Motion Planning and Control of a Swarm of Boids in a 3-Dimensional Space

Bibhya Sharma, Jito Vanualailai, Jai Raj

Abstract—In this paper, we propose a solution to the motion planning and control problem for a swarm of three-dimensional boids. The swarm exhibit collective emergent behaviors within the vicinity of the workspace. The capability of biological systems to autonomously maneuver, track and pursue evasive targets in a cluttered environment is vastly superior to any engineered system. It is considered an emergent behavior arising from simple rules that are followed by individuals and may not involve any central coordination. A generalized, yet scalable algorithm for attraction to the centroid and inter-individual swarm avoidance is proposed. We present a set of new continuous time-invariant velocity control laws, formulated via the Lyapunov-based control scheme for target attraction and collision avoidance. The controllers provide a collision-free trajectory. The control laws proposed in this paper also ensures practical stability of the system. The effectiveness of the control laws is demonstrated via computer simulations.

Keywords—Swarm, Practical stability, Motion planning.

I. Introduction

SWARMING is based on many exciting, attractive and stimulating entities that cooperate in order to exhibit a desired behavior. Inspiration for the design of these behaviors is taken from the collective behavior of social insects such as ants, termites, bees, and wasps, as well as from the behavior of other animal societies such as flocks of birds or schools of fish [1]. Even though single members of these societies are unsophisticated individuals, they are able to achieve complex tasks in cooperation [1]. Coordinated behavior emerges from relatively simple actions or interactions between the individuals [2]. The swarming behavior is a complex emergent behavior that occurs when individual agents follow simple behaviorial rules.

The fact that certain engineering problems can be solved in an ingenious way by roughly mimicking this natural phenomenon [3], [4], has led to greater efforts by mathematicians, engineers, computer scientists, physicists and biologists, in recent years, to seek better understanding of self-organization in organisms, and the formation and the persistence of aggregations [5], [6].

The emerging swarm behavior and its principles are now being used by scientists and researchers in many new approaches such as in optimization and in control of robots [7], [8], [9]. The use of robots with the concept of swarming is significantly increasing in the manufacturing arena, not only for productivity enhancement but also for greater versatility and flexibility [10].

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In literature, the flocking models are built within a framework of three basic rules of steering namely separation, alignment and cohesion.

These flocking rules describe how an individual maneuvers based on the positions and velocities of its nearby flock mates [11], [12]. Although the rules governing each member of a flock are seemingly basic, the collective motion is strikingly spectacular. The superposition of these rules results in the flock mates moving in a particular formation [13], with a common heading whilst ensuring all possible collision and obstacle avoidances [14], that is, basically a life-like behavior emerges from the flocking rules.

This paper considers the navigation problem of a three-dimensional swarm via an artificial potential fields (APF) method: Lyapunov based control scheme (LbCS). It will be shown that the LbCS is effective in designing the continuous time-invariant velocity control laws. In essence, we design a motion planner derived form the LbCS, that guarantees the establishment and maintenance of a geometrical formation of a swarm of boids, considering all practical limitations and constraints. This paper, will in general showcase and mimic the patterns arising from the emergent behavior of the swarms into various forms of simulations.

We will use the following two terms from [15] in this paper as we develop our Lyapunov-like function for system (2):

1) A *cohesive* group is a group in which the distances between individuals are bounded from above (members of a cohesive group tend to stay together and avoid dispersing).
2) A *well-spaced* group is a group which does not collapse into a tight cluster, i.e., where some minimal bin size exists such that each bin contains at most one individual. Moreover, the size of such a bin is independent of the number of individuals in a group.

II. A Three-Dimensional Swarm Model and its Practical Stability

We shall construct a model of a swarm with n individuals moving with the velocity of the swarm's centroid. At time $t \geq 0$, let $(x_i(t), y_i(t), z_i(t))$, $i = 1, 2, \ldots, n$, be the planar position of the ith individual, which we shall define as a point mass residing in a disk of radius $r_i > 0$,

$$b_i = \left[(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3) \in \mathbb{R}^3 : (\mathbf{z}_1 - x_i)^2 + (\mathbf{z}_2 - y_i)^2 + (\mathbf{z}_3 - z_i)^2 \leq r_i^2\right].$$

The sphere is described in [15] as a *bin*, and in [16] as a *private or safety area* of each individual. We shall use the former term, with *bin size* being the radius r_i of the sphere.

Let us define the *centroid of the swarm* as

$$(x_c, y_c, z_c) = \left(\frac{1}{n} \sum_{k=1}^{n} x_k, \frac{1}{n} \sum_{k=1}^{n} y_k, \frac{1}{n} \sum_{k=1}^{n} z_k \right).$$

At time $t \geq 0$, let $(v_i(t), w_i(t), u_i(t)) := (x_i'(t), y_i'(t), z_i'(t))$ be the instantaneous velocity of the ith point mass.

Using the above notations, we have thus a system of first-order ODEs for the ith individual, assuming the initial condition at $t = t_0 \geq 0$:

$$\left.\begin{aligned} x_i'(t) &= v_i(t) \\ y_i'(t) &= w_i(t) \\ z_i'(t) &= u_i(t) \\ x_{i0} := x_i(t_0), y_{i0} &:= y_i(t_0), z_{i0} := z_i(t_0). \end{aligned}\right\} \quad (1)$$

Suppressing t, we let $\mathbf{x}_i = (x_i, y_i, z_i) \in \mathbb{R}^3$ and $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n) \in \mathbb{R}^{3n}$ be our state vectors. Also, let

$$\mathbf{x}_0 = \mathbf{x}(t_0) = \underbrace{(x_{10}, y_{10}, z_{10}, \ldots, x_{n0}, y_{n0}, z_{n0})}_{3n \text{ terms}}.$$

If $\mathbf{g}_i(\mathbf{x}) := (v_i, w_i, u_i) \in \mathbf{R}^3$ and $\mathbf{G}(\mathbf{x}) := (\mathbf{g}_1(\mathbf{x}), \ldots, \mathbf{g}_n(\mathbf{x})) \in \mathbb{R}^{3n}$, then our swarm system of n individuals is

$$\dot{\mathbf{x}} = \mathbf{G}(\mathbf{x}), \;\; \mathbf{x}_0 = \mathbf{x}(t_0). \quad (2)$$

If $\mathbf{G} \in C[\mathbb{R}^{3n}, \mathbb{R}^{3n}]$, then we can invoke the definition of the practical stability of system (2) as provided by [17], noting that we do not need the existence of an equilibrium point of the system. In the definition, $\mathbb{R}_+ := [0, \infty)$.

Definition 1: System (2) is said to be

(S1) *practically stable* if given (λ, A) with $0 < \lambda < A$, we have $\|\mathbf{x}_0 - \mathbf{x}^*\| < \lambda$ implies that $\|\mathbf{x}(t) - \mathbf{x}^*\| < A$, $t \geq t_0$ for some $t_0 \in \mathbb{R}_+$;

(S2) *uniformly practically stable* if *(S1)* holds for every $t_0 \in \mathbb{R}_+$.

The following comparison principle is adapted from [17] to analyse the practical stability of system (2),

$$\begin{aligned} K &= \{a \in C[\mathbb{R}_+, \mathbb{R}_+] : a(d) \text{ is strictly increasing} \\ &\quad \text{in } d \text{ and } a(d) \to \infty \text{ as } d \to \infty\}, \end{aligned}$$

$$S(\rho) = \{\mathbf{x} \in \mathbb{R}^{3n} : \|\mathbf{x} - \mathbf{x}^*\| < \rho\},$$

and, for any Lyapunov-like function $V \in C[\mathbb{R}_+ \times \mathbb{R}^{3n}, \mathbb{R}_+]$,

$$D^+V(t, \mathbf{x}) := \limsup_{h \to 0^+} \frac{V(t + h, \mathbf{x} + h\mathbf{G}(\mathbf{x})) - V(t, \mathbf{x})}{h},$$

for $(t, \mathbf{x}) \in \mathbb{R}_+ \times \mathbb{R}^{3n}$, noting that if $V \in C^1[\mathbb{R}_+ \times \mathbb{R}^{3n}, \mathbb{R}_+]$, then $D^+V(t, \mathbf{x}) = V'(t, \mathbf{x})$, where

$$V'(t, \mathbf{x}) = V_t(t, \mathbf{x}) + V_x(t, \mathbf{x})\mathbf{G}(\mathbf{x}).$$

Theorem 1: Lakshmikantham, Leela and Martynyuk [17]. Assume that

1. λ and A are given such that $0 < \lambda < A$;
2. $V \in C[\mathbb{R}_+ \times \mathbb{R}^{3n}, \mathbb{R}_+]$ and $V(t, \mathbf{x})$ is locally Lipschitzian in $\mathbf{x}$;
3. for $(t, \mathbf{x}) \in \mathbb{R}_+ \times S(A)$, $b_1(\|\mathbf{x} - \mathbf{x}^*\|) \leq V(t, \mathbf{x}) \leq b_2(\|\mathbf{x} - \mathbf{x}^*\|)$, $b_1, b_2 \in K$ and $D^+V(t, \mathbf{x}) \leq q(t, V(t, \mathbf{x}))$, $q \in C[\mathbb{R}_+^3, \mathbb{R}]$;
4. $b_2(\lambda) < b_1(A)$ holds.

Then the practical stability properties of the scalar differential equation

$$h'(t) = q(t, h), \;\; h(t_0) = h_0 \geq 0,$$

imply the corresponding practical stability properties of system (2).

III. Deployment of Lyapunov-based Control Scheme

The principal objective of this section is to utilize the Lyapunov-based control scheme to design the velocity controls, v_i, w_i,and u_i, such that the swarm of boids will be able to exhibit unique swarming behavior in certain direction. The control scheme appropriately combines these potential functions to form a Lyapunov-like function candidate – a platform to design the nonlinear velocity controllers for the swarm of boids. A dichotomy of potential functions will be designed in the following subsections: the attractive potential function for convergence and the repulsive potential function that repels the swarm from specified obstacles in a defined workspace.

A. Attraction to the Centroid

We can ensure that the individuals of the swarm are attracted towards each other and also form a cohesive group by having a measurement of the distance from the ith individual to the swarm centroid. This is the concept behind *flock centering*, which is one of the well-known three heuristic flocking rules of Reynolds' [11]. The rule stipulates that the individuals stay close to the nearest flock mates. It is therefore a form of attraction between individuals. Centering necessitates a measurement of the distance from the ith individual to the swarm centroid. Thus, we define

$$\begin{aligned} R_i(\mathbf{x}) \;\; := \;\; & \frac{1}{2} \left[\left(x_i - \frac{1}{n} \sum_{i=1}^{n} x_i \right)^2 \right. \\ & + \left(y_i - \frac{1}{n} \sum_{i=1}^{n} y_i \right)^2 \\ & \left. + \left(z_i - \frac{1}{n} \sum_{i=1}^{n} z_i \right)^2 \right], \;\; i \in \mathbb{N}. \end{aligned}$$

This will be part of a Lyapunov-like function for system (2), and as we shall see later, its role is to ensure that ith individual is attracted to the swarm centroid.

B. Inter-individual Collision Avoidance

The short range repulsion requirement between individuals necessitates first a measurement of the distance between the ith and the jth individuals, $j \neq i$, $i, j \in \mathbb{N}$. With (1) of the ith individual in mind and for the boids to avoid each other, we consider the function

$$\begin{aligned} Q_{ij}(\mathbf{x}) \;&:=\; \frac{1}{2}\Big[(x_i - x_j)^2 + (y_i - y_j)^2 \\ &\quad + (z_i - z_j)^2 - (r_i + r_j)^2\Big]. \end{aligned}$$

The function is an Euclidean measure of the distance between the individual boids, and will appear in the denominator of an appropriate term in the candidate Lyapunov-like function to be proposed.

IV. Design of the Velocity Controllers

The nonlinear control laws for system (1) will be designed using the LbCS. In parallel, the control scheme will then utilize Theorem 1 to provide the mathematical proof of the practical stability of the system (1).

A. Lyapunov-like Function

As per the LbCS, we combine the attractive and the repulsive potential functions. We introduce *tuning parameters (or control parameters)*, that is, let there be real numbers $\gamma_i > 0$, $\beta_{ij} > 0$, and define, for $i, j = 1, \ldots, n$, a Lyapunov-like function for system (1) as

$$L_i(\mathbf{x}) = \gamma_i R_i(\mathbf{x}) + \sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij} R_i(\mathbf{x})}{Q_{ij}(\mathbf{x})}. \tag{3}$$

Next, we consider a Lyapunov-like function for system (2) as

$$L(\mathbf{x}) := \sum_{i=1}^{n} L_i(\mathbf{x}_i).$$

It is clear that L is continuous and locally positive definite over the domain

$$D(L) := \left\{ \mathbf{x} \in \mathbb{R}^{3n} : \sum_{i=1}^{n} \sum_{\substack{j=1,\\ j\neq i}}^{n} Q_{ij}(\mathbf{x}) > 0 \right\}.$$

Note that $L(\mathbf{x}^*) = 0$. However, $\mathbf{x}^* \notin D(L)$ since

$$\sum_{i=1}^{n} \sum_{\substack{j=1,\\ j\neq i}}^{n} Q_{ij}(\mathbf{x}^*) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{\substack{j=1,\\ j\neq i}}^{n} (r_i + r_j)^2 < 0.$$

This is indeed a desirable situation since if $\mathbf{x}^* \in D(L)$, and if at some time $t \geq 0$, we have that $\mathbf{x} = \mathbf{x}^*$, then this implies that the swarm has collapsed onto itself, a biologically impossible situation. As such, we are not interested in the centroid, but in the behavior of our swarm in the vicinity of its centroid.

B. Nonlinear Velocity Controllers for the Swarm

The time-derivative of L along every solution of system (2) is the dot product of the gradient of L, given by,

$$\nabla L = \left(\frac{\partial L}{\partial x_1}, \frac{\partial L}{\partial y_1}, \frac{\partial L}{\partial z_1}, \cdots, \frac{\partial L}{\partial x_n}, \frac{\partial L}{\partial y_n}, \frac{\partial L}{\partial z_n} \right),$$

and the time-derivative of the state vector $\mathbf{x} = (x_1, y_1, z_1 \ldots, x_n, y_n, z_n)$.

Let there be real numbers $\mu_i > 0$, $\nu_i > 0$ and $\eta_i > 0$ such that

$$v_i = -\mu_i \frac{\partial L}{\partial x_i}, \quad w_i = -\nu_i \frac{\partial L}{\partial y_i} \quad \text{and} \quad u_i = -\eta_i \frac{\partial L}{\partial z_i}.$$

Then

$$\begin{aligned} \dot{L}(\mathbf{x}) \;&=\; -\sum_{i=1}^{n} \left[\mu_i \left(\frac{\partial L}{\partial x_i}\right)^2 + \nu_i \left(\frac{\partial L}{\partial y_i}\right)^2 \right. \\ &\qquad \left. + \eta_i \left(\frac{\partial L}{\partial z_i}\right)^2 \right] \\ &=\; -\sum_{i=1}^{n} \left[\frac{v_i^2}{\mu_i} + \frac{w_i^2}{\nu_i} + \frac{u_i^2}{\eta_i} \right] \leq 0, \end{aligned}$$

for all $\mathbf{x} \in D(L)$.

For the ith individual, system (1) therefore becomes

$$\begin{aligned} & x_i'(t) = v_i(t) = v_i(\mathbf{x}(t)) = -\mu_i \frac{\partial L}{\partial x_i}, \\ & y_i'(t) = w_i(t) = w_i(\mathbf{x}(t)) = -\nu_i \frac{\partial L}{\partial y_i}, \\ & z_i'(t) = u_i(t) = u_i(\mathbf{x}(t)) = -\eta_i \frac{\partial L}{\partial z_i}, \\ & x_{i0} = x_i(t_0), y_{i0} = y_i(t_0), z_{i0} = z_i(t_0), \\ & t_0 \geq 0, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \frac{\partial L}{\partial x_i} &= \left(\gamma_i + \sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}(\mathbf{x})} \right) \left(x_i - \frac{1}{n} \sum_{k=1}^{n} x_k \right) \\ &\quad - 2 \sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij} R_i(\mathbf{x})}{Q_{ij}^2(\mathbf{x})} (x_i - x_j), \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial y_i} &= \left(\gamma_i + \sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}(\mathbf{x})} \right) \left(y_i - \frac{1}{n} \sum_{k=1}^{n} y_k \right) \\ &\quad - 2 \sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij} R_i(\mathbf{x})}{Q_{ij}^2(\mathbf{x})} (y_i - y_j), \end{aligned}$$

and

$$\frac{\partial L}{\partial z_i} = \left(\gamma_i + \sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij}}{Q_{ij}(\mathbf{x})} \right) \left(z_i - \frac{1}{n}\sum_{k=1}^{n} z_k \right) - 2\sum_{\substack{j=1,\\ j\neq i}}^{n} \frac{\beta_{ij} R_i(\mathbf{x})}{Q_{ij}^2(\mathbf{x})}(z_i - z_j).$$

Define the $n \times n$ diagonal matrix

$$H = \text{diag}(\underbrace{\mu_1, \nu_1, \eta_1 \ldots, \mu_n, \nu_n, \eta_n}_{3n \text{ elements}}).$$

Then system (2) becomes the gradient system

$$\dot{\mathbf{x}} = \mathbf{G}(\mathbf{x}) = -H\,(\nabla L(\mathbf{x})), \quad \mathbf{x}_0 := \mathbf{x}(t_0), \quad t_0 \geq 0, \tag{5}$$

the ith term of which is given by (4). It is clear that $\mathbf{G} \in C[D(L), \mathbb{R}^{32n}]$.

V. Practical Stability Analysis

In this section, we shall prove the practical stability of system (5), using the method by Lakshmikantham, Leela and Martynyuk [17].

Theorem 2: System (5) is uniformly practically stable.

Proof. Since

$$\dot{L}(\mathbf{x}(t)) \leq 0,$$

we have

$$0 \leq L(\mathbf{x}(t)) \leq L(\mathbf{x}(t_0)) \;\; \forall \;\; t \geq t_0 \geq 0. \tag{6}$$

Accordingly, for comparative analysis, it is sufficient to consider the practical stability of the scalar differential equation

$$h'(t) = 0, \;\; h(t_0) =: h_0, \;\; t_0 \geq 0. \tag{7}$$

The solution is

$$h(t; t_0, h_0) = h_0,$$

so that relative to every point $h^* \in \mathbb{R}$, we have

$$h(t; t_0, h_0 - h^*) = h_0 - h^*,$$

so that for any given number $P_0 > 0$,

$$|h(t; t_0, h_0 - h^*)| \leq |h_0 - h^*| + P_0.$$

We shall next show that by applying Theorem 1, we can simultaneously derive the explicit form of $P_0 > 0$, with which it is easy to see that (S2) holds for equation (7) if

$$A = A(\lambda) := \lambda + P_0.$$

To apply Theorem 1, we restrict our domain to $D(L)$ over which we see that $L \in C[D(L), \mathbb{R}_+]$, and note that L is locally Lipschitzian in $D(L)$ since $dL/dt \leq 0$ in $D(L)$. Re-defining $S(\rho)$ as $S(\rho) = \{\mathbf{x} \in D(L) : \|\mathbf{x} - \mathbf{x}^*\| < \rho\}$, we get

$$S(A) = \{\mathbf{x} \in D(L) : \|\mathbf{x} - \mathbf{x}^*\| < \lambda + P_0\}.$$

Recalling that $\gamma_i > 0$, $i \in \mathbb{N}$, we let

$$\gamma_{\min} := \min_{i\in\mathbb{N}} \gamma_i \quad \text{and } \gamma_{\max} := \max_{i\in\mathbb{N}} \gamma_i.$$

Further, let

$$b_1(\|\mathbf{x} - \mathbf{x}^*\|) := \frac{1}{2}\gamma_{\min}\|\mathbf{x} - \mathbf{x}^*\|^2$$

and

$$b_2(\|\mathbf{x} - \mathbf{x}^*\|) := \frac{1}{2}\gamma_{\max}\left[\|\mathbf{x} - \mathbf{x}^*\| + L(\mathbf{x}_0)\right]^2,$$

noting that $b_1, b_2 \in K$. Then assuming $P_0 > 0$ we easily see that with (6) we have

$$b_1(\|\mathbf{x} - \mathbf{x}^*\|) \leq L(\mathbf{x}) \leq b_2(\|\mathbf{x} - \mathbf{x}^*\|),$$

for $\mathbf{x} \in S(A)$ since

$$\begin{aligned}\sum_{i=1}^{n} R_i(\mathbf{x}) &= \frac{1}{2}\sum_{i=1}^{n}\left[\left(x_i - \frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 + \left(y_i - \frac{1}{n}\sum_{i=1}^{n} y_i\right)^2 + \left(z_i - \frac{1}{n}\sum_{i=1}^{n} z_i\right)^2\right] \\ &= \frac{1}{2}\|\mathbf{x} - \mathbf{x}^*\|^2.\end{aligned}$$

Indeed, the inequality $b_2(\lambda) < b_1(A)$ yields

$$\frac{1}{2}\gamma_{\max}\left[\lambda + L(\mathbf{x}_0)\right]^2 < \frac{1}{2}\gamma_{\min}[\lambda + P_0]^2,$$

which holds if we choose

$$P_0 > \left[\left(\sqrt{\frac{\gamma_{\max}}{\gamma_{\min}}} - 1\right) + \sqrt{\frac{\gamma_{\max}}{\gamma_{\min}}} L(\mathbf{x}_0)\right].$$

Since $\gamma_{\max}/\gamma_{\min} \geq 1$ for any $\gamma_{\max}, \gamma_{\min} > 0$, and because of (6), it is clear that P_0 exists and $P_0 > 0$. Thus, with $q(t, z) \equiv 0$, we conclude the proof of Theorem 2.

VI. Computer Simulations

As part of the article, computer simulations were done using *"Mathematica Software"* to show the effectiveness of the proposed velocity control laws of the swarm model. The RK4 method was used to numerically integrate system (5) to confirm the emergent behavior of a sufficiently large number of individuals governed by the system. Extensive computer simulations show that for a sufficiently large number of individuals the proposed model (5) generates collective behaviors, some of which are similar to those reported in literature. Indeed, we shall utilize the same descriptions of the behaviors. However, in our case, we obtain them as a direct result of manipulating the cohesion parameters ($\gamma_i > 0, i \in \mathbb{N}$), which are a measure of the strength of attraction between an individual i and the swarm centroid, the coupling parameters ($\beta_{ij} > 0$, $i, j \in \mathbb{N}$, $i \neq j$), which are a measure of the strength of the interaction between individual i and individual j, and the convergence parameters ($\alpha_i^s > 0, s = 1, 2; i \in \mathbb{N}$), which are measures of rate convergence of the ith individual to the swarm centroid.

A. Scenario 1: Leader-following Behavior

For our first example, we expect an individual with a low cohesion parameter to be further away from a more compactly arranged group of individuals with similar but higher cohesion parameters. Because of the effects of the attraction and the inter-individual collision-avoidance functions, the individual with the lower cohesion parameter can either be following or leading the group. In this example, the cohesion parameters ($\gamma_i > 0, i \in \mathbb{N}$) are randomized between 0.01 and 1, and the coupling and convergence parameters are fixed. This means that some boids can be further away from a more compact group of individuals. Fig 1 shows some boids following a compact swarm in an aligned manner, and an outermost boid leading the swarm almost along the path of the centroid. We can assume that this is a leader-follower behavior.

Recently Justh and Krishnaprasad [18] and Morgan and Schwartz [19] proposed an individual-based continuum mechanics approach that utilizes the Frenet-Serret equations of motion to describe the position and orientation of interacting individuals in a swarm. Their models can be used to designate and control a leader, which then leads the swarm. The dynamics of their models – which result in an emergent behavior – depend on the initial conditions. Our approach differs in that the leader emerges from the swarm, and our system dynamics depend only on the system parameters, not on the initial conditions.

B. Scenario 2: A Spiral-Like Behavior

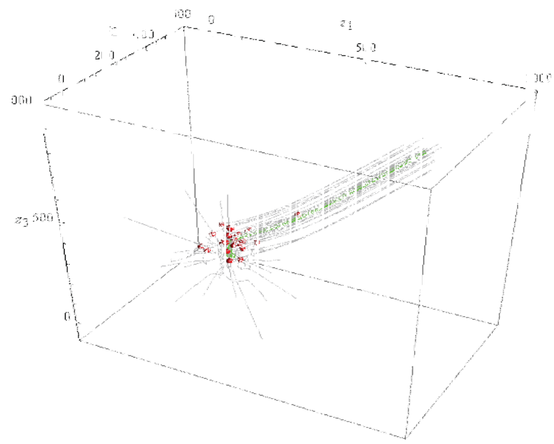
In our second example, we encounter an interesting behavior that is very similar to a spiral behavior. Using our model, the spiral behavior can be induced by allocating large randomized values of the coupling parameter to each individual. The simulations shows a cohesive group with individuals hovering about the centroid in a spiral fashion. As they change positions, the centroid traces out spiral curves.

From nature, we see that many millipedes defend themselves by rolling their bodies up into a ball or spiral. This behavior protects the legs and delicate underside of the animal, leaving only the hard plates of the body segments exposed [2]. At the beginning phase, the swarm members gradually aggregate and form a cohesive cluster. Then, they continuously move in the same direction as a group, and eventually evolve into a spiral motion.

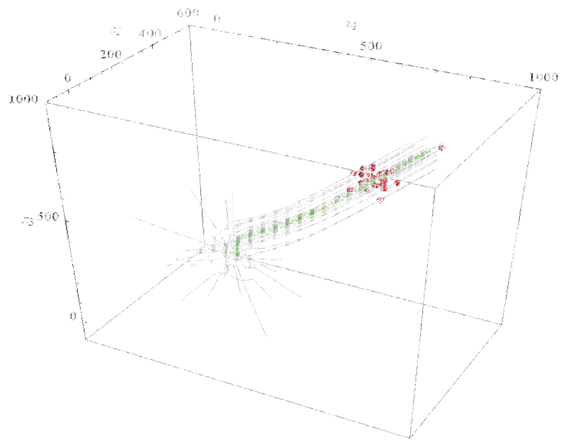
VII. Conclusion

This paper introduces a set of continuous, time-invariant velocity control laws, derived from the Lyapunov-based control scheme to show the emergent behavior arising from a swarm of boids and in general provides a solution to the motion planning and control of boids. The different emerging behaviors were a result of varying the control parameters in each of the case. The nonlinear control laws presented in this paper guarantees practical stability of the system which has been proved using the Lakshmikantham, Leela and Martynyuk

(a) Initial state of the swarm ($t = 0$).

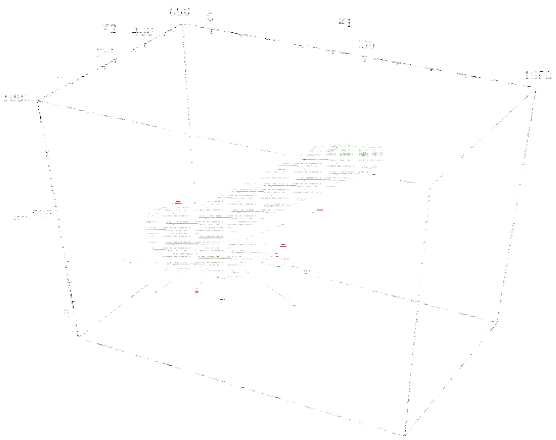


(b) State of the swarm at $t = 7$ units.

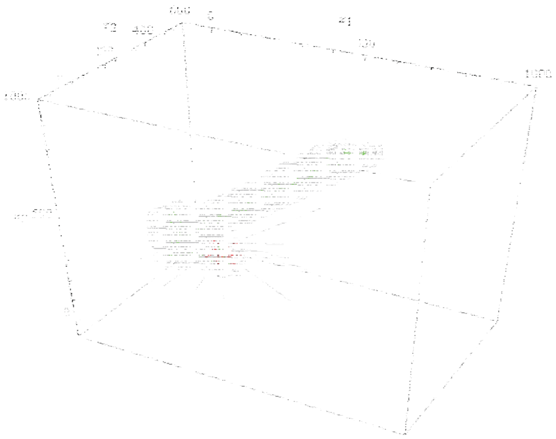


(c) State of the swarm at $t = 50$ units.

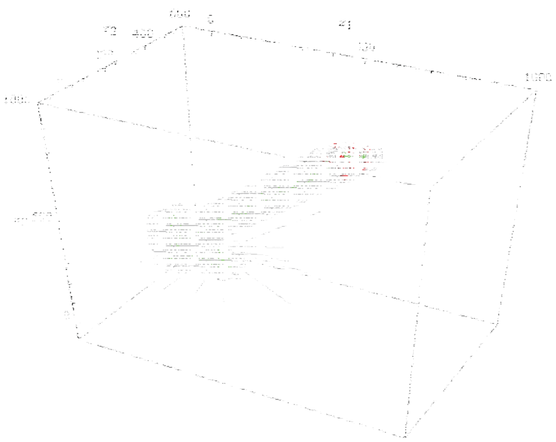
Fig. 1. Leader-following behavior. There are $n = 30$ individuals (shown in red), each with bin size 10, randomly positioned at the initial time $t = 0$. The parameters are $\alpha_i^s = 1$, $s = 1, 2, 3$, and $\beta_{ij} = 30$. The cohesion parameters γ_i are randomized between 0.01 and 1. The axes are $z_1(t)$, $z_2(t)$ and $z_3(t)$, respectively, for each individual i at time $t \geq 0$. The grey lines show the trajectories and of the individuals. The path of the centroid is given by the green line.



(a) Initial state of the swarm ($t = 0$).



(b) State of the swarm at $t = 10$ units.



(c) State of the swarm at $t = 100$ units.

Fig. 2. A spiral-like behavior. There are $n = 20$ individuals (shown in red), each with bin size 10, randomly positioned at the initial time $t = 0$. The parameters are $\alpha_i^s = 5$, $s = 1, 2, 3$, and $\gamma_i = 5$. The coupling parameters β_{ij} are randomized between and including 300 and 500. The axes are $z_1(t)$, $z_2(t)$ and $z_3(t)$, respectively, for each individual i at time $t \geq 0$. The grey lines shows the trajectories of the individuals. The path of the centroid is shown thick in green. The swarm is cohesive throughout.

method [20]. The efficiency of the control laws have been demonstrated through interesting simulations arising from the emergent behaviors of the swarm. This showed that the swarm model is a gradient system that is practically stable about the centroid.

Future work will attempt to extend the results of this paper and focus on the behavior of the swarms in the presence of obstacles.

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