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Dual-loop fractional-order IMC structure for unstable systems to handle ramp input signals

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ABSTRACT One major concern in control engineering is the problem of introducing an unstable system. Such systems are even more sensitive to ramp input changes, either set-point or disturbance. We have proposed an extended fractional-order IMC (FOIMC) control for an unstable system exhibiting a time delay. The complete design involves only three adjustable parameters, such as PID tuning. In the proposed structure, the inner loop control stabilises the system, whereas the fractional IMC filter improves the overall performance, together to tackle the ramp inputs. A systematic approach is developed to tune the required design parameters to obtain the desired peak of the sensitivity function and stability margins. The proposed control is simple and can easily calculate the FOIMC parameters from the explicit formulae. The method works under practical considerations, such as process parameter perturbations and load disturbances. The developed scheme is also tested on the nonlinear continuous stirred tank reactor system. The proposed control method results in a percentage enhancement of 61.9% under the perfectly ideal condition (when the process model is equal to the actual plant) whereas 81.3% enhancement is obtained when the process parameter variations are considered for the CSTR system.

INDEX TERMS Fractional order, IMC, ramp input, chemical process, robust system, maximum sensitivity.

I. INTRODUCTION

ENGINEERING applications such as chemical reactors, boilers, and liquid-level control tanks have shown the dynamics of integrating and unstable types. These processes are quite challenging to control. Several control techniques including PID [1] and its variants [2]–[7], Internal Model Control (IMC) [8], [9], Smith predictors [10], [11], and multi-loop architectures [6], [12]–[14] have been published in the literature for integrating and unstable processes. In [1], the analytical PID design is presented for unstable plants and achieved acceptable robustness while this method resulted in excessive overshoot in the setpoint tracking response, and therefore a second-order low-pass filter was used for its suppression. Irshad and Ali [15] suggested an optimal PI-PD control by defining a new formula of the objective function for non-self-regulating plants. However, it results in aggressive control efforts and inferior robustness while improving the desired performance index. Among the some, IMC is emphasised as an effective control tool in which overall performance is governed by a single design parameter [16]. Kumar et al. [17] designed IMC-PID for an unstable second-order system that exhibits inverse response behaviour. The setpoint weighting parameter was suggested to handle undesired overshoots in [17] and [18]. The IMC-PID stated above gives a fast set-point tracking response, however, the disturbance rejection performance is slow and inferior. These drawbacks of IMC-PID are overcome with fractional order PID (FOPID) as it contains two additional tuning factors in addition to the proportional, derivative, and integral gains [19]–[21]. Trivedi and Padhy [22] proposed an indirect design method of a fractional IMC system for various fractional-order models. Equilibrium optimiser-based frequency-shifted IMC-PD decoupled two-loop control is recently reported in [23] design for industrial plants to achieve improved performance and robustness. However, their tuning strategy seems a little lengthy and iterative. The fractional

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control is reported using the concept of Bode's ideals in [24] and [25]. In the recent past, tilted integral derivative (TID) and FOIMC have been effectively used to regulate the output of unstable processes in [26]–[28]. In [27], a fractional order TID controller was developed using the IMC approach for unstable systems with time delays. In that method, the output performed well for set-point tracking; however, it did not reject the step load disturbance that appeared at the plant input.

A recent study with a fractional-order ramp controller [29] presented for a stable noninteger plant. So far, it has been noticed that the modified IMC and FOIMC strategies referred to above have not suggested any systematic approach to select a filter or required parameters. Rather, these parameters are chosen randomly in the reported work, which may cause undesirable behaviour when applied to the real industrial system. In the lack of a specific method of selecting design parameters, the user will have to make some initial guesses followed by fine-tuning which is an iterative approach and time-consuming.

This paper presents a novel FOIMC architecture with a fractional order filter. The selection of unknown parameters is simple using a single designed constraint. The limitations of the previous methods are overcome with guaranteed stability during high-order input signals and perturbations. The key contributions are as follows.

- A double-loop FOIMC tuning method is proposed to handle ramp inputs for both set-point and disturbances.
- The tuning parameters are selected using simplified relation and based on the maximum sensitivity selection.
- The presented structure can provide guaranteed stability with any chosen fractional order of an IMC filter within the obtained stability region. Thus, it is capable of handling high model mismatch.
- The capability of the suggested scheme is justified on a non-linear CSTR system in the presence of step- and ramp-type disturbances.

The remainder of the paper is organized as follows. The content of Section II explains the proposed fractional-order IMC control. The results and investigation are given in Section III. Lastly, the conclusion is presented in Section IV.

II. PROPOSED FRACTIONAL-ORDER IMC STRUCTURE

The structure under consideration is shown in Fig. 1 where r_1 is the input to the setpoint and r_2 is the input to the plant P. The input load disturbance and measurement noise are considered by variables d and N, respectively. The proposed design of the complete architecture is a two-fold approach. First, the controller C_i is tuned to obtain the desired robustness (maximum sensitivity) of the inner loop (IL). Then, the fractional-order IMC (FOIMC) controller, namely C_f is designed to achieve the desired shape of the output loop (OL).

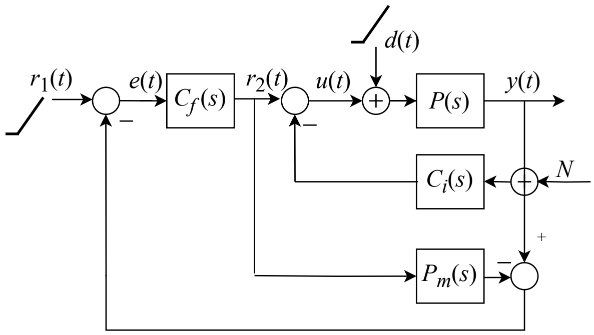

FIGURE 1: Proposed FOIMC structure.

A. INNER LOOP DESIGN

From Fig. 1, the following relation is obtained:

$$\frac{Y(s)}{R_2(s)} = \frac{P(s)}{1 + P(s)C_i(s)} \tag{1}$$

where the controller C_i is considered as $[K_p(1 + T_d s/(1 + T_d s/N)]$. Here K_p and T_d are the proportional and derivative gains, respectively. The integer number N represents the filter constant introduced to represent the controller's transfer function as a proper one. Now, let the dynamics of many industrial plants represented by the simple first-order transfer function be defined below.

$$P(s) = \frac{Ke^{-\theta s}}{Ts - 1} \tag{2}$$

where, K process gain, T time constant and θ time delay are known in our case. After substituting (2) into (1) and using the following approximation

$$e^{-\theta s} = \frac{1 - 0.5\theta s}{1 + 0.5\theta s} \tag{3}$$

we get,

$$\frac{Y(s)}{R_2(s)} = \frac{K(1+0.5\theta s)(T_d s/N+1)e^{-\theta s}}{(T_d s/N+1)(Ts-1)(1+0.5\theta) + KK_p(1-0.5\theta)[(T_d s/N+1)+T_d s]} \tag{4}$$

To obtain the analytical tuning formula of C_i, the term $(T_d s/N + 1)$ may be approximated as 1 as the derivative filter has considerably higher bandwidth than the process [30]. Thus, (4) becomes

$$\frac{Y(s)}{R_2(s)} = \frac{K(1+0.5\theta s)e^{-\theta s}}{[0.5T\theta - 0.5\theta KK_pT_d]s^2 + [(T-0.5\theta s) + KK_p(T_d - 0.5\theta s)]s + [KK_p - 1]} \tag{5}$$

Let us take the intended transfer function as [5], [31],

$$\left(\frac{Y(s)}{R_2(s)}\right)_d = \frac{K(1+0.5\theta s)e^{-\theta s}}{(1+\beta s)^2} \tag{6}$$

Comparing the denominator of (5) and (6), one obtains as

$$\beta^2 = (0.5T\theta - 0.5\theta KK_pT_d)/(KK_p - 1) \tag{7}$$

$$2\beta = ((T - 0.5\theta) + KK_p(T_d - 0.5\theta))/(KK_p - 1) \tag{8}$$

By solving (7) and (8), one can easily obtain the controller parameters for C_i as below.

$$\begin{aligned} K_p &= \frac{0.5\theta(T-0.5\theta+2\beta)+(0.5\theta T+\beta^2)}{K\beta^2+0.5\theta(2\beta K+0.5\theta K)}, \\ T_d &= \frac{0.5\theta T+\beta^2-\beta^2 K K_p}{0.5K\theta} \end{aligned} \tag{9}$$

B. OUTER LOOP DESIGN

Let us consider now designing C_f. To include the effect of IL, P_m is taken as the model equivalent and is defined below.

$$P_m = \frac{Y(s)}{R_2(s)} \tag{10}$$

Using (5),

$$P_m(s) = \frac{K(1+0.5\theta)e^{-\theta s}}{(Ts-1)(1+0.5\theta)+KK_p(1+T_d s)(1-0.5\theta)} \tag{11}$$

In the first step, decompose the model P_m into two parts as,

$$P_m(s) = P_{m+}(s)P_{m-}(s) \tag{12}$$

where $P_{m+}(s)$ contains all time delays and unstable zeros (non-invertible) and $P_{m-}(s)$ contains minimum phase elements (invertible). It can be written as,

$$P_{m-}(s) = \frac{K(1+0.5\theta)}{(Ts-1)(1+0.5\theta)+KK_p(1+T_d s)(1-0.5\theta)} \tag{13}$$

The FOIMC for OL is then expressed as the inverse of the invertible portion of the process model,

$$C_f(s) = P_{m-}^{-1} f(s) \tag{14}$$

In this approach, unlike the traditional IMC, the system performance is governed by the double-tuning parameters. There are various filter transfer functions proposed in the literature [9]. To make a flexible and tunable filter, the order is chosen as a real positive value. This way the modified new FO filter becomes,

$$f(s) = \frac{1}{(1+\lambda s^\alpha)^2}. \tag{15}$$

Here, the FO derivative α is an additional tuning parameter. By selecting the suitable values of the parameters α and λ, smooth and fast set-point tracking, rapid disturbance rejection response and adequate robustness are achieved simultaneously in our work. In terms of realization, the suggested FOIMC also has the same number of tunable parameters (three) as that of the widely used PID. With recent advances in fractional-order controllers, several approaches have been reported in the literature to implement the same. Some of these approaches are FOMCON toolbox [22], FPGA platform [32], and PIC16F876 (Microchip Technology) [33].

Now by substituting (13) and (15) into (14), the expression of C_f is obtained as below.

$$C_f(s) = \frac{(Ts-1)(1+0.5\theta)+KK_p(1+T_d s)(1-0.5\theta)}{K(1+0.5\theta)(1+\lambda s^\alpha)^2} \tag{16}$$

Using IMC, the overall transfer function becomes

$$\frac{y(s)}{r_1(s)} = C_f(s)P_m(s) \tag{17}$$

In the following section, the values of the parameters β, α and λ are obtained based on IL and OL maximum sensitivity requirements.

III. SELECTING TUNING PARAMETERS

The proposed FOIMC tuning has only three tuning parameters. The behaviour of the suggested control solely depends on values of β (IL), and α, λ (OL). They are required to be calculated appropriately for satisfactory performance and robustness. The literature shows that the general range for unstable systems M_s is between 1.5 to 3.5. This range ensures satisfactory robustness of the proposed control system in the face of undesired circumstances such as load fluctuations, noise, and mismatching of the plant model with the actual system. To ease in design and uniform condition, we have assumed the same M_s for both loops.

First, the explicit formulae for IL C_i in (9) is solved for β using the desired robustness M_s as,

$$M_s = \|1/(1+C_i(s)P(s))\|_\infty \tag{18}$$

Above expression involves unknown β which is calculated to meet the desired M_s.

Now, using (16) and (17), the complementary sensitivity function, $T_C(s) = y(s)/r_1(s)$ of the OL is obtained as,

$$T_C(s) = \frac{e^{-\theta s}}{(1+\lambda s^\alpha)^2} = \frac{1-\theta s}{(1+\lambda s^\alpha)^2} \tag{19}$$

Remark 1: If $T^{'}(s)$ is the corresponding open-loop transfer function of the equivalent unity feedback system, then $T^{'}(s) = T_C(s)/(1-T_C(s))$. Thus, the expression of maximum sensitivity for OL is $M_s = \left\|1/(1+T^{'}(s))\right\|_\infty$.

A. OL TUNING USING PHASE MARGIN

As per Remark 1 and using (19), one can obtain the expression,

$$T^{'}(j\omega) = \frac{1-j\omega\theta}{(1+\lambda\omega^\alpha j^\alpha)^2-(1-j\omega\theta)} \tag{20}$$

If ω_g is the gain cross over frequency, (20) reduces to

$$|1-j\omega_g\theta| = |(1+\lambda\omega_g^\alpha j^\alpha)^2-(1-j\omega_g\theta)| \tag{21}$$

Taking the magnitude of both sides of the above expression and simplifying it, we get

$$\begin{aligned} \lambda^4\omega_g^{4\alpha}\cos^2(\alpha\pi) + 4\lambda^2\omega_g^{2\alpha}\cos^2(\alpha\pi/2) \\ +4\lambda^3\omega_g^{3\alpha}\cos(\alpha\pi)\cos(\alpha\pi/2)+4 \\ +(\omega_g\theta)^2+\lambda^4\omega_g^{4\alpha}\sin^2(\alpha\pi)+4\lambda^2\omega_g^{2\alpha}\sin(\alpha\pi) \\ +4\omega_g\theta+2\lambda^2\omega_g^{2\alpha+1}\sin(\alpha\pi)+3=0 \end{aligned} \tag{22}$$

Also, the phase margin (PM) of the system given in (20) is

$$\mathrm{PM} = \pi - \tan^{-1}\omega_g\theta - \tan^{-1}\frac{2+\lambda^2\omega_g^{2\alpha}\sin(\alpha\pi)+\omega_g\theta}{\lambda^2\omega_g^{2\alpha}\cos(\alpha\pi)+2\lambda\omega_c^{\alpha}\cos(\alpha\pi/2)} \quad (23)$$

By solving (22) and (23), suitable λ can be determined for a given value of α and desired PM.

To understand the design steps and observe the effect of λ and α on M_s, taking the plant

$$P(s) = 2e^{-s}/(4s-1) \quad (24)$$

The selection procedure for λ and α is explained below from a stabile region that is obtained by taking PM $\in [30^o, 60^o]$.

Fig. 2 illustrates that the desired PM range can be obtained with $\alpha \in (1, 1.5)$ and $\lambda \in (0.1, 5.0)$. These ranges are obtained by conducting various simulations with varying α and λ and observing their effect on PM. Any set of values for λ and α may be selected from the shaded region for controller tuning. In addition, the effect of varying λ and α on OL M_s is investigated as seen in Fig. 3. It helps reduce the burden of parameter tuning. From this figure, λ can be chosen for a possible range of $\alpha \in (1, 1.5)$ to satisfy the user-defined M_s.

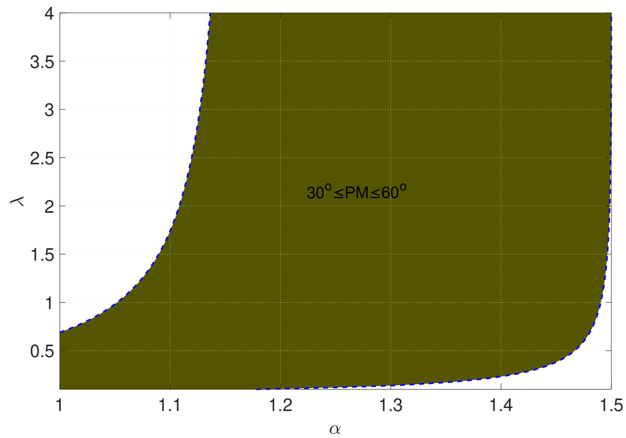

FIGURE 2: Stable range for α and λ with PM

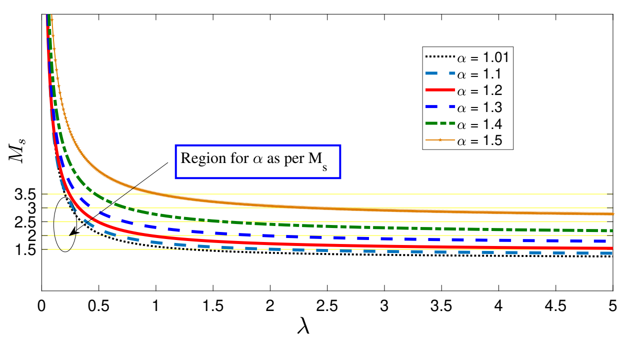

FIGURE 3: Possible values of λ and α as per M_s

TABLE 1: Design parameters of the proposed control

Example	M_s	K_p	T_d	α	λ	β
1	2.3	1.735	0.474	1.2	0.6	0.69
2	2.0	0.782	0.482	1.15	0.77	0.64
3	3.2	2.295	0.521	1.3	0.39	0.68
CSTR	2.0	1.195	9.234	1.1	3.5	14.5

B. SUMMARY OF TUNING APPROACH

- For a plant model (2), tune C_i using explicit formulae in (9) for β according to the desired M_s.
- Solve (22) and (23) to determine the appropriate λ according to M_s, PM and α.
- Tune C_f using (16).

IV. NUMERICAL STUDY

To verify the proposed control scheme, we have provided a comparative analysis with recently reported methods. The numerical comparisons are measured with indices such as integrals of absolute error (IAE), integrals of squared error (ISE), and integrals of time absolute error (ITAE). In addition, the ISU (integral of the squared control variable, u) is determined to estimate the input energy usage. The lower value of ISU signifies better controller tuning and therefore less wear and tear of movable parts in a plant. It is important to mention that the high-frequency noise can be mitigated with C_i by cascading with a low-pass filter as $1/(1+0.001s)$.

A. EXAMPLE: 1

The following process is considered to compare with other methods [2], [3], [5].

$$P(s) = \frac{2e^{-s}}{4s-1} \quad (25)$$

is considered. For fair comparisons, we have tuned the suggested controllers from the literature with the same M_s of 2.3.

Following the proposed approach, the controller parameters obtained are listed in Table 1 in the first row. Now, the unit step change in the setpoint input at time $t = 0$ and a step disturbance of -0.1 at $t = 20$, are assumed. The output and control signals from the proposed method and others are plotted in Figs. 4 and 5. It showed that the FOIMC control provided comparatively faster and smoother responses.

Let us assume a mismatch of +10% in both K and θ and -10% in T simultaneously to verify the robustness and keep the same controller. The corresponding responses are shown in Fig. 6. The numerical values of various indices are also tabulated in Table 2, and the smaller values demonstrated the excellent performance of our method.

Further, the capability of tracking the step-up or down type of command signal of various methods are compared. The resulting outputs are displayed in Fig. 7. It shows that the performance of all the methods are compared to each other except [3]. Large overshoots and undershoots appear in the output of the above cited work.

TABLE 2: Numerical comparisons

Ex.	Method	Normal					Perturbed				
		ISE	IAE	ITAE	ISU	%Δ	ISE	IAE	ITAE	ISU	%Δ
1	Proposed	2.5	4.0	32.9	26.4	13.9	2.6	4.5	42.0	25.0	5.7
	Kumar and Ajmeri, 2023	3.0	5.0	42.4	26.0		3.1	5.6	56.0	24.2	
	Kishore and Sree, 2018	3.0	5.0	39.8	37.7		4.2	5.9	47.0	39.8	
	Vanavil et al., 2015	2.5	4.6	45.0	26.4		2.5	4.8	47.3	24.0	
2	Proposed	3.0	4.8	49.0	5.6	19.2	3.5	5.1	54.5	6.0	8.5
	Peker and Kaya, 2023	5.5	8.1	106.0	5.9		6.8	10.3	167.0	6.4	
	Chakraborty et al., 2017	3.7	6.0	62.0	5.5		4.0	5.8	60.0	5.7	
	Zhang et al., 2020	2.6	5.5	69.0	6.5		3.4	6.0	70.0	7.6	
3	Proposed	22.6	28.5	740.0	992.0	62.7	29.7	29.5	769.0	984.0	62.8
	Alyoussef and Kaya, 2023	37.7	30.7	801.0	4070.0		47.0	32.8	866.0	4142.0	
	Ozyetkin et al., 2020	88.0	48.4	1274.0	3369.0		110.0	49.7	1318.0	3396.0	
CSTR	Proposed	1167.0	400.0	57040.0	52270.0	61.9	1223.0	401.0	56.5	52560.0	81.3
	Ajmeri and Ali, 2015	1416.0	739.0	238500.0	50470.0		1469.0	740.0	237200.0	50660.0	
	Begum et al., 2020	769.0	350.0	79620.0	320800.0		826.5	350.0	78760.0	321200.0	

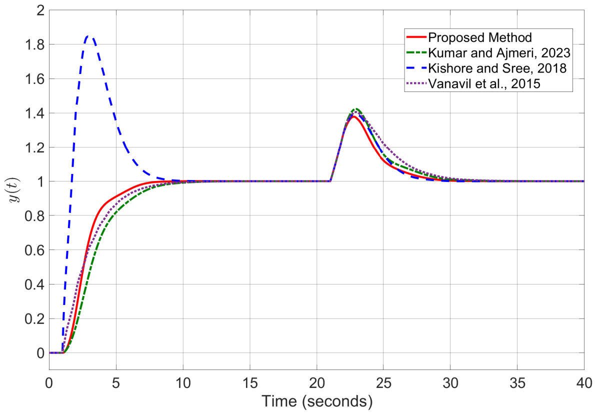

FIGURE 4: Outputs for Example 1 (Normal)

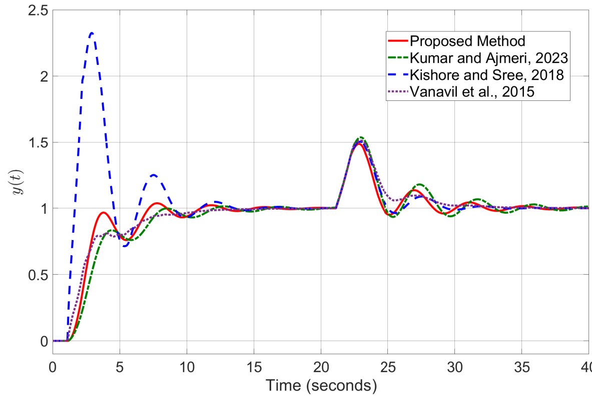

FIGURE 6: Outputs for Example 1 (Perturbed)

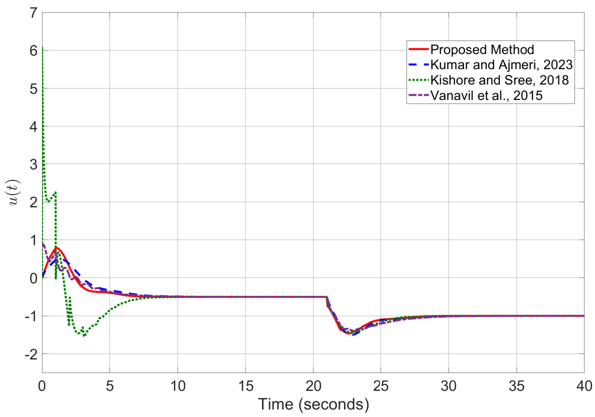

FIGURE 5: Control signals for Example 1 (Normal)

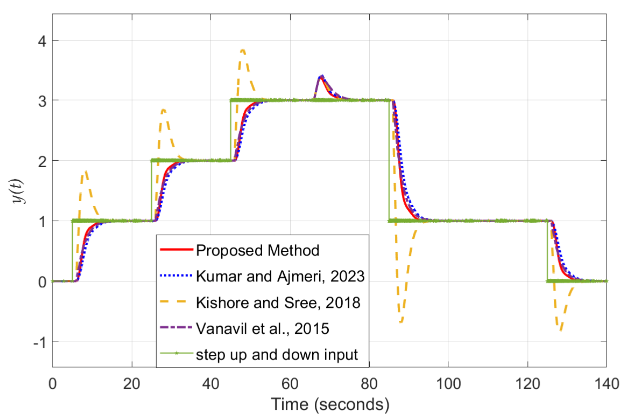

FIGURE 7: Outputs for Example 1 for a step up and down command

B. EXAMPLE: 2

Let us take an integrating process which is studied in the number of works [4], [7], [30] as given below:

$$P(s) = e^{-s}/s \tag{26}$$

The performance of the suggested control is contrasted with recent methods [4], [7]. Again, the input of the setpoint is taken as a unit step at $t = 0$ and the load disturbance of the step type with magnitude 1.5 at $t = 10$. Figs. 7 and 8 is displayed the closed-loop responses and the control signals, respectively, when the plant model exactly matches the process to be controlled. Fig. 9 shows the resulting signals under the perturbation of +25% in the time delay. The measured values from Table 2 and the output plotted depicted that the new control method would perform incredibly well compared to the others. It provided smaller IAE, ITAE and ISU under nominal and perturbed conditions. Note that Chakraborty et al. [30] gave a smaller ISU; however, the output is very sluggish. Their method took a longer time to attenuate the load disturbance.

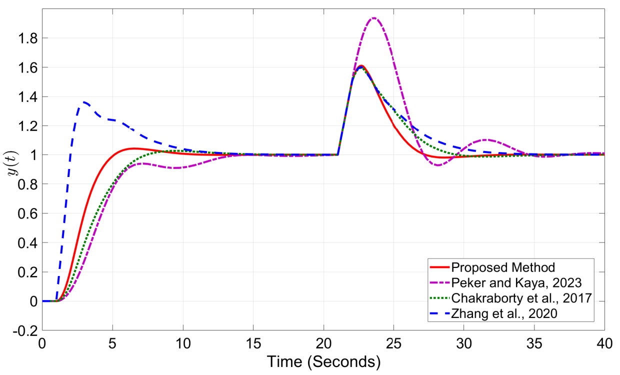

FIGURE 8: Outputs for Example 2 (Normal)

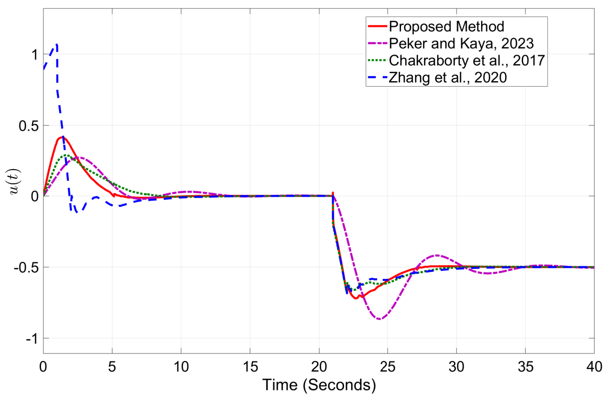

FIGURE 9: Control signals for Example 2 (Normal)

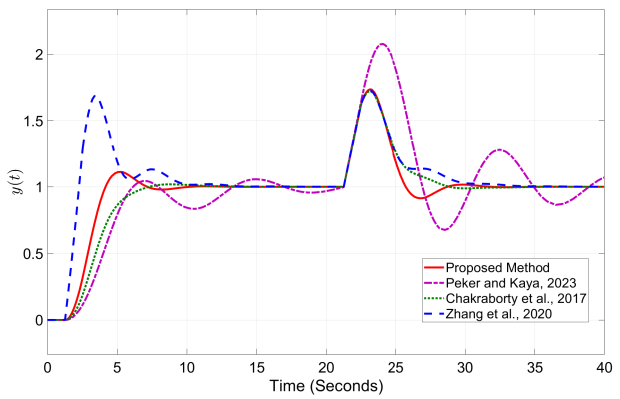

FIGURE 10: Outputs for Example 2 (Perturbed, +25% change in θ)

C. EXAMPLE: 3

Let us take a second-order process

$$P(s) = \frac{e^{-0.5s}}{(0.5s+1)(2s-1)} \tag{27}$$

The same process was considered in [34], [35] with four parameters PI-PD controller. A PI-PD [34] with controller settings was $K_p = 0.492$, $K_i = 0.340$, $K_f = 1.878$ and $K_d = 0.760$. The controller in [35] had parameters $K_p = 0.1955$, $K_i = 0.1906$, $K_f = 1.7222$ and $K_d = 0.4325$.

Note that the presented structure has only three tuning parameters. First, the process was approximated as

$$P(s) = \frac{e^{-1.067s}}{2.347s-1} \tag{28}$$

and chosing $M_s = 3.26$ for a fair comparison as in [34]. The parameters of our method are tabulated in Table 1. Now, at $t = 0$ a ramp input of 0.2 unit slope is applied at the set-point. Following a step disturbance of 5 at $t = 25$ is assumed at the input of the plant. The proposed method performed very well for the set-point and in the rejection of disturbances, as seen in Figs. 10 and 11. It is noted that the method [35] produces a steady-state error with ramp signal input. To test the mismatch of the model, a perturbation of +25% in θ was considered. Again, our method performed better compared to the others, as shown in Fig. 12. The values of various indices are tabulated in Table 2. That also proved the proposed control outperformed with fewer tuning parameters for ramp-type references.

In our work, the percentage enhancement ($\%\Delta$) achieved by the suggested control scheme is determined using the following relation:

$$\%\Delta = \frac{\sum_R - \sum_P}{\sum_R} \times 100 \tag{29}$$

where $\sum_R$ is the performance of the reported method corresponding to the smaller value of the sum-total of ISE, ITAE, IAE and ISU indices. The quantity $\sum_P$ is the sum of all these indices from the proposed FOIMC approach. Observing Table 2, one can see that percentage enhancement of 13.9, 19.2 and 62.7 are obtained with the proposed FOIMC, respectively, in the three studied examples for normal conditions. The proposed FOIMC gives percentage enhancement of 5.7, 8.5 and 62.8 in the above-said examples under the assumed perturbed conditions.

V. APPLICATION TO CSTR PROCESS

The isothermal chemical reactor is commonly used in the process industry. The process dynamics in terms of differential equation is described by [8]

$$\frac{dC}{dt} = \frac{Q}{V}(C_F - C) - \frac{k_1 C}{(k_2 C + 1)^2} \tag{30}$$

where Q and C_F represent inlet flow rate and concentration, respectively and C indicates the reactor's exit concentration. The parameter values are $Q = 0.0333L/s$, $k_1 = 10L/s$,

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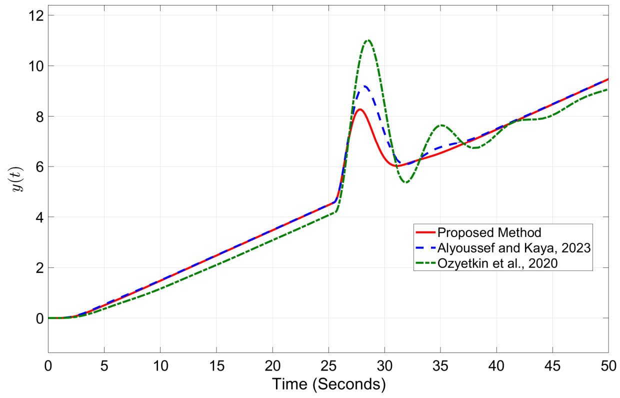

FIGURE 11: Outputs for Example 3 (Normal) with a ramp input

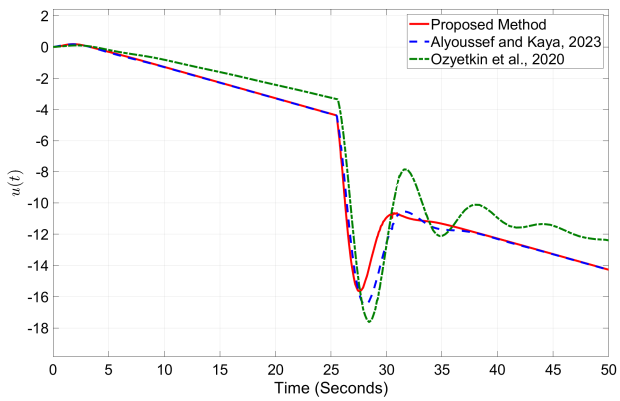

FIGURE 12: Control signals for Example 3 (Normal) with a ramp input

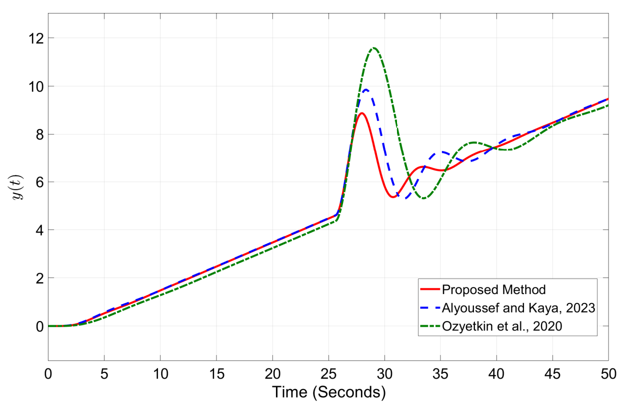

FIGURE 13: Outputs for Example 3 (Perturbed, +20% change in θ)

$k_2 = 10L/mol$, and the reactor volume is $V = 1L$. The desired steady state is taken as $C = 1.316$ with the nominal value $C_F = 3.288mol/L$.

For applying the proposed FOIMC as seen in Fig. 13, the nonlinear process model is first linearized and the feed concentration is taken as the controlled variable. The approximated model by assuming a process time delay of 20sec is obtained as

$$P(s) = 3.433e^{-20s}/(103.1s - 1) \tag{31}$$

It is to be noted that this model is used to obtain the control laws; however, the designed scheme is applied to the actual nonlinear system defined by (30). Now, the unity step changes along with step disturbance at $t = 1000$ sec is assumed. The results are plotted in Figs. 15 and 16. From these figures, it is observed that the method suggested in [12] yields a slow output and takes a long time to settle. Although Begum et al. [8] produces a sufficiently fast response, their method shows a very large ISU as seen in Table 2. In comparison, the presented FOIMC strategy results in acceptable performance and robustness with a small ISU value. Furthermore, robustness is also verified by assuming a model mismatch of +20% in θ. The corresponding plots are shown in Fig. 16. Table 2 shows that the presented control achieves $\%\Delta$ of 61.9 and 81.3 for normal and perturbed conditions, respectively.

To assess the robustness of the FOIMC scheme against noisy output, we have added white noise with a variance of 0.001 to the output. The corresponding outputs are plotted in Fig. 17. More interestingly, a slope disturbance ramp of 0.05 at 1000 sec is introduced before the plant input. The relevant outcomes are shown in Fig. 18. It shows that the proposed approach produces reduced steady-state offset during the ramp-type disturbances, thereby outperforming other controllers.

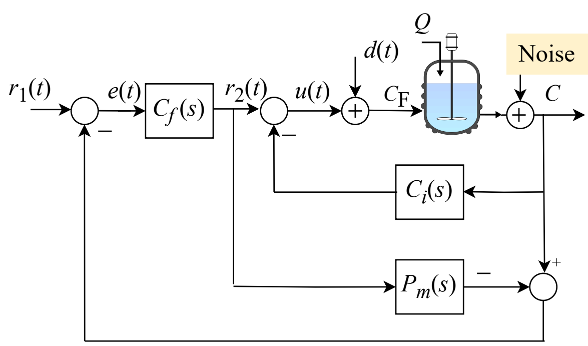

FIGURE 14: Scheme applied to CSTR process with a ramp disturbance and noise.

VI. CONCLUSION

This paper presented the extended FOIMC design for unstable systems, considering ramp-type input signals. Like the classical PID structure, the suggested FOIMC scheme has only three parameters, the selection of which is based on achieving the desired maximum sensitivity. In addition, the stability of the proposed control is guaranteed by choosing fractional IMC filter constants from the stability region corresponding to the PM $\in [30^o, 60^o]$. The proposed design helps the user find the controller settings in one attempt.

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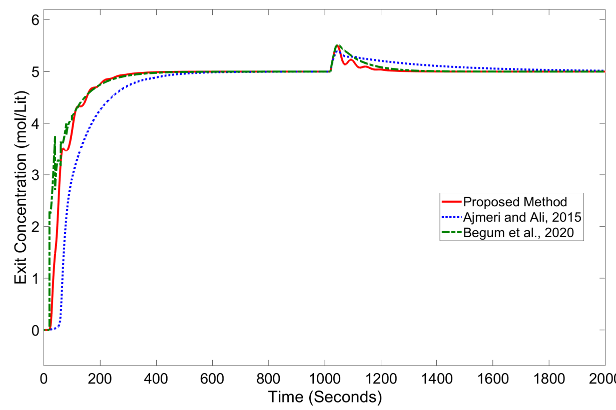

FIGURE 15: Exit concentration of CSTR under perfect modeling

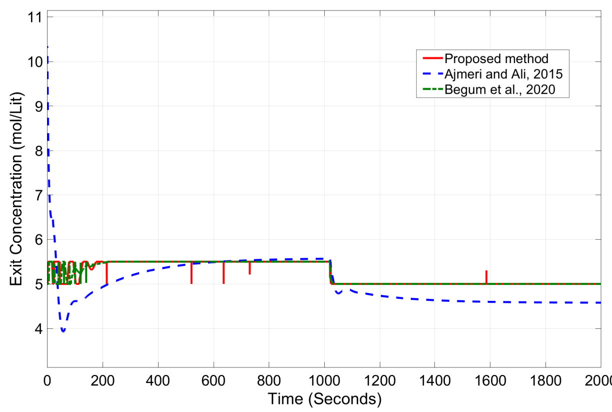

FIGURE 16: Inlet concentration of CSTR under perfect modeling.

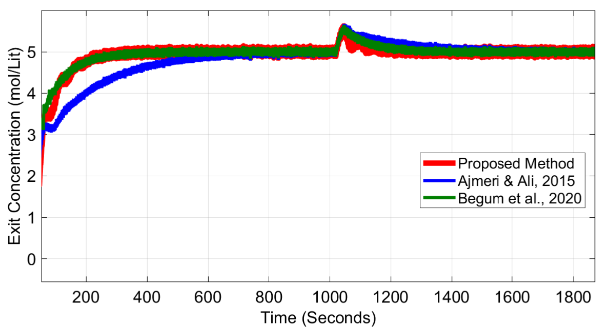

FIGURE 17: Exit concentration of CSTR in the presence of the considered noise.

The structure demonstrated stability and robustness with a high model mismatch and improved performance compared to existing approaches. The technique was implemented on a non-linear CSTR plant by considering the step and ramp load disturbances. It is concluded that the proposed approach resulted in 61.9% overall percentage enhancement under nominal conditions and 81.3% enhancement is obtained in the presence of the process parameter variations. Current

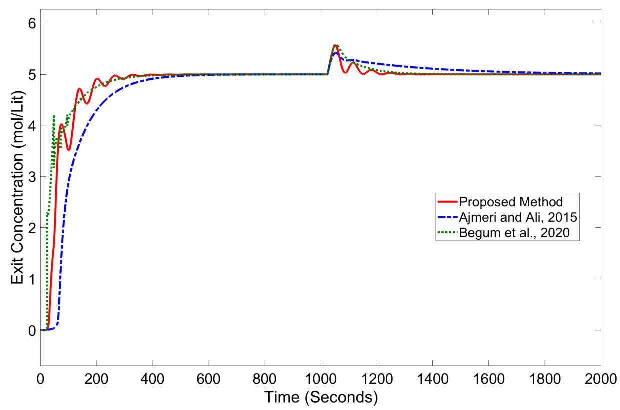

FIGURE 18: Exit concentration when perturbed θ +20% in the acutal CSTR.

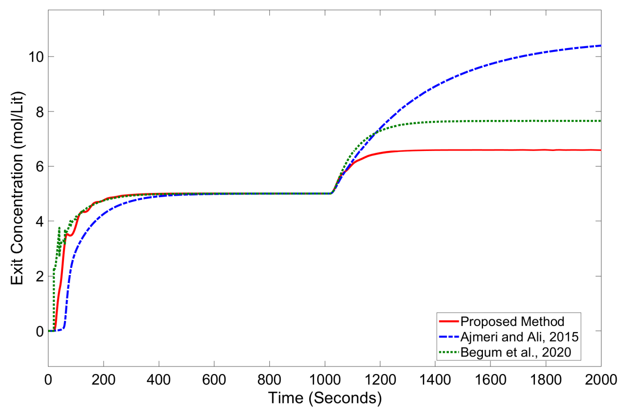

FIGURE 19: Exit concentration when a ramp disturbance input.

research is focused on the control of unstable low-order models. The suggested method can be extended for the higher-order integrating and unstable plants. Second, in the design approach, the fractional-order filter time constant was selected using the value α and the desired phase margin. After an extended numerical study, an attempt can be made to make an explicit formula to design filter parameters.

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