleration-based controllers, σ_1 and σ_2 along the system trajectory. Note that the controllers vanish as $t \to \infty$.

Table 1. Values of different parameters used in Simulation 1.

Robot dimension	$\ell_0 = 5$, $b = 3$, $\ell_1 = \ell_2 = \ell_3 = \ell_4 = 4$ units.
Clearance parameters	$\epsilon_1 = 0.3$ units and $\epsilon_2 = 0.2$ units.
Initial position	$(x_0(0), y_0(0)) = (-30, 50)$.
Initial orientation	$\theta_i(0) = 0$ for $i = 0, 1, 2, 3, 4$.
Initial orientation	$(\tau_1, \tau_2) = (50, -50)$.
Elliptic obstacles	Positions: $(o_{11}, o_{12}) = (0, -50)$, $(o_{21}, o_{22}) = (-30, 0)$, $(o_{31}, o_{32}) = (0, 20)$, $(o_{41}, o_{142}) = (40, -20)$, $(o_{51}, o_{52}) = (42, 20)$, $(o_{61}, o_{62}) = (-25, -25)$, $(o_{71}, o_{72}) = (-35, 30)$, $(o_{81}, o_{82}) = (20, 40)$. Sizes are random.
Line obstacles	Line 1: $(a_{11}, b_{11}) = (-10, -20)$, $(a_{21}, b_{21}) = (10, -20)$, Line 2: $(a_{12}, b_{12}) = (-10, -20)$, $(a_{22}, b_{22}) = (-10, 0)$, Line 3: $(a_{13}, b_{13}) = (10, -20)$, $(a_{23}, b_{23}) = (10, 0)$, Line 4: $(a_{14}, b_{14}) = (-10, 0)$, $(a_{24}, b_{24}) = (10, 0)$.
Control parameters	$\alpha_i = 0.01$, $\xi_{ijl} = \xi^*_{ijl} = 0.01$, $\varsigma_{ijk} = \varsigma^*_{ijk} = 0.01$ $\chi_{ij} = 0.1$, $\chi^*_{ij} = 0.1$
Convergence parameters	for $l = 1, 2, \ldots, 8$, $k = 1, 2, 3, 4$, $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$. $\delta_1 = 10$, $\delta_2 = 10$.
Physical limitations	$v_{max} = 5$ units/s, $\phi_{max} = 7\pi/18$ rad $\theta_{max} = 8\pi/18$ rad.
Workspace boundaries	$-60 \le z_1 \le 60$ and $-60 \le z_2 \le 60$

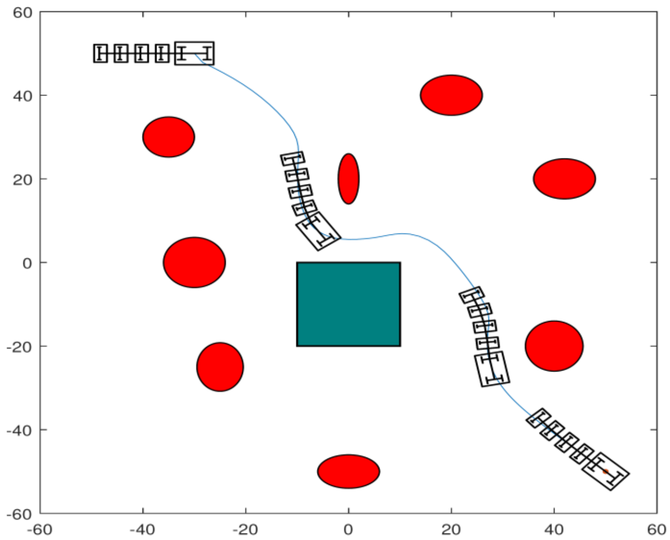

Fig. 3 Simulation result of a 4-trailer system.

The velocities and controllers converge to zero as the robot converges to its target. This is because the robot is required to stop at the target. The stability properties of the Lyapunov function as discussed in Section 6 ensures that $L(\mathbf{x})$ and $\dot{L}(\mathbf{x})$ must converge to zero at the target. Since the controllers σ_1 and σ_2 are functions of $L(\mathbf{x})$, the controllers (and the velocities) converge to zero as the robot converges to its target.

The evolution of the Lyapunov function and its time derivative along the robot's trajectory is shown in Fig. 5. The decreasing nature of the Lyapunov function, and the asymptotic convergence of $L(\mathbf{x})$ and $\dot{L}(\mathbf{x})$ as $t \to \infty$ numerically verifies the Lyapunov's stability properties.

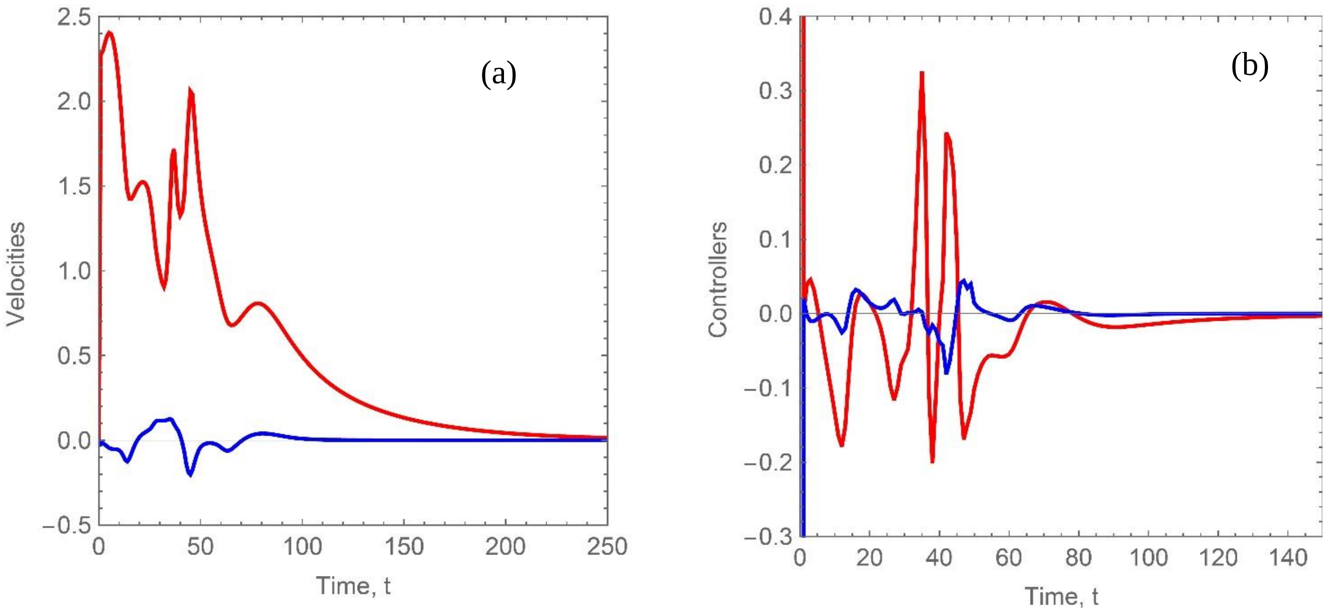

Fig. 4 Evolution of nonlinear velocities and controllers for the tractor-trailer trajectory (a) Velocities v in blue, ω in red. (b) Controllers σ_1 in blue, σ_2 in red.

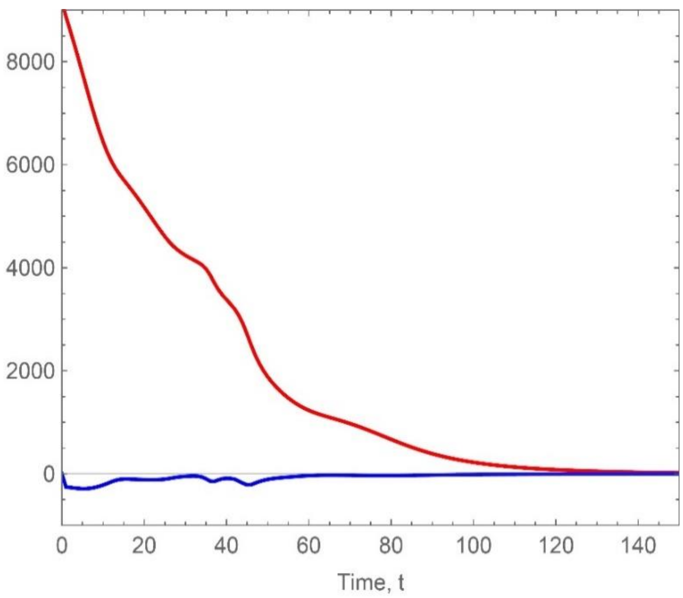

Fig. 5 Evolution of the Lyapunov function $L(\mathbf{x})$ (in red) and its time derivative $\dot{L}(\mathbf{x})$ (in blue) along the trajectory shown in Scenario 1.

7.2 Scenario 2

This scenario considers an 8-trailer system that is required to maneuver over a narrow bridge, avoid obstacles on its way and finally converge to a designated target. The lines that form the edges of the bridge are considered as line obstacles that need to be avoided by the system at all times. In addition, six elliptical obstacles of random sizes are added to the workspace. The robot's initial and target positions are $(-15, 50)$ and $(52, -56)$, respectively. Other parameters are similar to the ones used in Scenario 1.

As seen in Fig. 6, the 8-trailer system is able to avoid the edges of bridge and successfully maneuver through without hitting the edges of the bridge. It also avoids elliptical obstacles on its way and finally converges to its target.

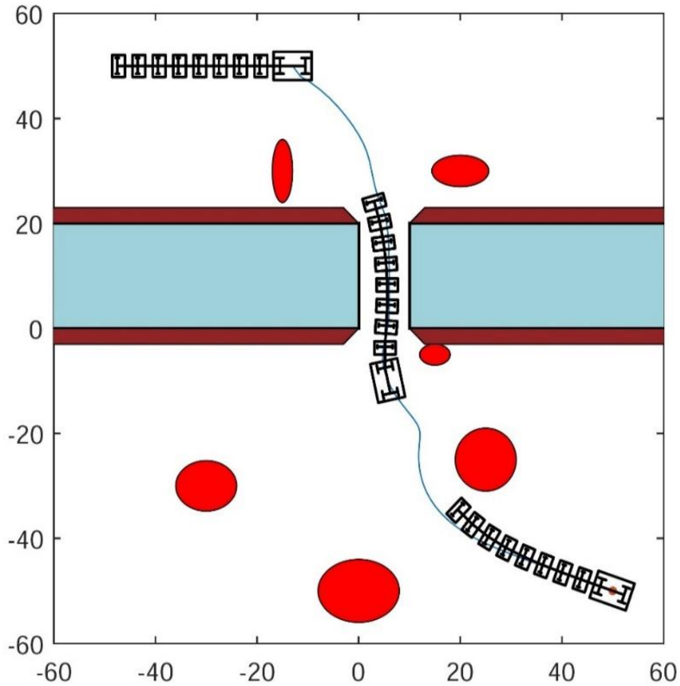

Fig. 6 Simulation result of an 8-trailer system from initial position (-15, 50) to target position (52, -56).

We have also generated the velocity profiles (Fig. 7(a)), graphs of the controllers (Fig. 7(b)) and the evolution of the Lyapunov function and its time derivative (Fig. 10) along the system trajectory. Similar behaviors are seen as observed for Scenario 1.

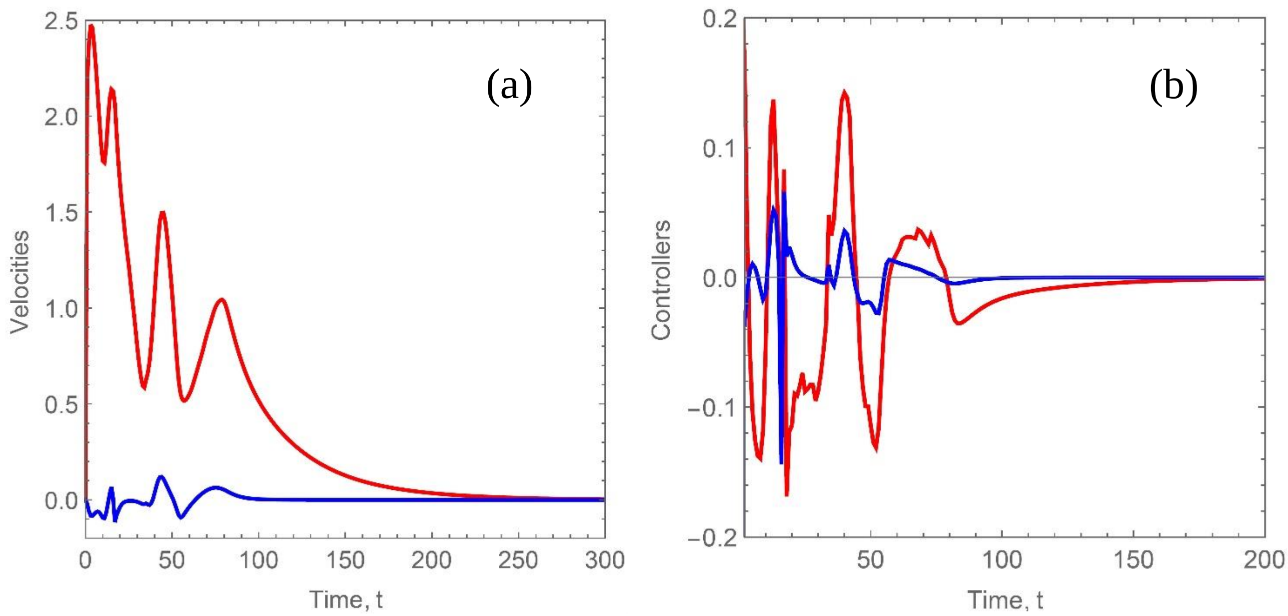

Fig. 7 Evolution of nonlinear velocities and controllers for the tractor-trailer trajectory (a) Velocities v in blue, ω in red. (b) Controllers σ_1 in blue, σ_2 in red.

7.3 Scenario 3

We now consider a 12-trailer system, where the tractor robot is required to move from an initial position of (−70, −50) to a target at (120, 0) which is inside a parking bay. Referring to Fig. 8, the two lines of the parking bay are considered as line obstacles that would guide the robot to enter and park correctly inside the bay. The workspace also includes a triangular obstacle (made up of lines) and six elliptic obstacles of random sizes and positions. The control, convergence and other parameters are similar to the ones used in Scenario 1.

Figure 8 shows the simulation result where the robot successfully avoids the line and elliptical obstacles, enters the parking bay and parks correctly at the target point.

The graphs of the velocities, non-linear acceleration controllers and the Lyapunov function are shown in Figs. 9(a), 9(b) and 11, respectively. Again, similar behaviors are noticed as obtained in previous scenarios.

8. Discussion

Tractor-trailer systems are integral to goods transfer via the road transport industry. However, drivers of such large goods vehicles continually face numerous risks in their work. One of the most common risks is the control of the tractor-trailer system. This paper presented stabilizing generalized acceleration controllers for a passive standard n-trailer system in an obstacle-ridden environment while observing system

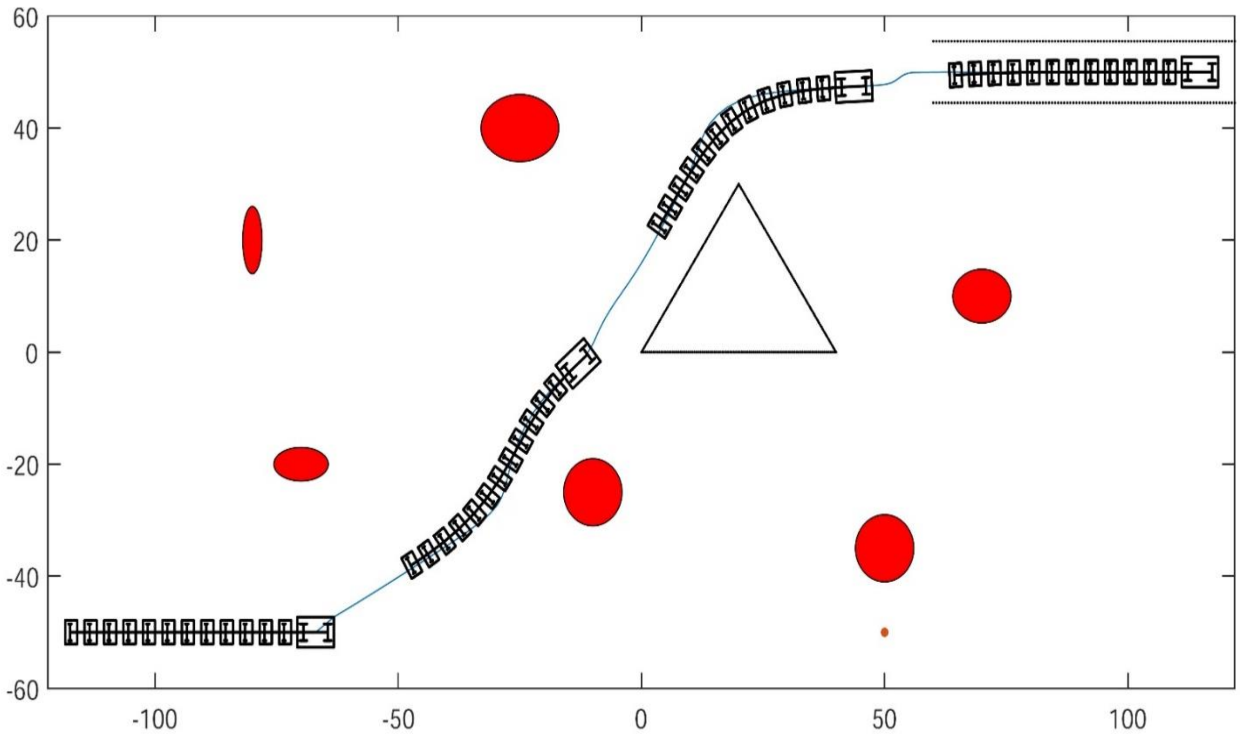

Fig. 8 Simulation result of a 12-trailer system.

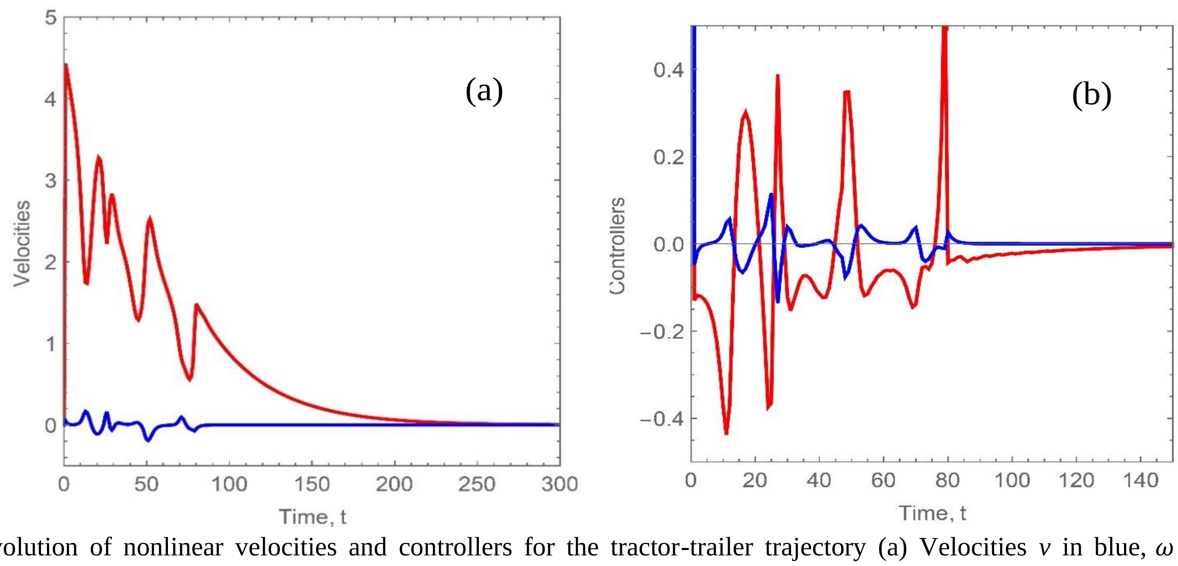

Fig. 9 Evolution of nonlinear velocities and controllers for the tractor-trailer trajectory (a) Velocities v in blue, ω in red, (b) Controllers σ_1 in blue, σ_2 in red.

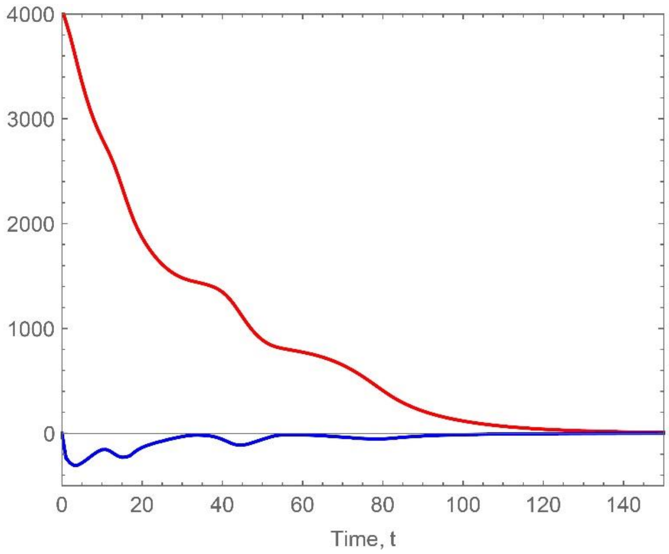

Fig. 10 Evolution of the Lyapunov function $L(\mathbf{x})$ (in red) and its time derivative $\dot{L}(\mathbf{x})$ (in blue) along the trajectory shown in Scenario 2.

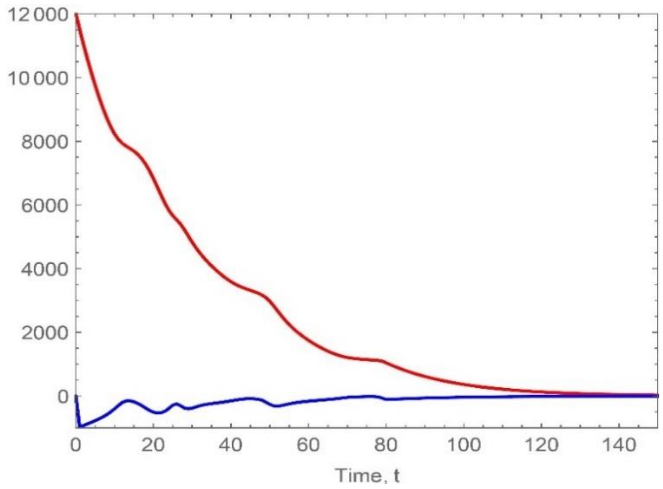

Fig. 11 Evolution of the Lyapunov function $L(\mathbf{x})$ (in red) and its time derivative $\dot{L}(\mathbf{x})$ (in blue) along the trajectory shown in Scenario 3.

constraints. Furthermore, the strategy of enclosing each body of the n-trailer system in a rectangular region gives rise to free space optimization, allowing the system to pass through narrow passages. In contrast, the motion control of a 1-trailer system, where each body of the tractor-trailer system has been enclosed in a circular protective region for obstacle avoidance, has been presented in Refs. [13] and [27]. The use of circles to inscribe the robotic system in protective regions is not an effective strategy to optimize the free space. Hence, the system faces issues passing through narrow passageways or making right and left turns on roads.

Simulation examples in scenarios 1, 2, and 3 show the effectiveness of the time-invariant, nonlinear, continuous controllers. The controllers presented in this paper are for a generalized passive standard n-trailer system and are robust, which allows the system to navigate autonomously and keeps it stable. Therefore, providing a solution to the control problem tagged with large goods transport tractor-trailer systems. However, there is still an issue tagged with the Lyapunov-based control scheme: local minima. In this research, the local minima issue of LbCS was addressed by selecting appropriate values for control and convergence parameters as demonstrated in Table 1. However, there is a limitation of the LbCS that is tagged with the selection of the control and convergence parameters. This research uses the brute-force method to select these parameters. This is in line with the work presented in literature by Refs. [2,13,15,29,30,31]. This limitation of LbCS can be considered as a future work.

While the proposed LbCS has been successfully validated through comprehensive simulations, translating these results to real-world applications presents specific challenges and opportunities for further development. The simulations demonstrate the system's ability to navigate constrained environments, but real-world implementation may encounter additional complexities such as sensor noise, mechanical imperfections, and unpredictable environmental dynamics.

8.1 Potential challenges in real-world applications

1. Sensor Limitations: In practice, sensor accuracy plays a crucial role in obstacle detection and avoidance. Real-time feedback from sensors such as LiDAR, cameras, or ultrasonic sensors could introduce noise or inaccuracies, which were not fully modeled in the simulations. Implementing robust filtering techniques would be necessary to ensure reliable operation.
2. Actuator Precision: The mechanical response of the tractor and trailer system's actuators may be smoother and more immediate than in simulations. Issues like actuator delay or wear over time can affect the stability of the controllers.
3. Environmental Uncertainty: Real-world environments often introduce dynamic and unpredictable changes, such as moving obstacles or uneven terrain. The system must be equipped to handle these variations, potentially requiring adaptive control schemes or real-time controller adjustments.

8.2 Feasibility and reliability of simulation results

The use of the LbCS ensures the theoretical stability of the system, which is a significant advantage in real-world scenarios. The controller's performance in simulation, particularly its ability to avoid obstacles and navigate narrow passages, indicates that the approach is feasible for real-world implementation, provided that the control parameters are tuned to accommodate real-world sensor data and actuation limits.

8.3 Potential applications

1.Warehouse Automation: The control scheme could be applied to autonomous robots in warehouse environments, where precise maneuvering through narrow aisles and obstacle avoidance are critical.
2. Autonomous Vehicles: The approach can be integrated into

larger autonomous vehicles for transporting goods in controlled environments such as factories, farms, or ports.
3. Agriculture: The method could assist in navigating farm equipment in dynamic, obstacle-ridden environments, helping to avoid damage to crops while maintaining efficient task completion.

9. Conclusion

In this paper, a new solution is proposed to control the motion of a standard n-trailer system in an obstacle-ridden bounded workspace. The 2-dimensional workspace is cluttered with fixed elliptical and line obstacles which the robot has to avoid along its way to the target. Each component of the n-trailer system is enclosed, for the first time, in rectangular regions to maximize freespace and that the minimum distance technique was utilized for the avoidance of obstacles. This also enables the robot to pass through narrow passages. For the extraction of nonlinear time-invariant continuous acceleration controllers, the Lyapunov-based control scheme is utilized. Stability of the system is proved via the Direct Method of Lyapunov and the proposed solution is numerically verified via simulations with 4, 8 and 12-trailer systems showing interesting scenarios. A limitation of this research is that we have not verified the result using experiments as this is a theoretical exposition.

Future work in this area can be motion control of multiple *n*-trailer system, inclusion of dynamic obstacles, and performing experiments with real robots. Moreover, proving asymptotic stability under suitable initial condition is a challenging and interesting problem which can be considered in future.

Conflict of interest

None declared.

References

[1] L. Battistuzzi, C. T. Recchiuto, & A. Sgorbissa, Ethical concerns in rescue robotics: a scoping review. *Ethics and Information Technology*, 2021, **23**, 863-875, doi: 10.1007/s10676-021-09603-0.

[2] R. Chand, R. P. Chand, S. A. Kumar, Switch controllers of an *n*-link revolute manipulator with a prismatic end-effector for landmark navigation. *PeerJ Computer Science*, 2022, **8**, e885, doi: 10.7717/peerj-cs.885.

[3] G. D. Cubber, D. Doroftei, K. Rudin, K. Berns, A. Matos, D. Serrano, J. Sanchez, S.Govindaraj, J.Bedkowski, R. Roda, E. Silva, S. Ourevitch, Introduction to the use of robotic tools for search and rescue. *Search and Rescue Robotics*, 2017, doi: 10.5772/intechopen.69489.

[4] M. Javaid, A. Haleem, R. P. Singh, R. Suman, Substantial capabilities of robotics in enhancing industry 4.0 implementation. *Cognitive Robotics*, 2021, **1**, 58-75, doi: 10.1016/j.cogr.2021.06.001.

[5] J. Leu, Y., Wang, M. Tomizuka, S. Di Cairano, Improved A-search guided tree for autonomous trailer planning. 2022 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2022, 7190-7196, doi: 10.1109/IROS47612.2022.9982121.

[6] F. Ingrand and M. Ghallab, Deliberation for autonomous robots: A survey. *Artificial Intelligence*, 2017, **247**, 10-44, doi: 10.1016/j.artint.2014.11.003.

[7] N. Lashkari, M. Biglarbegian, S. Yang, Optimal design of formation tracking control for a tractor-trailer robotic system with omni-directional wheels. In IEEE 19th International Conference on Intelligent Transportation Systems, 2016, 1826-1831, doi: 10.1109/ITSC.2016.7795806.

[8] M. Rahimi, H. Liu, I. D. Cardenas, A. Starr, A. Hall, R. Anderson, A review on technologies for localisation and navigation in autonomous railway maintenance systems. *Sensors*, 2022, **22**, 4185, doi: 10.3390/s22114185.

[9] D. N. Bertolani, V. P. Bacheti, M. Sarcinelli-Filho. Controlling a unicycle mobile platform pushing 1 and 2 passive trailers. *Journal of intelligent & robotic systems*, 2021, **103**, 43, doi: 10.1007/s10846-021-01478-z.

[10] A. Khanpoor, A. K. Khalaji, S. A. Moosavian. Modeling and control of an underactuated tractor-trailer wheeled mobile robot. *Robotica*, 2017, **35**, 2297-2318. doi: 10.1017/S0263574716000886.

[11] Y. Dong, H. Sun, Z. Liu, Z. Bao, Kinematics and constraint analysis of tractor-trailer mobile robot. In 2005 International Conference on Machine Learning and Cybernetics, 2005, **3**, 1380-1384, doi:10.1109/ICMLC.2005.1527159.

[12] A. Astolfi, P. Bolzern, A. Locatelli, Path-tracking of a tractor-trailer vehicle along rectilinear and circular paths: A Lyapunov-based approach. *IEEE transactions on robotics and automation*, 2004, **20**, 154-160, doi: 10.1109/TRA.2003.820928.

[13] J. Raj, K. Raghuwaiya, B. Sharma, J. Vanualailai, Motion control of a flock of 1-trailer robots with swarm avoidance. *Robotica*, 2021, **39**, 1-26, doi:10.1017/S0263574721000060.

[14] A. Benzerrouk, L. Adouane, P. Martinet, Dynamic obstacle avoidance strategies using limit cycle for the navigation of multi-robot system. In *2012* IEEE/RSJ IROS'12, 4th Workshop on Planning, Perception and Navigation for Intelligent Vehicles, 2012.

[15] A. Prasad, B. Sharma, J. Vanualailai, S. Kumar, Motion control of an articulated mobile manipulator in 3D using the Lyapunov-based control scheme. *International Journal of Control*, 2022, **95**, 2581-2595, doi: 10.1080/00207179.2021.1919927.

[16] L. Zeng and G. M. Bone, Mobile robot collision avoidance in human environments. *International Journal of Advanced Robotic Systems*, 2013, **10**, 41, doi: 10.5772/54933.

[17] M. Park, W. Chung, M. Kim, J. Song, Control of a mobile robot with passive multiple trailers. In IEEE International Conference on Robotics and Automation, 2004, **5**, 4369-4374, doi: 10.1109/ROBOT.2004.1302405.

[18] G. Rigatos, K. Busawon, M. Abbaszadeh, A nonlinear optimal control approach for the truck and N-trailer robotic system. *IFAC Journal of Systems and Control*, 2022, **20**, 100191, doi: 10.1016/j.ifacsc.2022.100191.

[19] S. A. Moosavian, E. Papadopoulos, Modified transpose Jacobian control of robotic systems. *Automatica*, 2007, **43**, 1226-1233, doi: 10.1016/j.automatica.2006.12.029.
[20] A. K. Khalaji and S. A. Moosavian, Modified transpose Jacobian control of a tractor-trailer wheeled robot. *Journal of Mechanical Science and Technology*, 2015, **29**, 3961-3969, doi: 10.1007/s12206-015-0841-3.
[21] K. Tanaka, K. Yamauchi, H. Ohtake, H. Wang, Sensor reduction for backing-up control of a vehicle with triple trailers, *IEEE Transactions on Industrial Electronics*, 2008, **56**, 497-509, doi: 10.1109/TIE.2008.2003218.
[22] Y. Deng, T. Li, M. Xie, S. Zhang, Distributed multi-robot-trailer scheduling based on communication between charging stations for robot being towed to recharge. *Electronics*, 2023, **12**, 1402, doi : 10.3390/electronics12061402.
[23] H. Zhao, W. Chen, S. Zhou, F. Zheng, Y. Liu, Localization and motion planning of industrial tractor–trailers vehicles. *IEEE Transactions on Control Systems Technology*, 2023, **31**, 2928-2936, doi: 10.1109/TCST.2023.3275497.
[24] D. N. Bertolani, V. P. Bacheti, A. S. Brandão, R. Carelli, M. Sarcinelli-Filho, Controlling a multi-articulated robot in backward movement using an adaptive controller. *Mechatronics*, 2023, **92**, 102967, doi: 10.1016/j.mechatronics.2023.102967.
[25] T. Zhao, W. Huang, P. Xu, W. Zhang, P. Li, Y. Zhao, A simple curvature-based backward path-tracking control for a mobile robot with N trailers. *Actuators,* 2014, **13**, 237, doi: 10.3390/act13070237.
[26] J. Cheng, Y. Zhang, Z. Wang, A direct Lyapunov approach for tracking control of mobile robot with two on-axle hitching trailers. In The 27th Chinese Control and Decision Conference, 2015, 3429-3432, doi: 10.1109/CCDC.2015.7162515.
[27] J. Raj, K. Raghuwaiya, S. Singh, B. Sharma, J. Vanualailai, Swarming intelligence of 1-trailer systems. In Advanced Computer and Communication Engineering Technology: Proceedings of ICOCOE, 2015, 2016, 251-264, doi: 10.1007/978-3-319-24584-3_21.
[28] R. Chand, S. A. Kumar, R. P. Chand, Navigation of an n-link revolute robotic arm via hierarchal landmarks. In 2021 3rd Novel Intelligent and Leading Emerging Sciences Conference (NILES), 2021, 188-192, doi:10.1109/NILES53778.2021.9600539.
[29] S. A. Kumar, J. Vanualailai, A. Prasad, Assistive technology: autonomous wheelchair in obstacle-ridden environment. *PeerJ Computer Science*, 2021, **7**, e725, doi: 10.7717/peerj-cs.725.
[30] J. Raj, K. Raghuwaiya, J. Vanualailai, B. Sharma, Path planning of multiple mobile robots in a dynamic 3D environment. In Advances in Computer, Communication and Computational Sciences: Proceedings of IC4S 2019, 2021, 209-219, doi: 10.1007/978-981-15-4409-5_19.
[31] J. Vanualailai, J. Raj, K. Raghuwaiya, Autonomous quadrotor maneuvers in a 3D complex environment. In Advances in Computer, Communication and Computational Sciences: Proceedings of IC4S 2019, 2021, 221-231, doi: 10.1007/978-3-030-75275-0_75.
[32] J. Vanualailai, B. Sharma, S. I. Nakagiri, An asymptotically stable collision-avoidance system. *International Journal of Non-Linear Mechanics*, 2008, **43**, 925-932, doi: 10.1016/j.ijnonlinmec.2008.06.012.
[33] A. K. Khalaji and S. A. Moosavian, Robust adaptive controller for a tractor–trailer mobile robot. *IEEE/ASME Transactions On Mechatronics*, 2013, **19**, 943-953, doi: 10.1109/TMECH.2013.2261534.
[34] S. Zhou, H. Zhao, W. Chen, Z. Miao, Z. Liu, H. Wang, Y. Liu, Robust path following of the tractor-trailers system in GPS-denied environments. *IEEE Robotics and Automation Letters*, 2019, **5**, 500-507, doi: 10.1109/LRA.2019.2956380.
[35] J. H. Lee, W. Chung, M. Kim, J. B. Song, A passive multiple trailer system with off-axle hitching, *International Journal of Control, Automation, and Systems*, 2004, **2**, 289-297.
[36] O. Khatib, Real-time obstacle avoidance for manipulators and mobile robots. *The International Journal of Robotics Research*, 1986, **5**, 90-98. doi:10.1177/027836498600500106.
[37] B. Sharma, J. Vanualailai, S. Singh, Lyapunov-based nonlinear controllers for obstacle avoidance with a planar n-link doubly nonholonomic manipulator. *Robotics and Autonomous Systems*, 2012, **60**, 1484-1497, doi: 10.1016/j.robot.2012.07.014.
[38] B. Sharma, J. Vanualailai, S. Singh, Tunnel passing maneuvers of prescribed formations. *International Journal of Robust and Nonlinear Control*, 2014, **24**, 876-901, doi: 10.1002/rnc.2923.

